TMDs in diquark spectator model

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References:
arXiv:hep-ph/9704335,arXiv:hep-ph/0209085, arXiv:hep-ph/0201296,
arXiv:hep-ph/0310319,arXiv:0807.0323,
https://inspirehep.net/files/2dc0be6b78d490f716d88a17aa689ff3

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Cross-section to TMDs

$$\frac{d\sigma}{dQ^2\, d^2\mathbf{q_T}} = \sum_{i,j} H_{ij}(Q) \int_0^1 d\xi_a d\xi_b \int d^2\mathbf{b_T} e^{i\mathbf{b_T}.\mathbf{q_T}} f_{i/P_a}(\xi_a, \mathbf{b_T}) f_{j/P_b}(\xi_b, \mathbf{b_T})$$
$$\times \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}, \frac{\mathbf{q_T}}{Q}\right)\right]$$

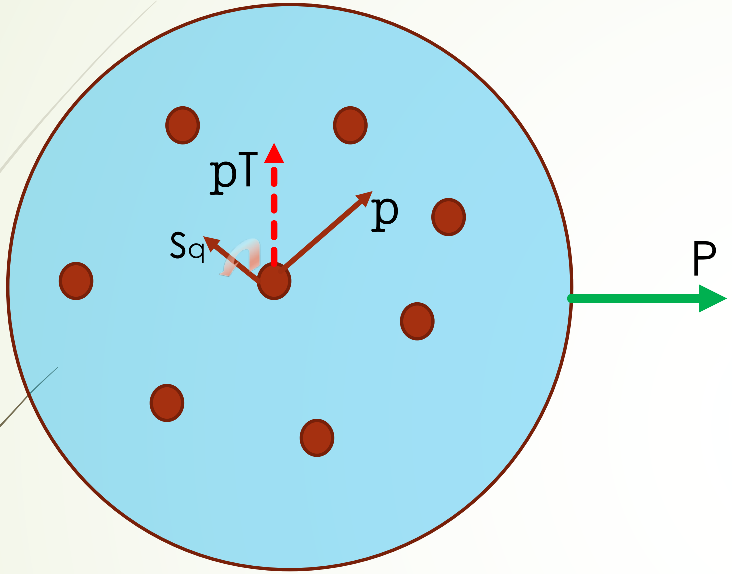
Momentum-space version of $f_{i/P_a}(\xi_a, \mathbf{b_T})$ (or $f_{j/P_b}(\xi_b, \mathbf{b_T})$) was decomposed into 8 leading TMD PDFs

$$\Phi(x, k_T, P, S) = f_1(x, k_T^2)\frac{\not{P}}{2} + \frac{h_{1T}(x, k_T^2)}{4}\gamma_5[\not{S}_T, \not{P}] + \frac{S_L}{2} g_{1L}(x, k_T^2)\gamma_5 \not{P} + \frac{k_T.S_T}{2M} g_{1T}(x, k_T^2)\gamma_5 \not{P}$$
$$+ S_L h_{1L}^{\perp}(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M} + \frac{k_T.S_T}{2M} h_{1T}^{\perp}(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M}$$
$$+ i h_1^{\perp}(x, k_T^2)\frac{[\not{k}_T, \not{P}]}{4M} - \frac{\epsilon_T^{k_T S_T}}{4M} f_{1T}^{\perp}(x, k_T^2)\not{P}$$

+ 16 Terms (Twist 3) + 8 Terms (Twist 4) + ...

M. Constantinou et al arXiv: 2006.08636
Bomhof &Mulders et al arXiv:0709.1390
Lorce et al arXiv:1411.2550

Momenta in light-cone coordinates



A generic 4-vector

$$a = [a^-, a^+, \boldsymbol{a}_T]$$

Proton momentum (with no transverse component

$$P = \left[\frac{M^2}{2P^+}, P^+, \mathbf{0}\right]$$

Quark momentum

$$p = \left[\frac{p^2 + \boldsymbol{p}_T^2}{2xP^+}, xP^+, \boldsymbol{p}_T\right]$$

Quark correlator

ξ – Ligh-cone space-time coordinates

$$\Phi(x, \boldsymbol{p}_T; S) = \int \frac{d\xi^- d\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi} \langle P, S|\bar{\psi}(0)\,\mathcal{U}_{[0,\xi]}\,\psi(\xi)|P, S\rangle\Big|_{\xi^+=0}$$

For antiquark → $\psi \to \psi^c = C\overline{\psi}^T$

$$U_{[0,\xi]} = \mathcal{P}\, e^{-ig\int_0^\xi dw\cdot A(w)}$$

Gauge link path for SIDIS process:

$$[0,\xi] \equiv (0,0,\mathbf{0}_T) \to (0,\infty,\mathbf{0}_T) \to (0,\infty,\infty_T) \to (0,\infty,\boldsymbol{\xi}_T) \to (0,\xi^-,\boldsymbol{\xi}_T)$$

