TMDs in
in the Bag model

Ishara Fernando

University of Virginia

Reference:
https://arxiv.org/abs/1001.5467

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Background information of this work

- Considered SIDIS process
- Gaussian Ansatz for TMDs
- Only focused on Time-reversal (T) – even TMDs

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Definitions

Light-front quark correlator

$$\phi(x,\vec{p}_T)_{ij} = \int \frac{\mathrm{d}z^-\mathrm{d}^2\vec{z}_T}{(2\pi)^3}\, e^{ipz}\, \langle N(P,S)|\bar{\psi}_j(0)\,\mathcal{W}(0,\ z;\ \text{path})\,\psi_i(z)|N(P,S)\rangle\Big|_{z^+=0,\,p^+=xP^+}$$

Projections of the quark correlator

$$\frac{1}{2}\,\mathrm{tr}\left[\gamma^+\,\phi(x,\vec{p}_T)\right] = f_1 - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N}\, f_{1T}^\perp$$

$$\frac{1}{2}\,\mathrm{tr}\left[\gamma^+\gamma_5\,\phi(x,\vec{p}_T)\right] = S_L\, g_1 + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N}\, g_{1T}^\perp$$

$$\frac{1}{2}\,\mathrm{tr}\left[i\sigma^{j+}\gamma_5\,\phi(x,\vec{p}_T)\right] = S_T^j\, h_1 + S_L\,\frac{p_T^j}{M_N}\, h_{1L}^\perp + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2}\, h_{1T}^\perp + \frac{\varepsilon^{jk}p_T^k}{M_N}\, h_1^\perp$$

+ sub-leading terms

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+ sub-leading terms

Parton distribution functions: $j^a(x) = \int \mathrm{d}^2\vec{p}_T\, j^a(x,\vec{p}_T^{\,2})$

$$j = f_1,\, g_1,\, h_1,\, e,\, ,g_T,\, h_L$$

And (1)-moments:

$$j^{(1)q}(x,k_\perp) = \frac{k_\perp^2}{2M_N^2}\, j^q(x,k_\perp)\,, \quad j^{(1)q}(x) = \int \mathrm{d}^2k_\perp\, \frac{k_\perp^2}{2M_N^2}\, j^q(x,k_\perp)$$

(1/2)-moments $\quad f_1^{(1/2)q}(x) = \int \mathrm{d}^2k_\perp\, \frac{k_\perp}{2M_N}\, f_1^q(x,k_\perp)$

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Definitions

Quark field defined in the MIT bag model:

$$\Psi_\alpha(\vec{x},t) = \sum_{n>0,\kappa=\pm1,m=\pm1/2} N(n\kappa)\{b_\alpha(n\kappa m)\psi_{n\kappa jm}(\vec{x},t) + d_\alpha^\dagger(n\kappa m)\psi_{-n-\kappa jm}(\vec{x},t)\}$$

Where,

$$\psi_{n,-1,\frac{1}{2}m}(\vec{x},t) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} i j_0(\frac{\omega_{n,-1}|\vec{x}|}{R_0})\chi_m \\ -\vec{\sigma}\cdot\hat{x}\; j_1(\frac{\omega_{n,-1}|\vec{x}|}{R_0})\chi_m \end{pmatrix} e^{-i\omega_{n,-1}t/R_0}$$

For the lowest mode, we have $n=1$, $\kappa=-1$, and $\omega_{1,-1}\approx 2.04$ denoted as $\omega\equiv\omega_{1,-1}$ momentum space wave function for the lowest mode,

$$\varphi_m(\vec{k}) = i\sqrt{4\pi}NR_0^3\begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k}\; t_1(k)\chi_m \end{pmatrix}$$

$$N = \left(\frac{\omega^3}{2R_0^3(\omega-1)\sin^2\omega}\right)^{1/2}$$

The two functions t_i, $i=0,1$ are defined as

$$t_i(k) = \int_0^1 u^2 du j_i(ukR_0) j_i(u\omega)$$

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Definitions

$$A = \frac{16\omega^4}{\pi^2(\omega-1)j_0^2(\omega)\,M_N^2}\,, \quad k = \sqrt{k_z^2 + k_\perp^2}\,, \quad k_z = xM_N - \omega/R_0\,, \quad \widehat{k}_z = \frac{k_z}{k}\,, \quad \widehat{M}_N = \frac{M_N}{k}$$

bag radius is fixed such that $R_0 M_N = 4\omega$

$$N_u = 2\,, \quad N_d = 1\,, \quad P_u = \frac{4}{3}\,, \quad P_d = -\frac{1}{3}$$

