TMDs from LQCD (A review on the progress from lattice QCD)

Ishara Fernando

02/03/2021

Mainly based on [arXiv:2010.02445(2020), arXiv:2006.08636(2020) and references therein] by Martha Constantinou

INTRODUCTION

- Hadron Structure → PDFs, GPDs and TMDs
  (number densities with probablilistic interpretations within parton model)
- The mechanism that enables accessing Hadron Structure is "asymptotic freedom" which enables the factorization of cross-sections (at high energy) into hard-part and soft-part.
- Example: Cross-section for inclusive unpolarized DIS

Soft-part: Non-perturbative part

$$\sigma_{\mathrm{DIS}}(x, Q^2) = \sum_i \left[H^i_{\mathrm{DIS}} \otimes f_i\right](x, Q^2)$$

Hard-part (Calculable)

- Distribution functions depend on "processes" which reflect to renormalization scale.
- That's not an issue because data sets can be evolved in the same scale using perturbation theory. Thus, data sets from different processes can be analyzed.

INTRODUCTION

RECENT PROGRESS FROM LATTICE QCD COMMUNITY

- Kaon distribution amplitudes (arXiv: 2003.14128)
- x dependence of PDfs for the Decuplet (Spin 3/2) Delta baryons (arXiv: 2002.12044)
- Exploration of machine learning methods for Pion & Kaon PDFs (arXiv: 2005.13955)

- Lattice calculations of matrix elements with non- local operators related to TMDs, their renormalization
- First explorations of TMDs from lattice QCD
ArXiv: 0908.1283, 1011.1213, 1111.4249, 1506.07826, 1706.03406, 1701.01536, PoS SPIN2018, 047 (2018), and an overview 2006.08636(2020)

INTRODUCTION

- PDFs, GPDs and TMDs are light-cone correlation functions and cannot be accessed from the Euclidean formulation of lattice QCD. Can be accessed via Mellin moments, and OPE, but it's challenging.
- Alternative methods: Use of smeared operators, Large momentum effective theory (LaMET), etc.
- Thus, lattice QCD studies are based on calculating hadron matrix elements of the type

$$\widetilde{\Phi}^{[\Gamma]} \equiv \frac{1}{2}\langle P', S'|\bar{q}(-b/2)\Gamma\,\mathcal{U}[-b/2, b/2]q(b/2)|P, S\rangle$$

- An arbitrary Dirac structure 'Γ' is allowed for, and the states can also carry definite spin in addition to momentum.
- In a concrete lattice calculation, the staple-shaped gauge connection between the quark operators $\bar{q}$, q, summarized here by U, has finite extent; in the following, the vector v specifies the direction of the staple legs, with their length scaled by the parameter η. For η = 0, the path becomes a straight link between the quark operators.
- Standard TMD observables are obtained by extrapolating the obtained data to η → ∞.
- The TMD observables considered in the following are, however, appropriate ratios in which the soft factors (divergences) are canceled.

SIVERS SHIFT

- For example, in the Γ = γ+ channel, for a proton, $\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]} = \widetilde{A}_{2B} + i m_N \epsilon_{ij} b_i S_j \widetilde{A}_{12B}$
- $\widetilde{A}_{iB}$ essentially correspond to Fourier- transformed TMD PDFs. Through them, one can finally define observables such as the generalized Sivers shift

$$\langle k_T \rangle_{TU}(b^2, b\cdot P, \hat{\zeta}, \eta v\cdot P, \ldots) = -m_N \frac{\widetilde{A}_{12B}(b^2, b\cdot P, \hat{\zeta}, \eta v\cdot P, \ldots)}{\widetilde{A}_{2B}(b^2, b\cdot P, \hat{\zeta}, \eta v\cdot P, \ldots)}$$

- Generalizing these calculations to a scan of the (b · P)- dependence allows one to access also the dependence on x, which is Fourier conjugate to (b · P) .
- A complementary approach to the x-dependence of TMD PDFs based on quasi-TMD PDFs