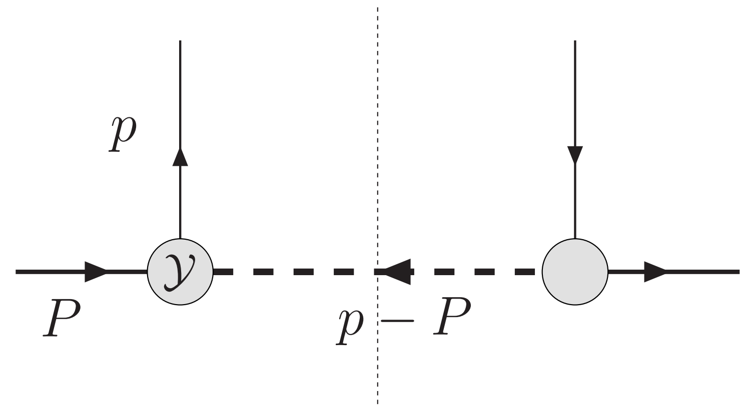
Gauge link path for Drell-Yan process:
Runs in the opposite direction via $\infty \to -\infty$

Gauge link (Wilson line):
Which connects two different space-time points by all possible gluon field paths 'A', coupling to qaurks with coupling constant 'g'

Sign difference in SIDIS and DY

The TMDs can be written in-terms of these correlators…

diquark spectator model for T-even terms



After inserting complete set of intermediate states and truncating the summation to a single on-shell spectator state with mass 'Mx' gives,

$$\Phi(x, \boldsymbol{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(0)}(S)\Big|_{p^2=\tau(x,\boldsymbol{p}_T)}$$

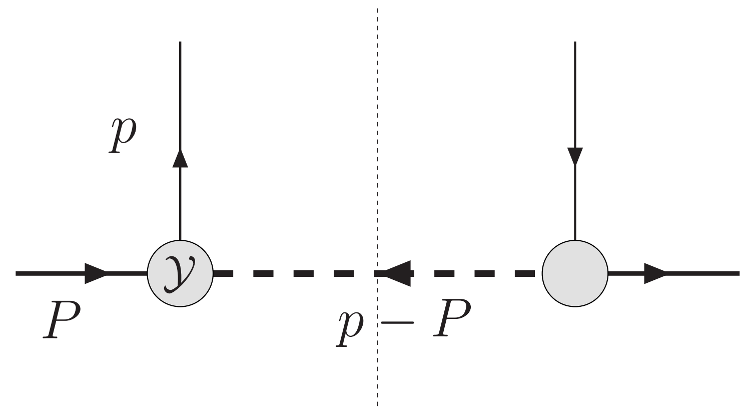
p is the momentum of the active quark, m its mass, and the on-shell condition $(P-p)^2 = M_X^2$

Therefore, quark-off shell condition is, $$p^2 \equiv \tau(x, \boldsymbol{p}_T) = -\frac{\boldsymbol{p}_T^2 + L_X^2(m^2)}{1-x} + m^2$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M^2$$

Notice that the Proton (nucleon) can couple to a quark (active) and to a spectator diquark with spin 0 or 1 as well as isospin 0 and 1.

Diquark spectator model for T-even functions



$$\Phi(x, \boldsymbol{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(0)}(S)\Big|_{p^2=\tau(x,\boldsymbol{p}_T)}$$

Therefore, tree-level scattering amplitude:

$$\mathcal{M}^{(0)}(S) = \langle P-p|\psi(0)|P,S\rangle = \begin{cases} \dfrac{i}{\not p - m}\, \mathcal{Y}_s\, U(P,S) & \text{scalar diquark,} \\ \dfrac{i}{\not p - m}\, \varepsilon^*_\mu(P-p,\lambda_a)\, \mathcal{Y}^\mu_a\, U(P,S) & \text{axial-vector diquark} \end{cases}$$

4-vector polarization of the spin-1 vector diquark with momentum $P-p$ and helicity states λ_a

When summing over all possible polarization states:

$$d^{\mu\nu} = \sum_{\lambda_a} \varepsilon^{*\mu}_{(\lambda_a)}\, \varepsilon^{\nu}_{(\lambda_a)}$$

$$d^{\mu\nu}(P-p) = \begin{cases} -g^{\mu\nu} + \dfrac{(P-p)^\mu n_-^\nu + (P-p)^\nu n_-^\mu}{(P-p)\cdot n_-} - \dfrac{M_a^2}{[(P-p)\cdot n_-]^2}\, n_-^\mu\, n_-^\nu \\ -g^{\mu\nu} + \dfrac{(P-p)^\mu\,(P-p)^\nu}{M_a^2} \\ -g^{\mu\nu} + \dfrac{P^\mu\, P^\nu}{M_a^2} \\ -g^{\mu\nu} \end{cases}$$

Different choices for different diquark models

Nucleon-quark-diquark vetex

$$\mathcal{Y}_s = i g_s(p^2)\, \mathbb{1} \qquad \mathcal{Y}^\mu_a = i\frac{g_a(p^2)}{\sqrt{2}}\, \gamma^\mu\, \gamma_5$$

$$g_X(p^2) = \begin{cases} g_X^{p.l.} & \text{point-like,} \\ g_X^{dip}\, \dfrac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2} & \text{dipolar,} \\ g_X^{exp}\, e^{(p^2-m^2)/\Lambda_X^2} & \text{exponential} \end{cases}$$