Assumed SU(6) spin-flavor symmetry of the proton wave function,

- Spin-independent TMDs for a give flavor = flavor factor (N) x flavor less term
- Spin-dependent TMDs for a give flavor = spin-flavor factor (P) x flavor less term

Since there are no explicit gluon degrees of freedom, T-odd TMDs vanish in this model

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Results

T-even leading twist TMDs are given by

$$
\begin{aligned}
f_1^q(x,k_\perp) &= N_q A\left[t_0^2 + 2\widehat{k}_z\, t_0 t_1 + t_1^2\right] \\
g_1^q(x,k_\perp) &= P_q\, A\left[t_0^2 + 2\widehat{k}_z\, t_0 t_1 + (2\widehat{k}_z^2 - 1)\, t_1^2\right] \\
h_1^q(x,k_\perp) &= P_q\, A\left[t_0^2 + 2\widehat{k}_z\, t_0 t_1 + \widehat{k}_z^2\, t_1^2\right] \\
g_{1T}^{\perp q}(x,k_\perp) &= P_q\, A\left[\;2\widehat{M}_N (t_0 t_1 + \widehat{k}_z\, t_1^2)\right] \\
h_{1L}^{\perp q}(x,k_\perp) &= P_q\, A\left[-2\widehat{M}_N (t_0 t_1 + \widehat{k}_z\, t_1^2)\right] \\
h_{1T}^{\perp q}(x,k_\perp) &= P_q\, A\left[-2\widehat{M}_N^2\, t_1^2\right]
\end{aligned}
$$

"Lorentz-invariance relations" (LIRs)

$$
\begin{aligned}
g_T(x) &\overset{\text{LIR}}{=} g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)}(x) \\
h_L(x) &\overset{\text{LIR}}{=} h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x) \\
h_T(x) &\overset{\text{LIR}}{=} -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)}(x)\,, \\
g_L^{\perp}(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)}(x) &\overset{\text{LIR}}{=} 0\,, \\
h_T(x,p_T) - h_T^{\perp}(x,p_T) &\overset{\text{LIR}}{=} h_{1L}^{\perp}(x,p_T)\,,
\end{aligned}
$$

Certain relations among TMDs must be valid in any quark model of the nucleon lacking gluon degrees of freedom

"no-gluon models" the absence of the Wilson-link

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Results

Linear relations in bag model:

$$\mathcal{D}^q\, f_1^q(x,k_\perp) + g_1^q(x,k_\perp) = 2h_1^q(x,k_\perp)$$
$$\mathcal{D}^q\, e^q(x,k_\perp) + h_L^q(x,k_\perp) = 2g_T^q(x,k_\perp)$$
$$\mathcal{D}^q\, f^{\perp q}(x,k_\perp) = h_T^{\perp q}(x,k_\perp)$$

$$g_{1T}^{\perp q}(x,k_\perp) = -\,h_{1L}^{\perp q}(x,k_\perp)$$
$$g_T^{\perp q}(x,k_\perp) = -\,h_{1T}^{\perp q}(x,k_\perp)$$
$$g_L^{\perp q}(x,k_\perp) = -\,h_T^q(x,k_\perp)$$

$$g_1^q(x,k_\perp) - h_1^q(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp)$$
$$g_T^q(x,k_\perp) - h_L^q(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp)$$
$$h_T^q(x,k_\perp) - h_T^{\perp q}(x,k_\perp) = h_{1L}^{\perp q}(x,k_\perp)$$

Dilution factor

$$\mathcal{D}^q = \frac{P_q}{N_q}$$

Highlighted relation:

$$g_1^q(x) - h_1^q(x) = g_T^q(x) - h_L^q(x)$$

- Involves only collinear PDFs
- For the 1st Mellin moments, this relation is valid model independently
- Useful to compare OAM from different models which satisfy this relation

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Orbital Angular Momentum

In the absence of gauge-fields:

$$\hat{L}^i_q(0,z) = \bar{\psi}_q(0)\varepsilon^{ikl}\hat{r}^k\hat{p}^l\psi_q(z)$$

where $\hat{r}^k = i\,\frac{\partial}{\partial p^k}$ and $\hat{p}^l = p^l$

The following quantity is defined following the general definition of matrix element(s)

$$L^j_q(x,p_T) = \int \frac{\mathrm{d}z^-\mathrm{d}^2\vec{z}_T}{(2\pi)^3}\, e^{ipz}\, \langle N(P,S^3)|\hat{L}^i_q(0,z)|N(P,S^3)\rangle\Big|_{z^+=0,\, p^+=xP^+}$$

In a longitudinally polarized nucleon, $L^3_q(x,p_T)\mathrm{d}^2\vec{p}_T\mathrm{d}x$ tells how much OAM of a quark
Which carries longitudinal momentum fraction 'x' and transverse momentum pT,
Contributes to the nucleon spin.