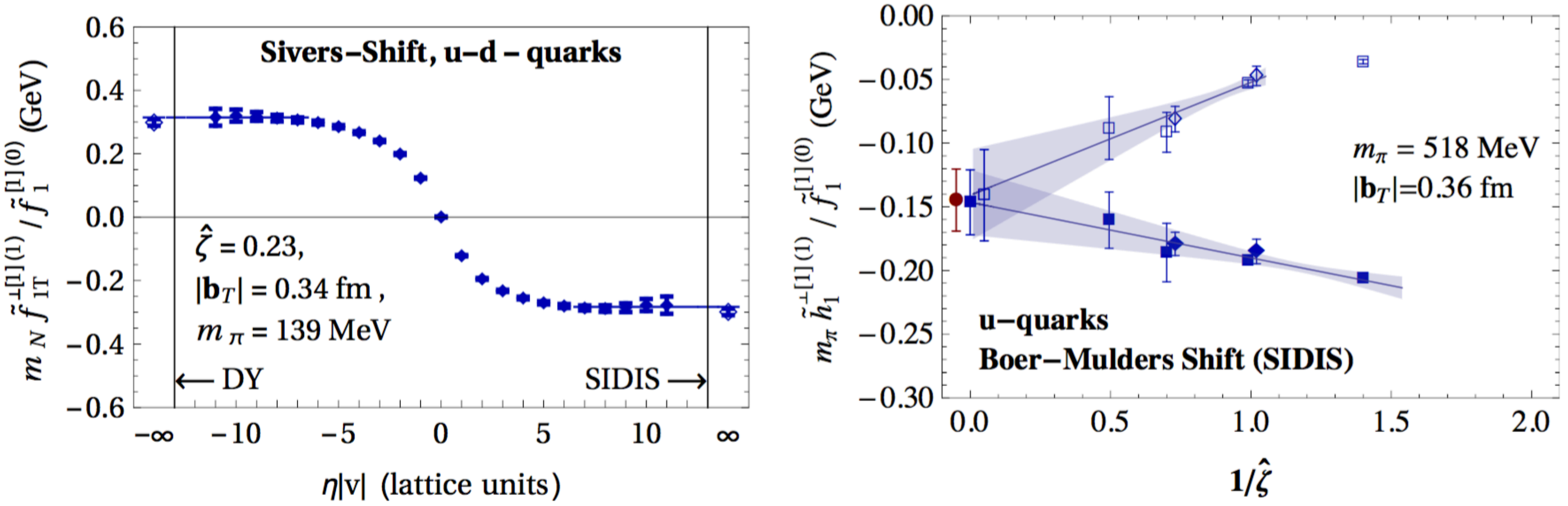

FIG. 30 Left: Proton Sivers shift as a function of staple length for fixed b_T and $\hat{\zeta}$; $\eta \to \infty$ defines the SIDIS limit. Right: Extrapolation of the SIDIS-limit data for the pion Boer-Mulders shift to large $\hat{\zeta}$ at fixed b_T (Engelhardt *et al.*, 2016). Open symbols represent a partial contribution that dominates at large $\hat{\zeta}$, providing further insight into the approach to the asymptotic regime.

OAM

- With nonzero transverse momentum transfer ΔT = P ′ – P , one can correlate quark transverse momentum with position;
  ΔT is Fourier conjugate to the quark impact parameter rT.
  This allows one to directly access longitudinal quark orbital angular momentum (OAM), ⟨rT × kT⟩
- The choice of gauge link U corresponds to different decompositions of proton spin. A staple link extending to infinity, such as used in standard TMD PDF studies, yields Jaffe-Manohar OAM, whereas the η = 0 limit yields Ji OAM