Free parameters

Some results for T-even functions

Unpolarized quark distribution:

$$f_1(x,\boldsymbol{p}_T) = \frac{1}{4}\,\mathrm{Tr}\left[\left(\Phi(x,\boldsymbol{p}_T,S)+\Phi(x,\boldsymbol{p}_T,-S)\right)\gamma^+\right] + \mathrm{h.c.}$$
$$= \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S)+\overline{\mathcal{M}}^{(0)}(-S)\,\mathcal{M}^{(0)}(-S)\right)\gamma^+\right] + \mathrm{h.c.}$$

Using the choices (mentioned in the previous slide) for a dipolar form factor,

$$f_1^{q(s)}(x,\boldsymbol{p}_T) = \frac{g_s^2}{(2\pi)^3}\,\frac{[(m+xM)^2+\boldsymbol{p}_T^2]\,(1-x)^3}{2\,[\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2)]^4}$$
$$f_1^{q(a)}(x,\boldsymbol{p}_T) = \frac{g_a^2}{(2\pi)^3}\,\frac{[\boldsymbol{p}_T^2\,(1+x^2)+(m+xM)^2\,(1-x)^2]\,(1-x)}{2\,[\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2)]^4}$$

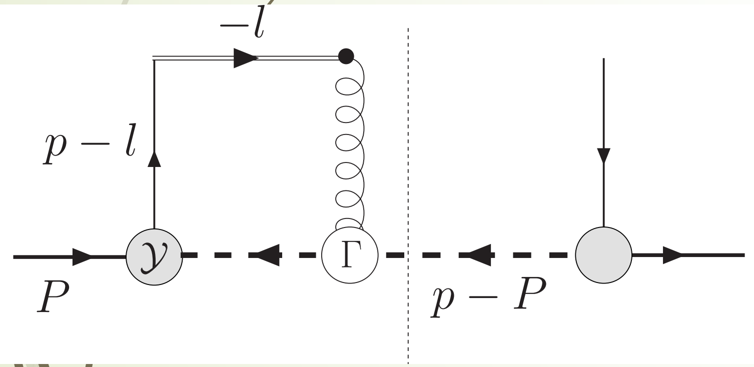
Similarly the results for the rest of T-even functions can be obtained….

Diquark spectator model for T-odd functions

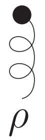
$$\frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x, \boldsymbol{p}_T^2) = -\frac{1}{4} \mathrm{Tr}\left[\left(\Phi(x, \boldsymbol{p}_T, S) - \Phi(x, \boldsymbol{p}_T, -S) \right) \gamma^+ \right] + \mathrm{h.c.}\ ,$$

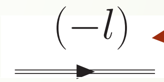
$$\frac{\varepsilon_T^{ij} p_{Tj}}{M} h_1^{\perp}(x, \boldsymbol{p}_T^2) = \frac{1}{4} \mathrm{Tr}\left[\left(\Phi(x, \boldsymbol{p}_T, S) + \Phi(x, \boldsymbol{p}_T, -S) \right) i\sigma^{i+} \gamma_5 \right] + \mathrm{h.c.}$$

At tree-level, these functions vanish because no interaction between the active quark and diquark (no interference between the scattering amplitudes); but one can generate such a non-zero contribution by considering the interference between tree-level scattering amplitude and One-gluon-exchange between active quark and diquark. [corresponds to leading-twist one-gluon-exchange operator of the gauge-link]



single-gluon-exchange scattering amplitude in eikonal approximation

Feynman Rules: 

$= -ie_c\, n_-^{\rho}$ 

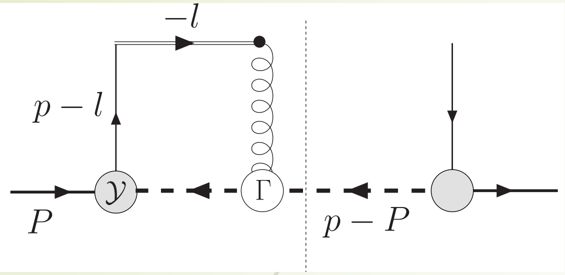
$= \frac{i}{-l^+ + i\epsilon}$

This sign is for SIDIS

$$\Phi^{(1)}(x, \boldsymbol{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \left(\overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(1)}(S) + \overline{\mathcal{M}}^{(1)}(S)\, \mathcal{M}^{(0)}(S) \right) \Big|_{p^2 = \tau(x, \boldsymbol{p}_T)}$$

$$\mathcal{M}^{(1)}(S) = \begin{cases} -\int \frac{d^4 l}{(2\pi)^4} \frac{ie_c\, \Gamma_{s\,\rho}\, n_-^{\rho} (\not{p} - \not{l} + m)\, \mathcal{Y}_s\, U(P,S)}{(D_1 + i\varepsilon)(D_2 - i\varepsilon)(D_3 + i\varepsilon)(D_4 + i\varepsilon)} & \text{scalar diquark,} \\ -\int \frac{d^4 l}{(2\pi)^4} \frac{ie_c\, \varepsilon_{\sigma}^{*}(P-p, \lambda_a)\, \Gamma_{a\,\rho}^{\nu\sigma}\, n_-^{\rho} (\not{p} - \not{l} + m)\, d_{\mu\nu}(p-l-P)\, \mathcal{Y}_a^{\mu}\, U(P,S)}{(D_1 + i\varepsilon)(D_2 - i\varepsilon)(D_3 + i\varepsilon)(D_4 + i\varepsilon)} & \text{axial-vector diquark} \end{cases}$$