In the bag model: $L^3_q(x,p_T) = (-1)\, h^{\perp(1)q}_{1T}(x,p_T)$

Total Angular momentum J^3_q = S^3_q + L^3_q

$$S^3_q = \frac{1}{2}\int \mathrm{d}x g^q_1(x)$$

$$L^3_q = \int \mathrm{d}x \int \mathrm{d}^2\vec{p}_T L^3_q(x,p_T)$$

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Orbital Angular Momentum

$$
\begin{aligned}
2J_q^3 &= \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \Big[g_1^q(x,k_\perp) - 2\, h_{1T}^{\perp(1)q}(x,k_\perp) \Big] \\
&= P_q \frac{A}{M_N} \int \mathrm{d}^3 k \Big[t_0^2 + 2\widehat{k}_z\, t_0 t_1 + (2\widehat{k}_z^2 - 1 + 2\frac{k_\perp^2}{k^2})\, t_1^2 \Big] \\
&= P_q \frac{A}{M_N} \int \mathrm{d}^3 k \big[t_0^2 + t_1^2 \big] \\
&= P_q
\end{aligned}
$$

$$N_u = 2\,, \quad N_d = 1\,, \quad P_u = \frac{4}{3}\,, \quad P_d = -\frac{1}{3}$$

$$J_u^3 + J_d^3 = \frac{1}{2}$$

This supports SU(6) light-cone quark model result $L_q^3 = (-1) \int \mathrm{d}x\, h_{1T}^{\perp(1)q}(x)$

In the bag model for SU(6) symmetry,
for unpolarized TMDs the d-quark distributions are factor 2 smaller than the u-quark distributions.
In the case of the polarized TMDs, the d-quark distributions are factor 4 smaller and have opposite sign compared to the u-quark distributions.

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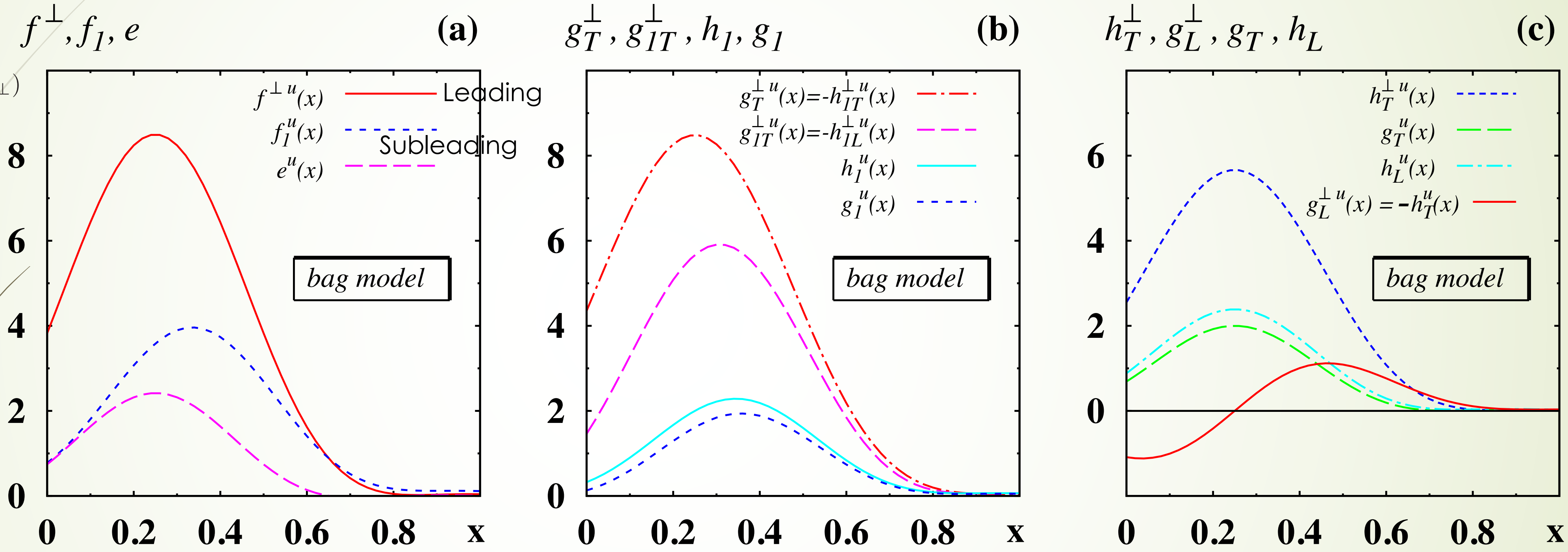
Results: Integrated TMDs

