Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass

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Based on [PHYSICAL REVIEW D 101, 094513 (2020)] by C. Alexandrou arXiv: 2003.08486

FORMALISM

- A key object for the study of the spin decomposition is the QCD energy-momentum tensor (EMT)

$$T^{\mu\nu} = \boxed{\bar{T}^{\mu\nu}} + \hat{T}^{\mu\nu}$$

Gauge invariant parts of
the Traceless part

$$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu;g} + \bar{T}^{\mu\nu;q}$$

Gluon-part Quark-part

$$\bar{T}^{\mu\nu;g} = F^{\{\mu\rho}F^{\nu\}}{}_{\rho} \qquad \bar{T}^{\mu\nu;q} = \bar{\psi}i\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi$$

$F^{\mu\nu}$ is the gluon field-strength tensor

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FORMALISM

- Angular Momentum Density M^{0ij} in terms of the EMT → $M^{\alpha\mu\nu} = \bar{T}^{\alpha\nu}x^{\mu} - \bar{T}^{\alpha\mu}x^{\nu}$
- i^th component of the angular momentum operator

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}(x)$$

- For gluons: $\vec{J}^g = \int d^3x(\vec{x} \times (\vec{E} \times \vec{B}))$
- For quarks: $\vec{J}^q = \int d^3x \left[\underbrace{\bar{\psi}\frac{\vec{\gamma}\gamma^5}{2}\psi}_{\text{Intrinsic Spin}} + \overbrace{\bar{\psi}(\vec{x} \times i\vec{D})\psi}^{\text{OAM}} \right]$
- Ji's Sum rule (gauge invariant) $\vec{J} = \vec{J}^g + \vec{J}^q = \vec{J}^g + \left(\underbrace{\frac{\vec{\Sigma}^q}{2}}_{\text{Intrinsic Spin}} + \underbrace{\vec{L}^q}_{\text{OAM}} \right)$

$q^+ = q + \bar{q}$ to denote the sum from quark and antiquark

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FORMALISM

- LQCD computation

Generalized Form Factors in Minkovski Space

$$\langle N(p',s')|T^{\mu\nu;q,g}|N(p,s)\rangle =$$

$$= \bar{u}_N(p',s')\left[A_{20}^{q,g}(q^2)\gamma^\mu P^\nu + B_{20}^{q,g}(q^2)\frac{i\sigma^{\mu\rho}q_\rho P^\nu}{2m_N} + C_{20}^{q,g}\frac{q^\mu q^\nu}{m_N}\right]u_N(p,s)$$

Ensemble	c_{SW}	β	N_f	a [fm]	V	am_π	$m_\pi L$	am_N	m_N/m_π	m_π [GeV]	L [fm]
cB211.072.64	1.69	1.778	$2+1+1$	0.0801(4)	$64^3\times 128$	0.05658(6)	3.62	0.3813(19)	6.74(3)	0.1393(7)	5.12(3)
cA2.09.48	1.57551	2.1	2	0.0938(3)(1)	$48^3\times 96$	0.06208(2)	2.98	0.4436(11)	7.15(2)	0.1306(4)(2)	4.50(1)

Contributions from two types of diagrams:
(i) The operator couples directly to a valence quark, known as the connected contribution
(ii) The operator couples to a sea quark resulting in a quark loop, known as the disconnected contribution.

In this case: "Physical limit" (usually done at higher pion mass and then doing extrapolation)