Diquark spectator model for T-odd functions



$$D_1 = l^2 - m_g^2,$$
$$D_2 = l^+,$$
$$D_3 = (p-l)^2 - m^2,$$
$$D_4 = (P-p+l)^2 - M_X^2$$

$$\Phi^{(1)}(x, \boldsymbol{p}_T, S) \sim \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \left(\overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(1)}(S) + \overline{\mathcal{M}}^{(1)}(S)\, \mathcal{M}^{(0)}(S) \right)\Big|_{p^2=\tau(x,\boldsymbol{p}_T)}$$

$$\mathcal{M}^{(1)}(S) = \begin{cases} -\displaystyle\int \frac{d^4 l}{(2\pi)^4} \frac{i e_c\, \Gamma_{s\,\rho}\, n_-^\rho\, (\not{p} - \not{l} + m)\, \mathcal{Y}_s\, U(P,S)}{(D_1 + i\varepsilon)\,(D_2 - i\varepsilon)\,(D_3 + i\varepsilon)\,(D_4 + i\varepsilon)} & \text{scalar diquark,} \\ -\displaystyle\int \frac{d^4 l}{(2\pi)^4} \frac{i e_c\, \varepsilon_\sigma^*(P-p,\lambda_a)\, \Gamma^{\nu\sigma}_{a\,\rho}\, n_-^\rho\, (\not{p} - \not{l} + m)\, d_{\mu\nu}(p-l-P)\, \mathcal{Y}_a^\mu\, U(P,S)}{(D_1 + i\varepsilon)\,(D_2 - i\varepsilon)\,(D_3 + i\varepsilon)\,(D_4 + i\varepsilon)} & \text{axial-vector diquark} \end{cases}$$

$$\frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \boldsymbol{p}_T^2) = -\frac{1}{4} \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \mathrm{Tr}\left[\left(\mathcal{M}^{(1)}(S) \overline{\mathcal{M}}^{(0)}(S) - \mathcal{M}^{(1)}(-S) \overline{\mathcal{M}}^{(0)}(-S) \right) \gamma^+ \right] + \text{h.c.}\,,$$

$$\frac{\varepsilon_T^{ij} p_{Tj}}{M} h_1^\perp(x, \boldsymbol{p}_T^2) = \frac{1}{4} \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \mathrm{Tr}\left[\left(\mathcal{M}^{(1)}(S) \overline{\mathcal{M}}^{(0)}(S) + \mathcal{M}^{(1)}(-S) \overline{\mathcal{M}}^{(0)}(-S) \right) i\sigma^{i+} \gamma_5 \right] + \text{h.c.}$$

$$f_{1T}^{\perp\, q(s)}(x, \boldsymbol{p}_T^2) = -\frac{g_s}{4} \frac{1}{(2\pi)^3} \frac{M\, e_c^2}{2(1-x)P^+} \frac{(1-x)^2}{[\boldsymbol{p}_T^2 + L_s^2(\Lambda_s^2)]^2}\, 2\,\mathrm{Im}\, J_1^s$$

$$f_{1T}^{\perp\, q(a)}(x, \boldsymbol{p}_T^2) = \frac{g_a}{4} \frac{1}{(2\pi)^3} \frac{M\, e_c^2}{4(1-x)P^+} \frac{(1-x)^2}{[\boldsymbol{p}_T^2 + L_a^2(\Lambda_a^2)]^2}\, 2\,\mathrm{Im}\, J_1^a\,,$$

$$h_1^{\perp\, q(s)}(x, \boldsymbol{p}_T^2) = f_{1T}^{\perp\, q(s)}(x, \boldsymbol{p}_T^2)$$

$$h_1^{\perp\, q(a)}(x, \boldsymbol{p}_T^2) = -\frac{1}{x} f_{1T}^{\perp\, q(a)}(x, \boldsymbol{p}_T^2).$$