$f^{\perp q}(x) = \int d^2k_\perp f^{\perp q}(x, k_\perp)$



These are predicted behavior of unpolarized (a) and polarized (b & c) of Integrated TMDs for u-quarks

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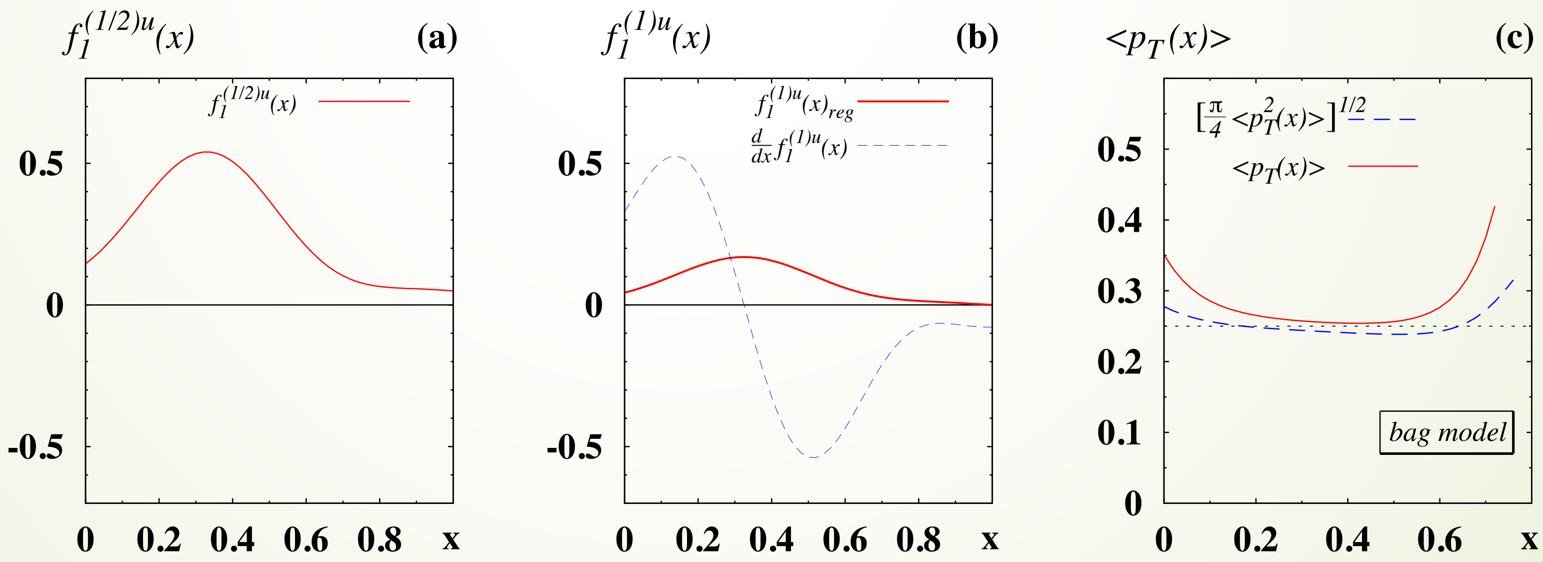
Results: Transverse momentum of unpolarized quarks

for a generic TMD $j^q(x, k_\perp)$

$$\langle p_T \rangle = \frac{\int \mathrm{d}x \int \mathrm{d}^2k_\perp \; k_\perp \, j^q(x,k_\perp)}{\int \mathrm{d}x \int \mathrm{d}^2k_\perp \; j(x,k_\perp)} \, , \quad \langle p_T^2 \rangle = \frac{\int \mathrm{d}x \int \mathrm{d}^2k_\perp \; k_\perp^2 \, j^q(x,k_\perp)}{\int \mathrm{d}x \int \mathrm{d}^2k_\perp \; j(x,k_\perp)}$$

$$\langle p_T(x) \rangle = 2M_N \, \frac{f_1^{(1/2)q}(x)}{f_1^q(x)} \qquad \langle p_T^2(x) \rangle = 2M_N^2 \, \frac{f_1^{(1)q}(x)_{reg}}{f_1^q(x)}$$