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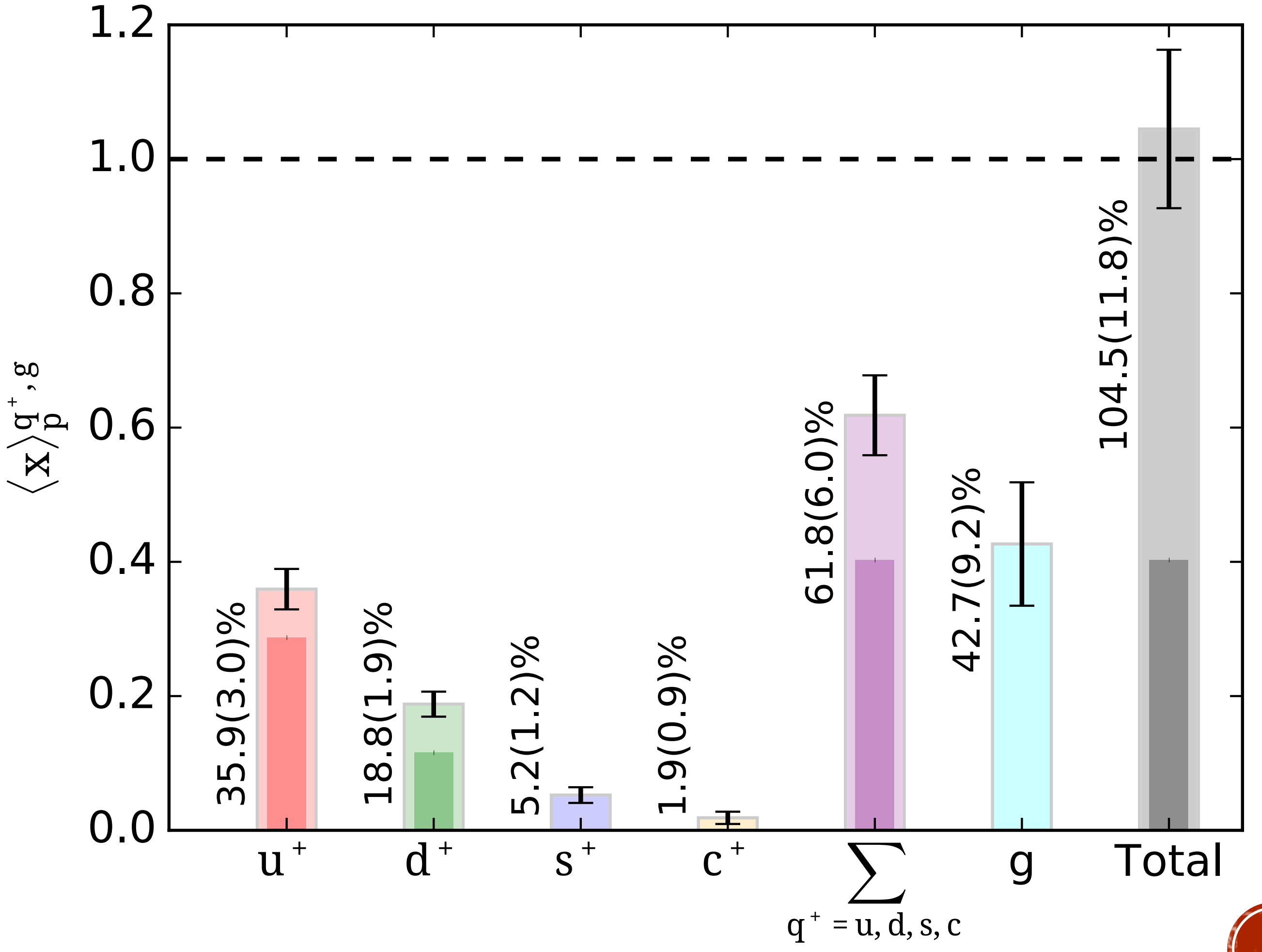
RESULTS

The decomposition of the proton average momentum fraction <x>

$$q^{+} = q + \bar{q}$$

The momentum fraction carried by quarks in the proton is found to be 0.618(60) and by gluons 0.427(92), the sum of which gives 1.045(118) confirming the momentum sum rule

$$\sum_{q=u,d,s,c} \langle x \rangle_R^{q^+} + \langle x \rangle_R^{g} = 104.5(11.8)\%$$