Note: for scalar diquarks
Sivers function = Boer-Mulders function

Diquark spectator model for T-odd functions

$$f_{1T}^{\perp\, q(s)}(x, \boldsymbol{p}_T^2) = -\frac{g_s}{4}\frac{1}{(2\pi)^3}\frac{M\, e_c^2}{2(1-x)P^+}\frac{(1-x)^2}{[\boldsymbol{p}_T^2 + L_s^2(\Lambda_s^2)]^2}\, 2\,\mathrm{Im}\, J_1^s$$

$$f_{1T}^{\perp\, q(a)}(x, \boldsymbol{p}_T^2) = \frac{g_a}{4}\frac{1}{(2\pi)^3}\frac{M\, e_c^2}{4(1-x)P^+}\frac{(1-x)^2}{[\boldsymbol{p}_T^2 + L_a^2(\Lambda_a^2)]^2}\, 2\,\mathrm{Im}\, J_1^a\,,$$

$$h_1^{\perp\, q(s)}(x, \boldsymbol{p}_T^2) = f_{1T}^{\perp\, q(s)}(x, \boldsymbol{p}_T^2)$$

$$h_1^{\perp\, q(a)}(x, \boldsymbol{p}_T^2) = -\frac{1}{x}\, f_{1T}^{\perp\, q(a)}(x, \boldsymbol{p}_T^2).$$

$$J_1^s = \int \frac{d^4l}{(2\pi)^4}\frac{g_s\big((p-l)^2\big)}{(D_1+i\varepsilon)\,(D_2-i\varepsilon)\,(D_3+i\varepsilon)\,(D_4+i\varepsilon)}\, 4i\Big(l^+ + 2(1-x)P^+\Big)\Big(l^+M - P^+(m+xM)\,\frac{\boldsymbol{l}_T\cdot\boldsymbol{p}_T}{\boldsymbol{p}_T^2}\Big)$$

$$2\,\mathrm{Im}\, J_1^s = \int \frac{d^4l}{(2\pi)^4}\frac{g_s\big((p-l)^2\big)}{D_1\, D_3}\, 4\,\left(l^+ + 2(1-x)P^+\right)\left(l^+M - P^+(m+xM)\,\frac{\boldsymbol{l}_T\cdot\boldsymbol{p}_T}{\boldsymbol{p}_T^2}\right)(2\pi i)\,\delta(D_2)\,(-2\pi i)\,\delta(D_4)$$

$$= -4P^+\,(m+xM)\,(1-x)\, g_s\,\mathcal{I}_1 \,.= g_s\,\frac{P^+\,(m+xM)\,(1-x)^2}{\pi L_s^2(\Lambda_s^2)\,[\boldsymbol{p}_T^2 + L_s^2(\Lambda_s^2)]}$$

$$2\,\mathrm{Im}\, J_1^a = -8P^+\, x\,(m+xM)\, g_a\,\mathcal{I}_1^{dip} = g_a\,\frac{2P^+\, x(1-x)\,(m+xM)}{\pi L_a^2(\Lambda_a^2)\,[\boldsymbol{p}_T^2 + L_a^2(\Lambda_a^2)]}$$

For dipolar form factor

Diquark spectator model for T-odd functions

$$f_{1T}^{\perp\,q(s)}(x,\boldsymbol{p}_T^2) = -\frac{g_s^2}{4}\,\frac{M\,e_c^2}{(2\pi)^4}\,\frac{(1-x)^3\,(m+xM)}{L_s^2(\Lambda_s^2)\,[\boldsymbol{p}_T^2+L_s^2(\Lambda_s^2)]^3}$$

$$f_{1T}^{\perp\,q(a)}(x,\boldsymbol{p}_T^2) = \frac{g_a^2}{4}\,\frac{M\,e_c^2}{(2\pi)^4}\,\frac{(1-x)^2\,x\,(m+xM)}{L_a^2(\Lambda_a^2)\,[\boldsymbol{p}_T^2+L_a^2(\Lambda_a^2)]^3}$$

$$h_1^{\perp\,q(s)}(x,\boldsymbol{p}_T^2) = f_{1T}^{\perp\,q(s)}(x,\boldsymbol{p}_T^2)$$

$$h_1^{\perp\,q(a)}(x,\boldsymbol{p}_T^2) = -\frac{1}{x}\,f_{1T}^{\perp\,q(a)}(x,\boldsymbol{p}_T^2)\,.$$

A smoking gun! ☺ (or may be not ☹): Angular motion of Partons (for axial-vector diquark spectator)

Also, transverse-momentum dependent moments can be calculated:

$$f_{1T}^{\perp(1)}(x) = \int d\boldsymbol{p}_T\,\frac{\boldsymbol{p}_T^2}{2M^2}\,f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

$$f_{1T}^{\perp(1/2)}(x) = \int d\boldsymbol{p}_T\,\frac{|\boldsymbol{p}_T|}{2M}\,f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

Numerical Approach

Extracting the unknown parameters by fitting to known distribution functions (Following are some example data sets)

- Normalization of distributions $$\pi \int_0^1 dx \int_0^\infty d\boldsymbol{p}_T^2 \, f_{1\,\mathrm{norm}}^{q(X)}(x, \boldsymbol{p}_T^2) = 1$$

$$f_1^u = \frac{3}{2} f_{1\,\mathrm{norm}}^{u(s)} + \frac{1}{2} f_{1\,\mathrm{norm}}^{u(a)}$$
$$f_1^d = f_{1\,\mathrm{norm}}^{d(a')} \, .$$