A similar result From light-cone Constituent model

(1)-moment is computed with a finite cutoff $\Lambda_{\mathrm{cut}} \gg M_N$

$\langle p_T(x) \rangle \approx 0.25\,\mathrm{GeV}$ for $0.2 \lesssim x \lesssim 0.5$

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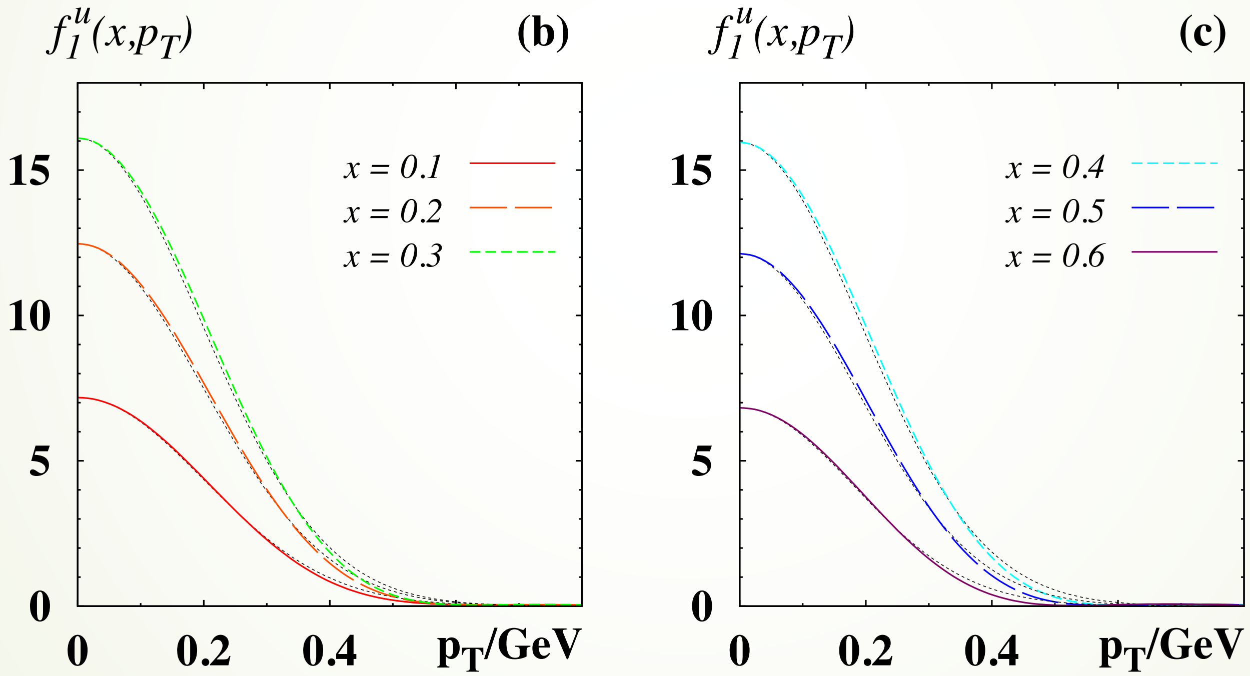
The Gaussian model in the Bag model

$$f_1^q(x, p_T) = f_1^q(x)\ \exp(-p_T^2/\langle p_T^2(x)\rangle_{\text{Gauss}})/(\pi\langle p_T^2(x)\rangle_{\text{Gauss}})$$



Solid-line

$$\langle p_T^2(x)\rangle = 2M_N^2\ \frac{f_1^{(1)q}(x)_{reg}}{f_1^q(x)}$$

Dashed-line

$$\langle p_T^2(x)\rangle_{\text{Gauss}} = \pi\ \frac{f_1^q(x,0)}{f_1^q(x)}$$

Also, other TMD pT dependence were examined in this work

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Thank you