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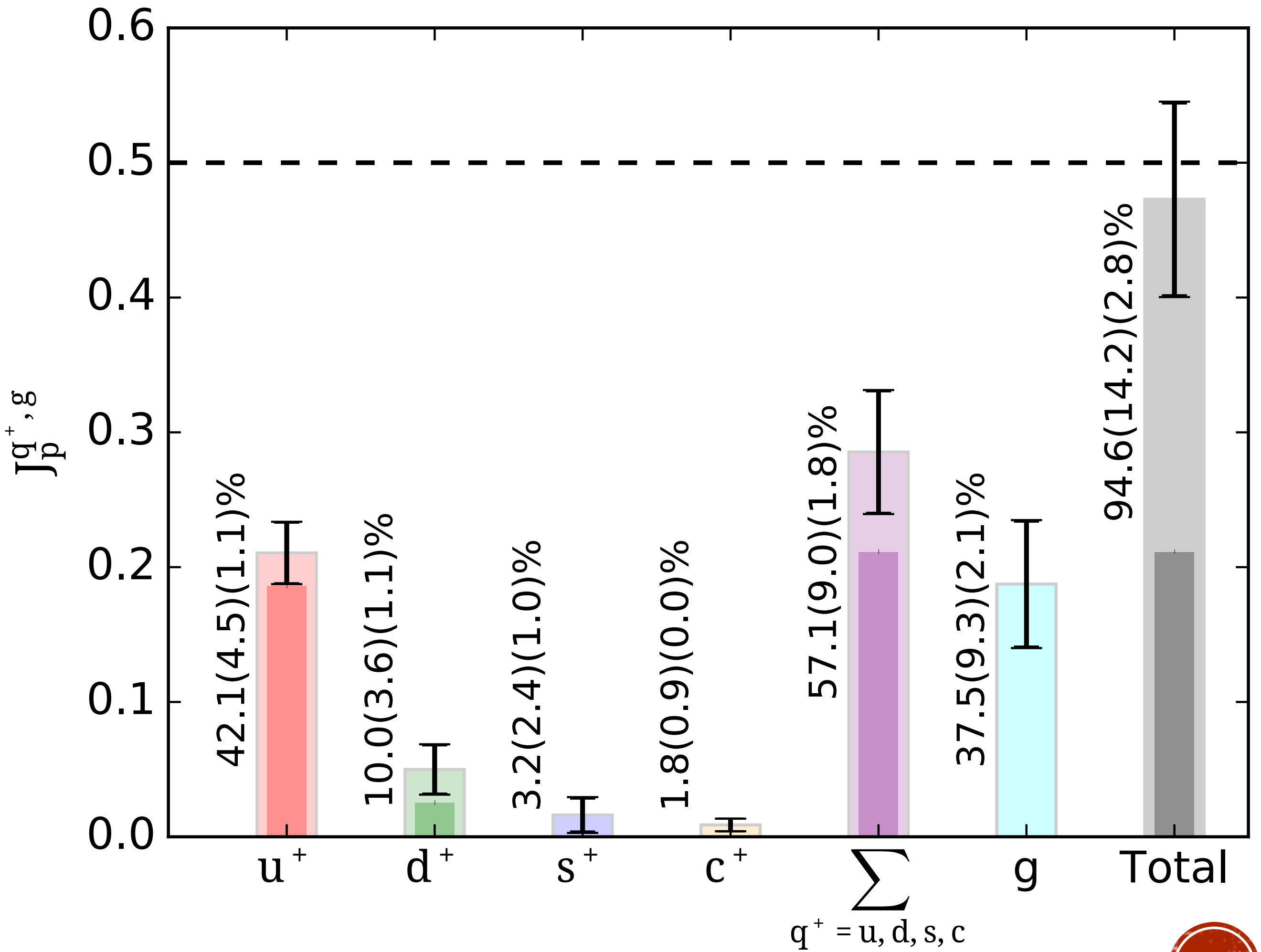
RESULTS