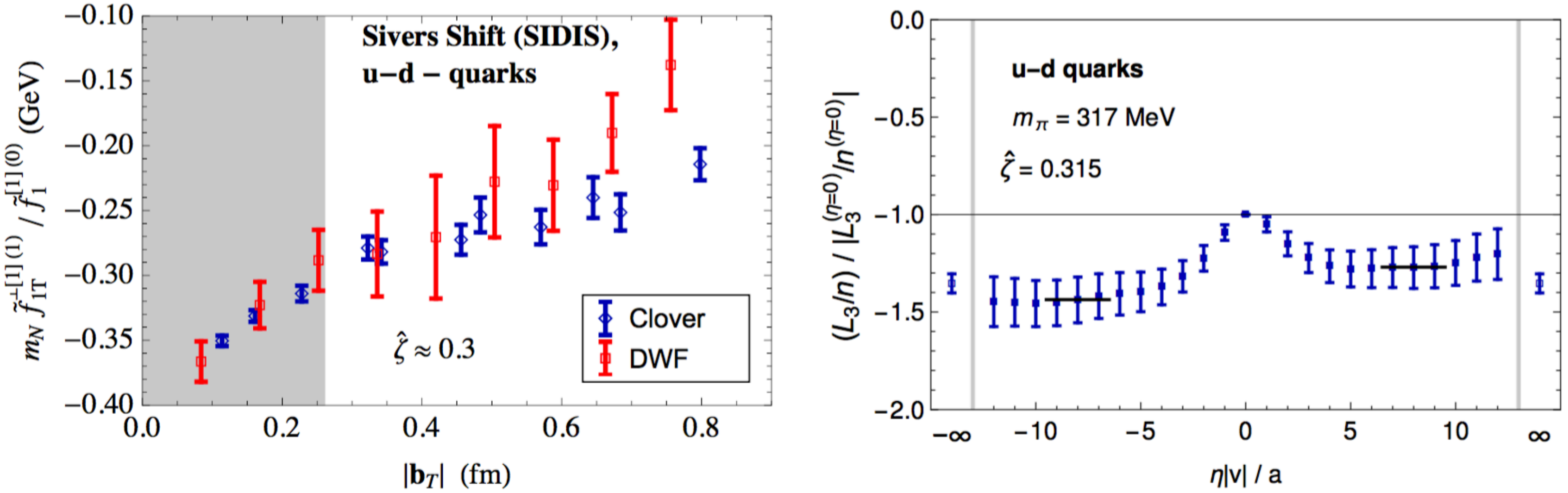

FIG. 32 Left: Comparison between SIDIS-limit data for the proton Sivers shift obtained for two distinct lattice discretizations, as a function of b_T at fixed $\hat{\zeta}$ (Yoon *et al.*, 2017). The data are compatible within uncertainties, suggesting that no significant violations of multiplicative renormalization are present. Right: Longitudinal quark OAM in the proton L_3 as a function of staple length at fixed $\hat{\zeta}$ (Engelhardt *et al.*, 2018). The limit $\eta = 0$ yields Ji OAM, $\eta \to \pm\infty$ Jaffe-Manohar OAM. The ratio of L_3 to the number of valence quarks n is evaluated to cancel multiplicative renormalizations, analogous to Eq. (63). Data are shown in units of the absolute value of Ji OAM.

Quasi-TMD approach

- An alternative approach to calculating TMD PDFs from lattice QCD using so-called quasi-TMD PDFs
- The light-cone TMD PDFs involve matrix elements of non-local operators containing a staple-shaped Wilson line.
- Method: calculating correlation functions with space-like separated partons, as done for PDFs and GPDs, and then properly match them to their light-cone counterparts.
- First attempts:
  ArXiv: 1405.7640, 1801.05930, 1811.00026, 1901.03685, 1910.08569, 1910.11415, 1911.03840, 2002.07527

$$\tilde{f}_{i/P}(x, \mathbf{b_T}, \mu, P^z) = \lim_{\substack{a\to 0\\ \eta\to\infty}} \tilde{Z}^i_{\mathrm{uv}}(\mu, a) \times \tilde{f}^{0\,(u)}_{i/P}\big(x, \mathbf{b_T}, a, \eta, P^z\big)\tilde{\Delta}^q_S(\mathbf{b_T}, a, \eta)$$

$$\tilde{f}^{0\,(u)}_{i/P}(x, \mathbf{b_T}, a, \eta, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} N_{\tilde{\Gamma}} \times \big\langle P\big|\bar{q}(b/2)\tilde{\mathcal{U}}\tfrac{\tilde{\Gamma}}{2}q(-b/2)\big|P\big\rangle$$

where $b^\mu = (0, \mathbf{b_T}, b^z)$, and the Wilson line path $\tilde{\mathcal{U}}$ is chosen such that it connects $b/2 \to (0, \mathbf{b_T}/2, \eta) \to (0, -\mathbf{b_T}/2, \eta) \to -b/2$. For unpolarized TMD PDFs, the Dirac structure can be chosen as $\tilde{\Gamma} = \gamma^0, \gamma^3$, with the normalization factor $N_{\gamma^0} = 1, N_{\gamma^3} = P^z/P^0$.

