June 7, 2022
Fermilab - UVA Seminar

Global extraction of quark unpolarized Transverse Momentum Distributions at N3LL

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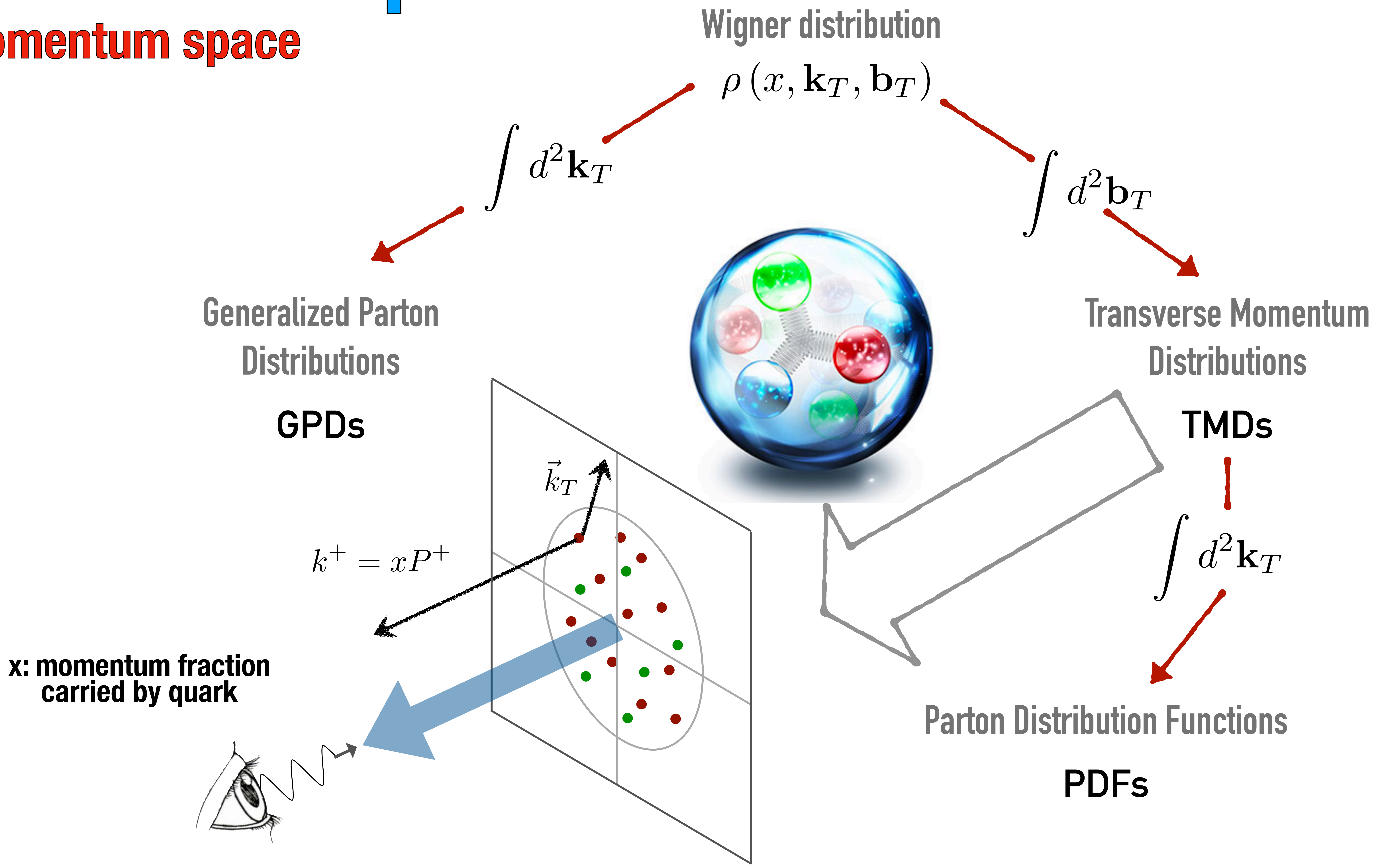
Work done in collaboration with:
A. Bacchetta, V. Bertone, G. Bozzi, M. Cerutti, F. Piacenza, M. Radici, A. Signori

EIC Center at Jefferson Lab
3DSPIN MAPPING THE PROTON IN 3D
European Research Council
Istituto Nazionale di Fisica Nucleare
UNIVERSITÀ DI PAVIA