The decomposition of the proton spin J

$$q^{+} = q + \bar{q}$$

	$\langle x \rangle$	J	$\frac{1}{2}\Delta\Sigma$	L
u^{+}	0.359(30)	0.211(22)(5)	0.432(8)	−0.221(26)(5)
d^{+}	0.188(19)	0.050(18)(5)	−0.213(8)	0.262(20)(5)
s^{+}	0.052(12)	0.016(12)(5)	−0.023(4)	0.039(13)(5)
c^{+}	0.019(9)	0.009(5)(0)	−0.005(2)	0.014(10)(0)
g	0.427(92)	0.187(46)(10)		
Tot.	1.045(118)	0.473(71)(14)	0.191(15)	0.094(51)(9)

The major contribution comes from the up quark amounting to about 40% of the proton spin. The down, strange, and charm quarks have relatively smaller contributions. All quark flavors together constitute about 60% of the proton spin. The gluon contribution is significant, namely about 40% of the proton spin, providing the missing piece to obtain in total 94.6(14.2)(2.8)% of the proton spin.



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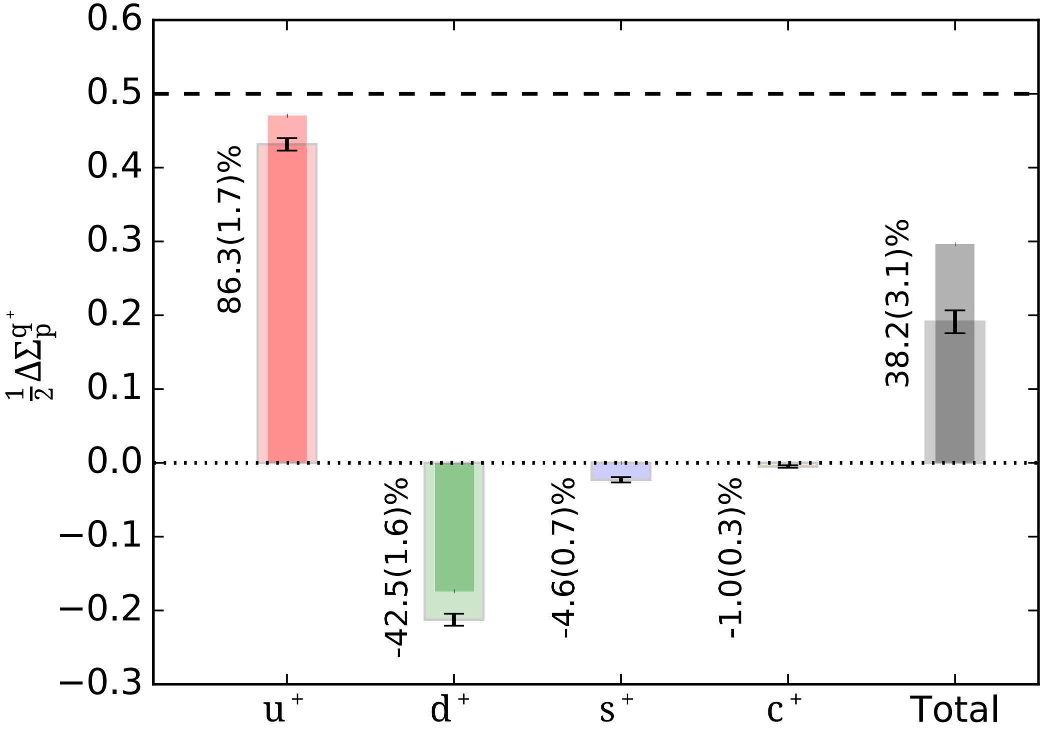
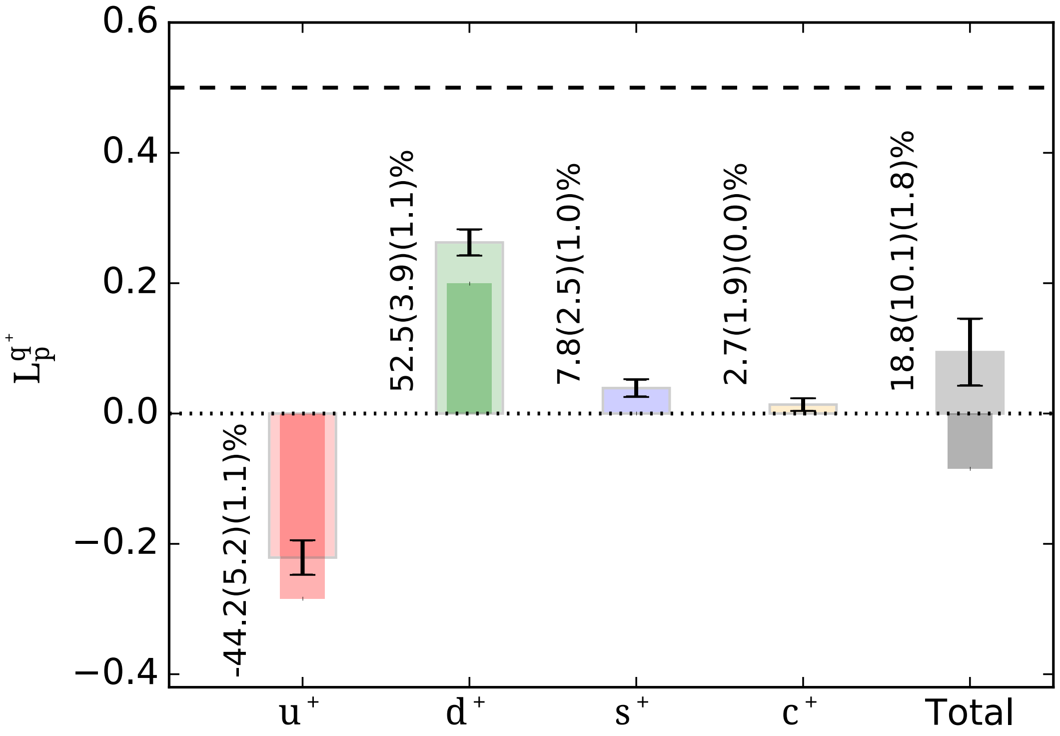
RESULTS