Quasi-TMD approach (cont…)

- However, the relation between quasi-TMD PDFs and TMD PDFs has been argued to be spin-independent.
- The key relation between quasi-TMD PDFs and TMD PDFs is,

$$\tilde{f}_{\rm ns}(x, \mathbf{b_T}, \mu, P^z) = C_{\rm ns}(\mu, xP^z) g_q^S(b_T, \mu) \\ \times \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_{\rm ns}(x, \mathbf{b_T}, \mu, \zeta)$$

- the quasi-TMD PDF f˜ in the nonsinglet ‘ns’ = u − d channel is related to the TMD PDF fns through a perturbative kernel Cns, which is known at one loop.
- The Collins-Soper kernel γζ (see the next slide) is required to relate TMD PDFs at different hadron energies, and thus its nonperturbative determination from lattice QCD is of key interest.

DETERMINING THE NON-PERTURBATIVE COLLINS-SOPPER KERNAL

TMD Evolution Formula

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu_0, \zeta_0) \times \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln\frac{\zeta}{\zeta_0}\right]$$

$$\gamma_\zeta^i(\mu, b_T) = \frac{2}{\ln(\zeta_1/\zeta_2)} \ln \frac{f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta_1)}{f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta_2)}$$

In lattice calculations: $\mu_0^2 \sim \zeta_0 \sim \mathcal{O}(4\ \mathrm{GeV}^2)$

Drell-Yan production: $\mu^2 \sim \zeta \sim m_Z^2 \approx (91\ \mathrm{GeV})^2$.

- x : the fraction of the hadron momentum carried by the struck parton
- bT: the Fourier-Conjugate of the transverse momentum qT
- \mu: Virtuality scale
- \zeta: a scale related to the momentum of the hadron / hard-scale of the scattering process.
- First exponential: \mu evolution
- Second exponential: Collins-Sopper evolution (which includes the Collins-Sopper Kernal [often denoted by K])

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\mathrm{ns}}(\mu, xP_2^z)\, \tilde{f}_{\mathrm{ns}}(x, \mathbf{b_T}, \mu, P_1^z)}{C_{\mathrm{ns}}(\mu, xP_1^z)\, \tilde{f}_{\mathrm{ns}}(x, \mathbf{b_T}, \mu, P_2^z)}$$