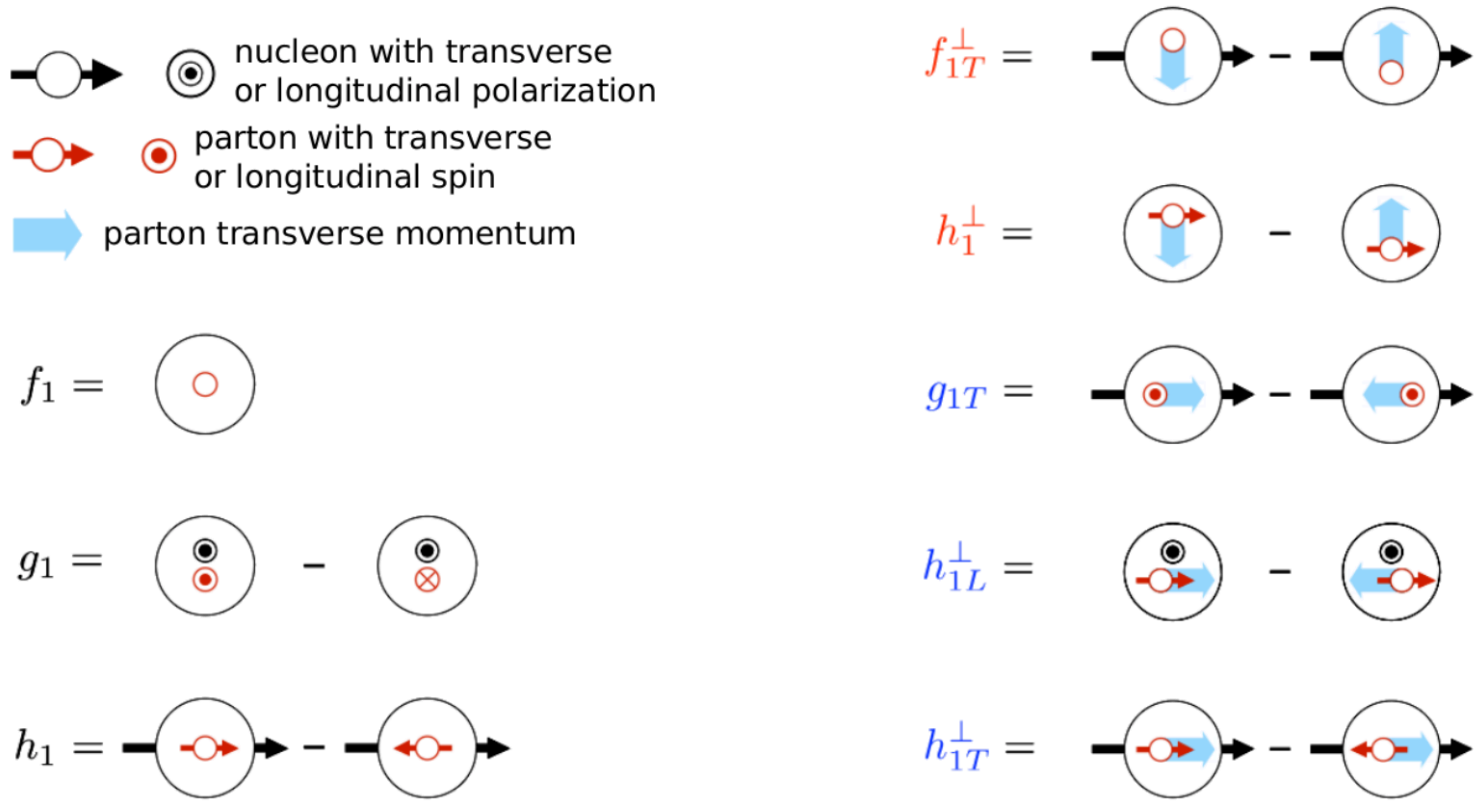
TMDs: 3D maps

in momentum space



Transverse Momentum Distributions

TMD PDFs		quark polarization		
		U	L	T
nucleon polarization	U	f_1		$h_1^\perp$
	L		$g_{1\mathrm{L}}$	$h_{1\mathrm{L}}^\perp$
	T	$f_{1\mathrm{T}}^\perp$	$g_{1\mathrm{T}}$	$h_1, h_1^\perp$