- For un-polarized distributions → ZEUS2002 (ZEUS, S. Chekanov et al., Phys. Rev. D67, 012007 (2003), hep-ex/0208023)

- For helicity distributions → GRSV2000 (M. Gluck, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D63, 094005 (2001), hep-ph/0011215)

Selected Results

Un-polarized distributions

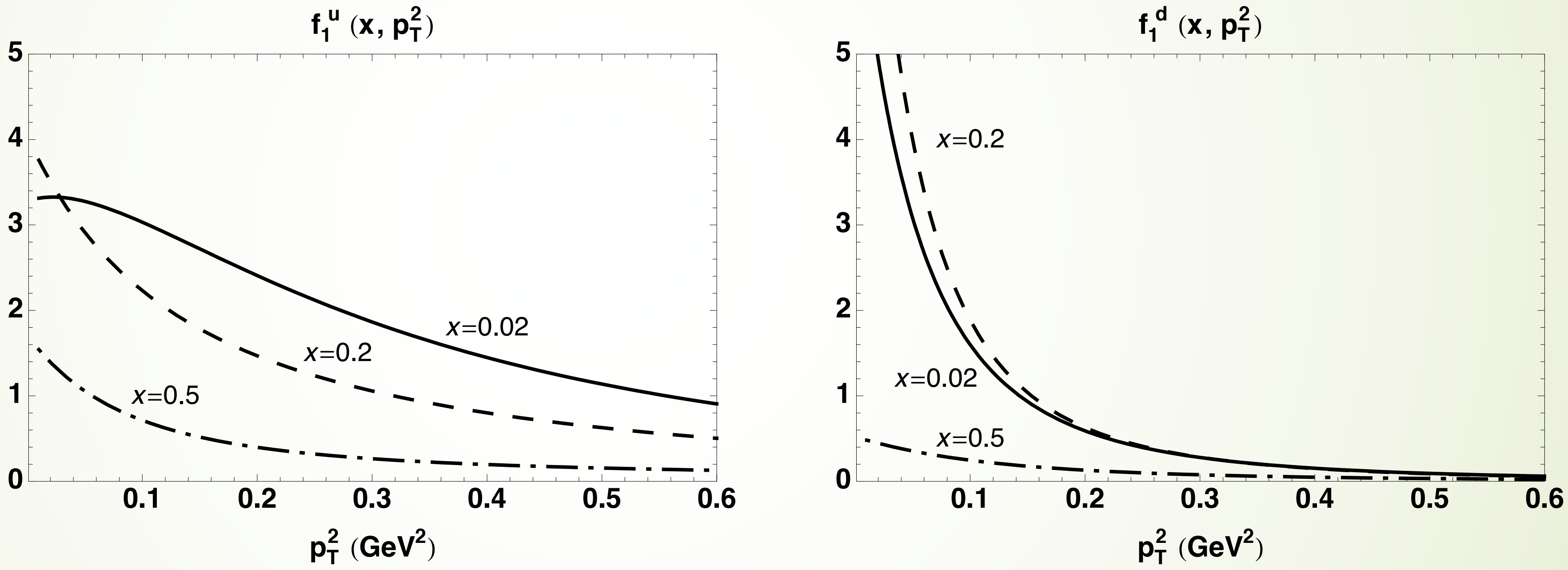


FIG. 5: The $\boldsymbol{p}_T^2$ dependence of the unpolarized distribution $f_1(x, \boldsymbol{p}_T^2)$ for up (left panel) and down quark (right panel). Different lines correspond to different values of x. The downturn of the function f_1^u at relatively small x is due to wavefunctions with nonzero orbital angular momentum.