Ebert et al. ArXiv:1811.00026

DETERMINING THE NON-PERTURBATIVE COLLINS-SOPPER KERNAL

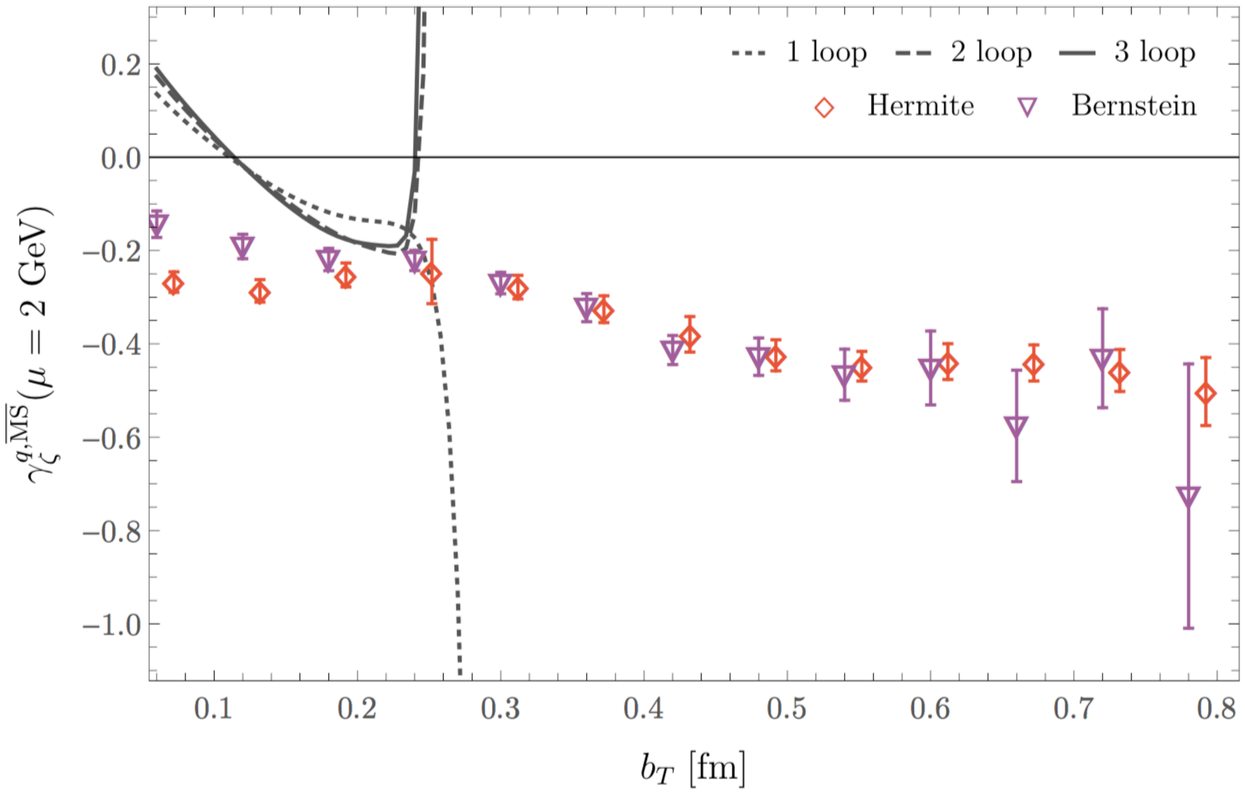

Fig. 21. Lattice results on the Collins-Soper evolution kernel as a function of b_T. The interpolation of the unsubtracted quasi-TMD PDF using Hermite and Bernstein polynomial bases are shown with red diamonds and purple triangles, respectively. Results from perturbation theory [194, 195] are shown with dashed and solid lines. Source: Ref. [193]. Article published under the terms of the Creative Commons Attribution 4.0 International license.

This indicates non-negligible systematic effects in the lattice data, such as power corrections in the small-b_T region. The challenges of the inverse problem arise in these calculations too, as the Fourier transform over b_T is required. **However, an extensive study of the systematic uncertainties is necessary to control unwanted effects.**

The calculation is performed in quenched LQCD with pion mass of 1.207 GeV. The hadron is boosted with P_3 = 1.29, 1.94, 2.58 GeV and for transverse parton momentum q_T in the range of 250 MeV and 2 GeV

P. Shannahan et al. ArXiv: 2003.06063 (2020)

Approach to Global Fits with some remarks

- The limited kinematic coverage for the differential cross sections provided by experimental data leads to a challenging inverse problem; it is unlikely, if not impossible, that one can completely fix the TMD PDFs as continuous functions over the full range of kinematics.
- To pin down TMD PDFs to the best possible accuracy, we need data from multiple observables, experimentally measured or calculated within Lattice QCD, that are related to the same universal set of TMD PDFs, covering a wide kinematic regime, and to perform QCD global analyses and fits, similar to what is done to extract the PDFs.
- TMD PDFs are associated with two momentum scales, Q and kT (or its Fourier conjugate bT), QCD evolution involves two coupled evolution equations and covers a two- dimensional phase space, e.g., (Q,bT), where bT $\in [0,\infty)$
- Therefore, the path used in solving these two coupled equations is not unique, which could affect the size of higher order corrections, leading to an additional scheme dependence of the TMD PDFs
- The fundamental and most important difference from DGLAP evolution is that evolution kernels for evolving TMD PDFs from an input scale Q_0 to any higher observed scale Q, referred as the Collins-Soper kernels, depend on the value of bT , and are not perturbatively calculable for the region where bT $>1/Q_0$.
- TMD observables at separations up to bT ~ 1 fm at hadron momenta up to about 2GeV will be accessible at the physical pion mass in the medium term, where it can be expected that moments in the momentum fraction x will continue to be obtained with better precision than the x-dependent quantities, even as determinations of the latter are developed.