Intrinsic quark-spin



Orbital Angular Momentum



The orbital angular momentum of the up quark is negative, reducing the total angular momentum contribution of the up quark to the proton spin. The con- tribution of the down quark to the orbital angular momentum is positive, almost canceling the negative intrinsic spin contri- bution resulting in a relatively small positive contribution to the spin of the proton.

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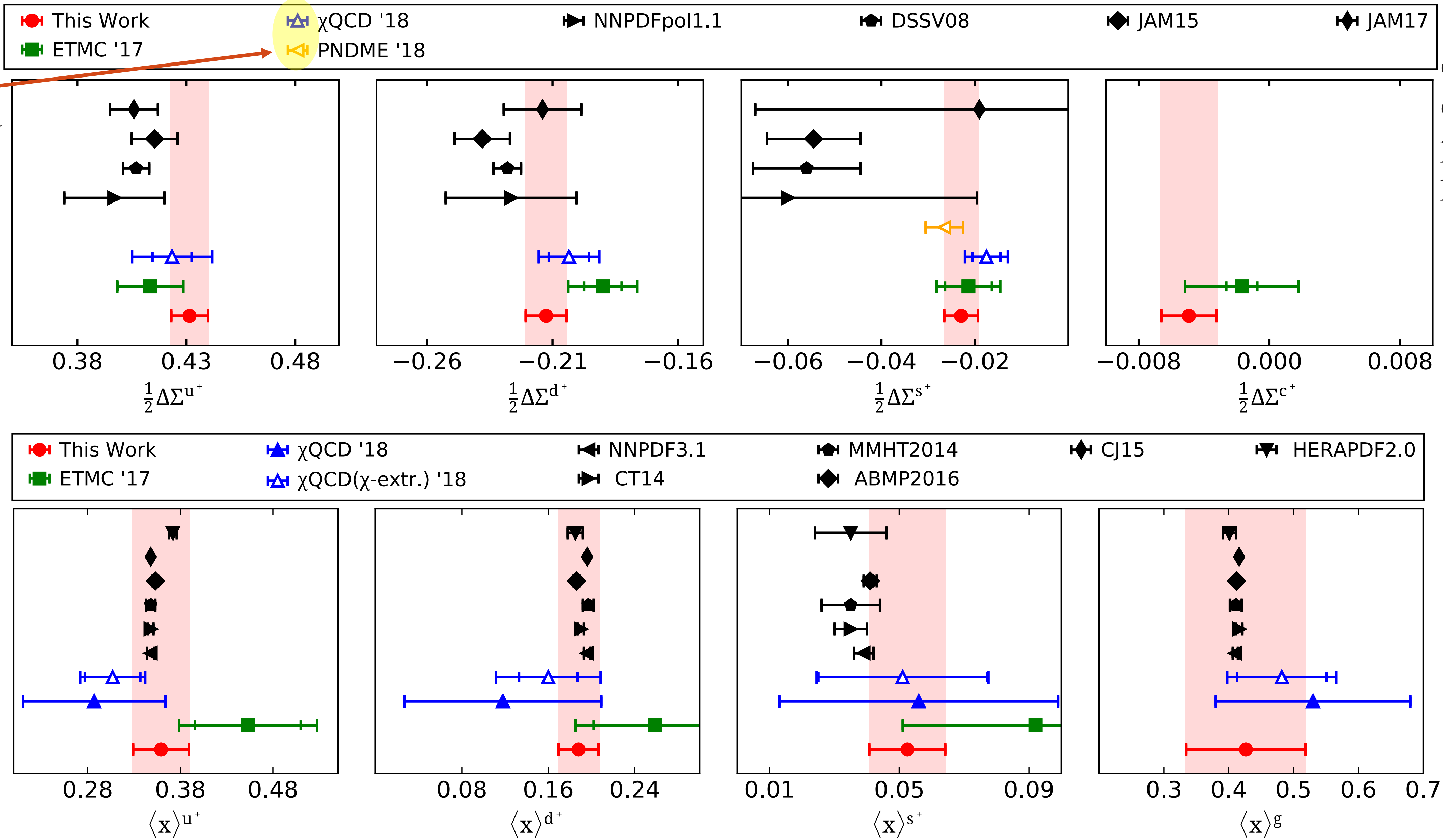
COMPARISON WITH OTHER LQCD CALCULATIONS

obtained after a chiral extrapolation



computed directly at the physical point

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RESULTS

The major outcomes of this work are the following:

(i) The contribution of quarks to the intrinsic proton spin is found to be $\frac{1}{2}\sum_{q=u,d,s,c} \Delta\Sigma^{q^+} = 0.191(15)$. This is in agreement with the upper bound of the COMPASS value $0.13 \leq \frac{1}{2}\Delta\Sigma \leq 0.18$ [89]. It is worth mentioning that our value for $\frac{1}{2}\Delta\Sigma^{c^+} = -0.005(2)$ is the most precise determination, not only as determined from lattice QCD but also from analyses of experimental data.

(ii) The verification of the momentum sum for the proton computing all the contributions: $\langle x\rangle^{u^+} + \langle x\rangle^{d^+} + \langle x\rangle^{s^+} + \langle x\rangle^{c^+} + \langle x\rangle^{g} = 0.359(30) + 0.188(19) + 0.052(12) + 0.019(9) + 0.427(92) = 1.045(118)$.

(iii) The full decomposition of the angular momentum of the proton. We find for the quark angular momentum $J^{u^+} + J^{d^+} + J^{s^+} + J^{c^+} + J^{g} = 0.211(22)(5) + 0.050(18)(5) + 0.016(12)(5) + 0.009(5)(0) + 0.187(46)(10) = 0.473(71)(14)$.

(iv) The computation of the quark orbital angular momentum obtaining $\sum_{q=u,d,s,c} L^{q^+} = 0.094(51)(9)$.

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THANK YOU