A. Bacchetta et al arXiv: 0807.0323

Selected Results

first-moments of Sivers distributions

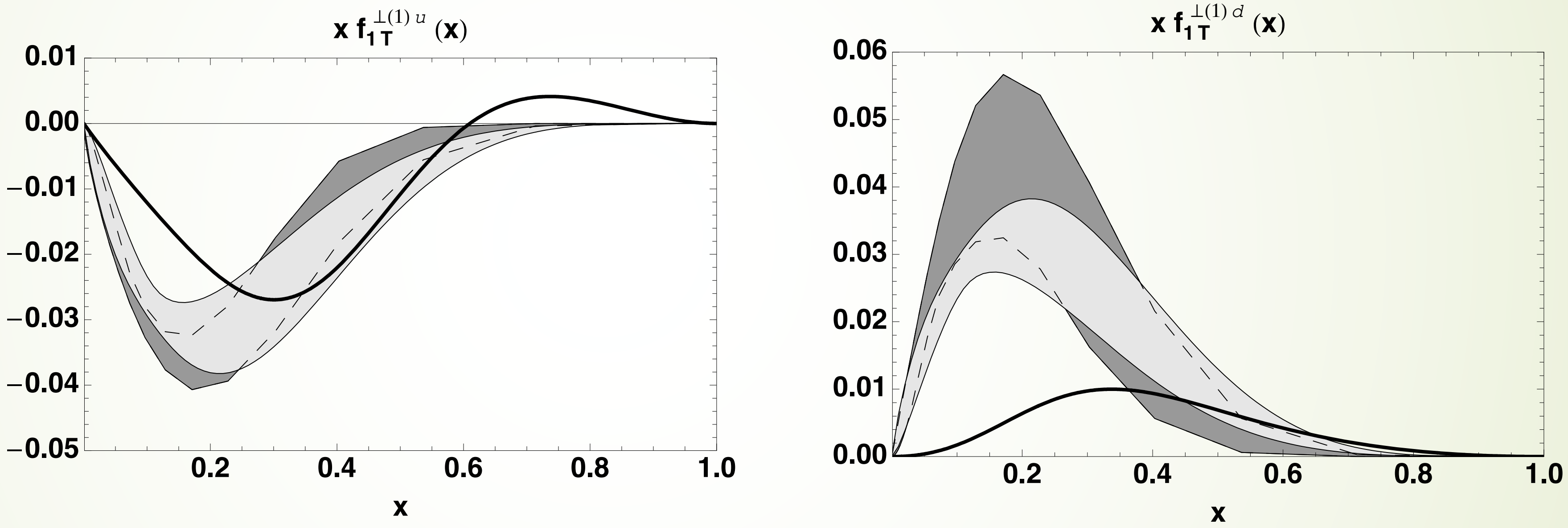
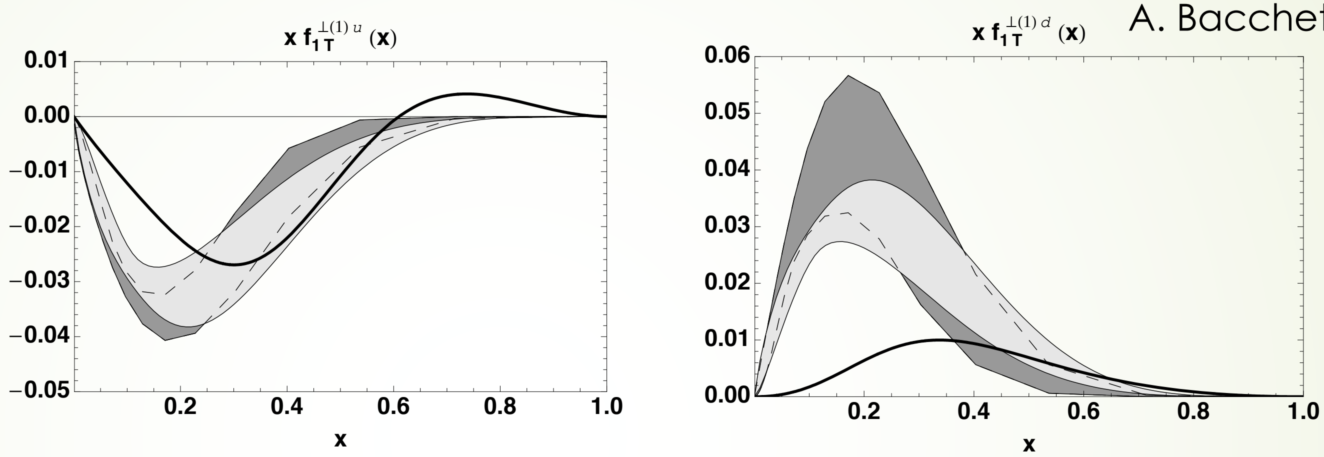


FIG. 10: The first $\boldsymbol{p}_T$-moment $x f_{1T}^{\perp(1)}(x)$ of the Sivers function; left (right) panel for up (down) quark. Solid line for the results of the spectator diquark model. Darker shaded area for the uncertainty band due to the statistical error of the quark parametrizations from Ref. [79], lighter one from Ref. [80].

A. Bacchetta et al arXiv: 0807.0323

Selected Results comparison (spec. diquark vs LFCQM)

A. Bacchetta et al arXiv: 0807.0323



"In particular, we found that the Sivers function for both up and down quarks is dominated by the interference of S and P wave components, while the P − D wave interference terms contribute at most by 20%. On the other side, the relative weight of the P − D wave interference terms increases in the case of the Boer-Mulders function, in particular for the down-quark component"

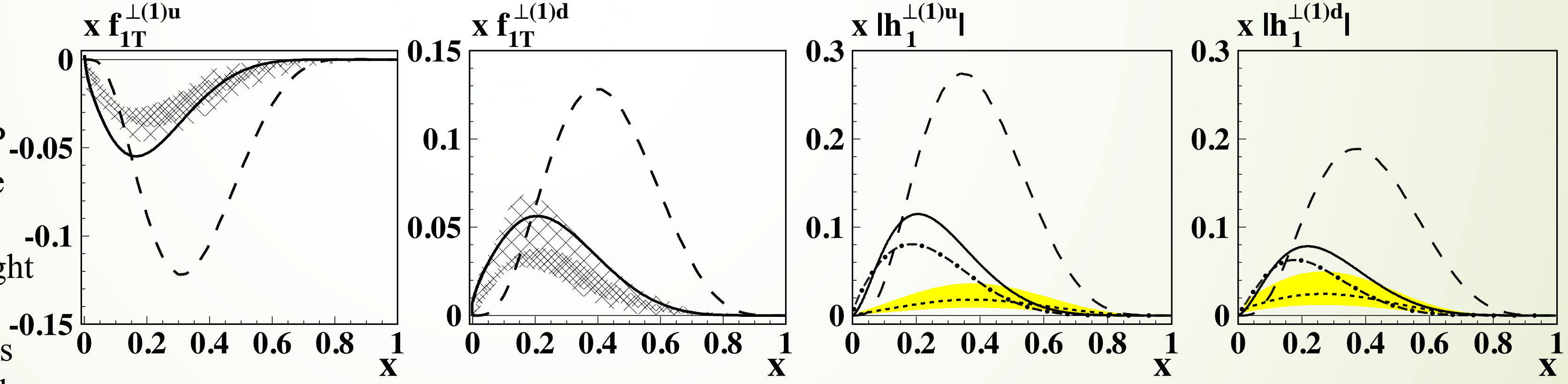


Fig. 1. Results for the first transverse-momentum moment of the Sivers and Boer Mulders functions for up and down quarks, as function of x. See text for the explanation of the different curves.

Pasquini et al arXiv:1008.0945

Selected Results

A. Bacchetta et al arXiv: 0807.0323

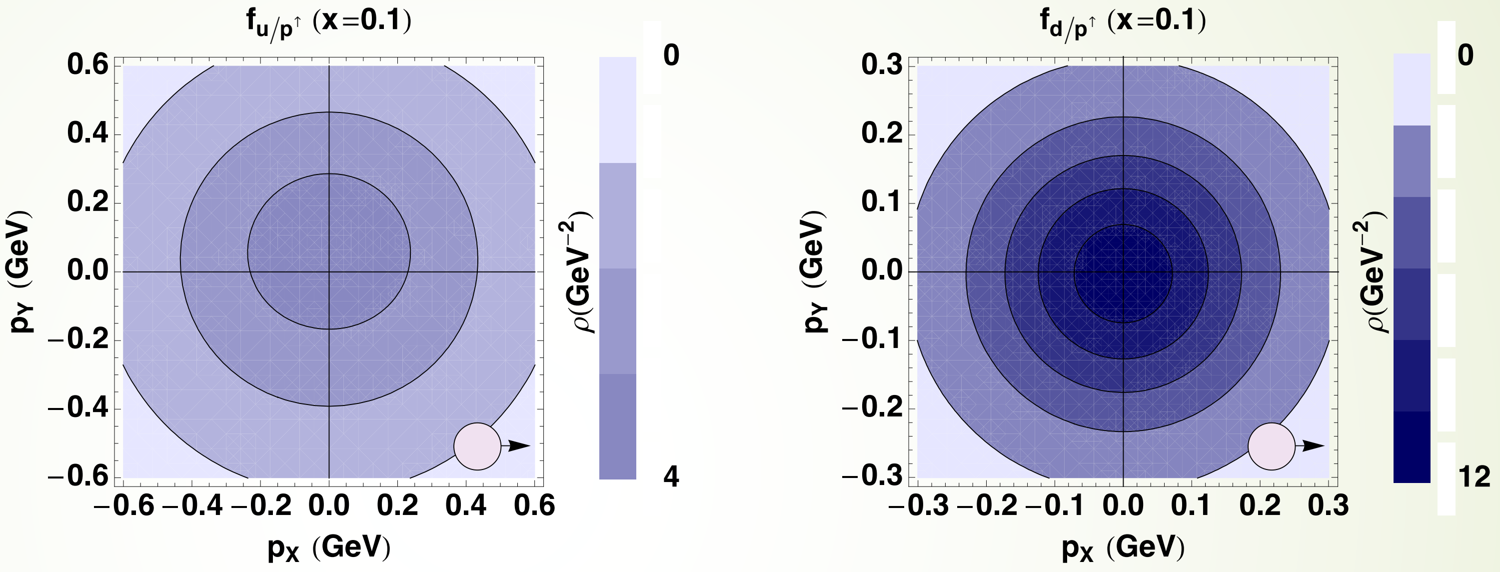


FIG. 11: The model result for the spin density of unpolarized quarks in transversely polarized protons (see text for the precise definition) in $\boldsymbol{p}_T$ space at $x = 0.1$. Left panel for up quark, right panel for down quark. The circle with the arrow indicates the direction of the proton polarization.

In a SIDIS experiment, typically P (proton momentum) is antialigned to the z-axis that points in the direction of the momentum transfer. Hence, if the proton polarization is chosen along the x-axis, the spin density shows an asymmetry in momentum space along the p_y- direction, whose size is driven by the Sivers function. In Fig. 11, we show $f_{q/p\uparrow}(0.1, p_T)$ for q = u (left panel) and q = d (right panel). Since the Sivers function for the up (down) quark is negative (positive), the density is deformed towards positive (negative) values of p_y.

Thank you