Transverse Momentum Distributions

TMD PDFs

		quark polarization		
		U	L	T
nucleon polarization	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	$h_1, h_1^\perp$

Parton Distribution Functions

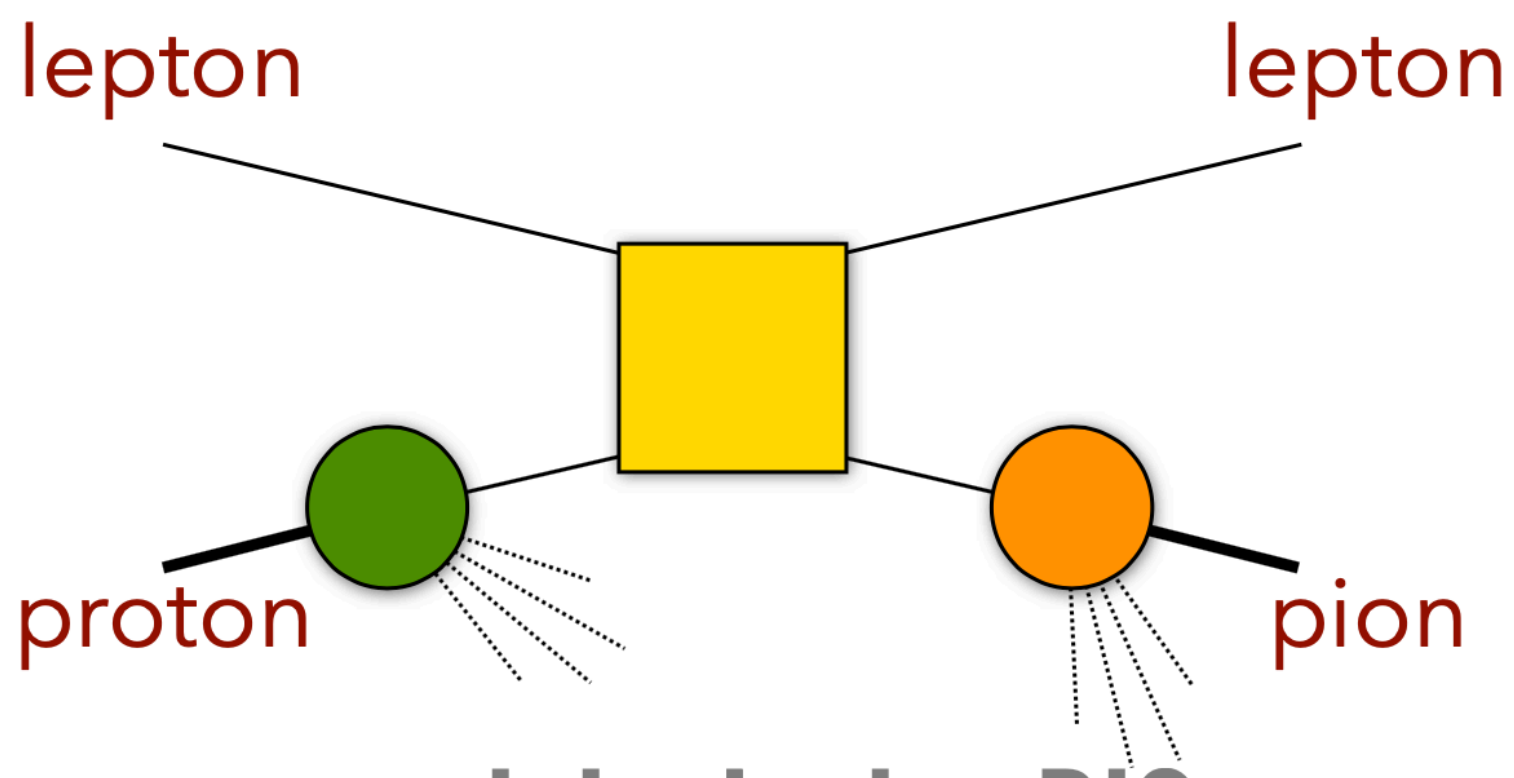
TMD FFs

		quark polarization		
		U	L	T
hadron polarization	U	D_1		$H_1^\perp$
	L		G_{1L}	$H_{1L}^\perp$
	T	$D_{1T}^\perp$	G_{1T}	$H_{1T}, H_{1T}^\perp$

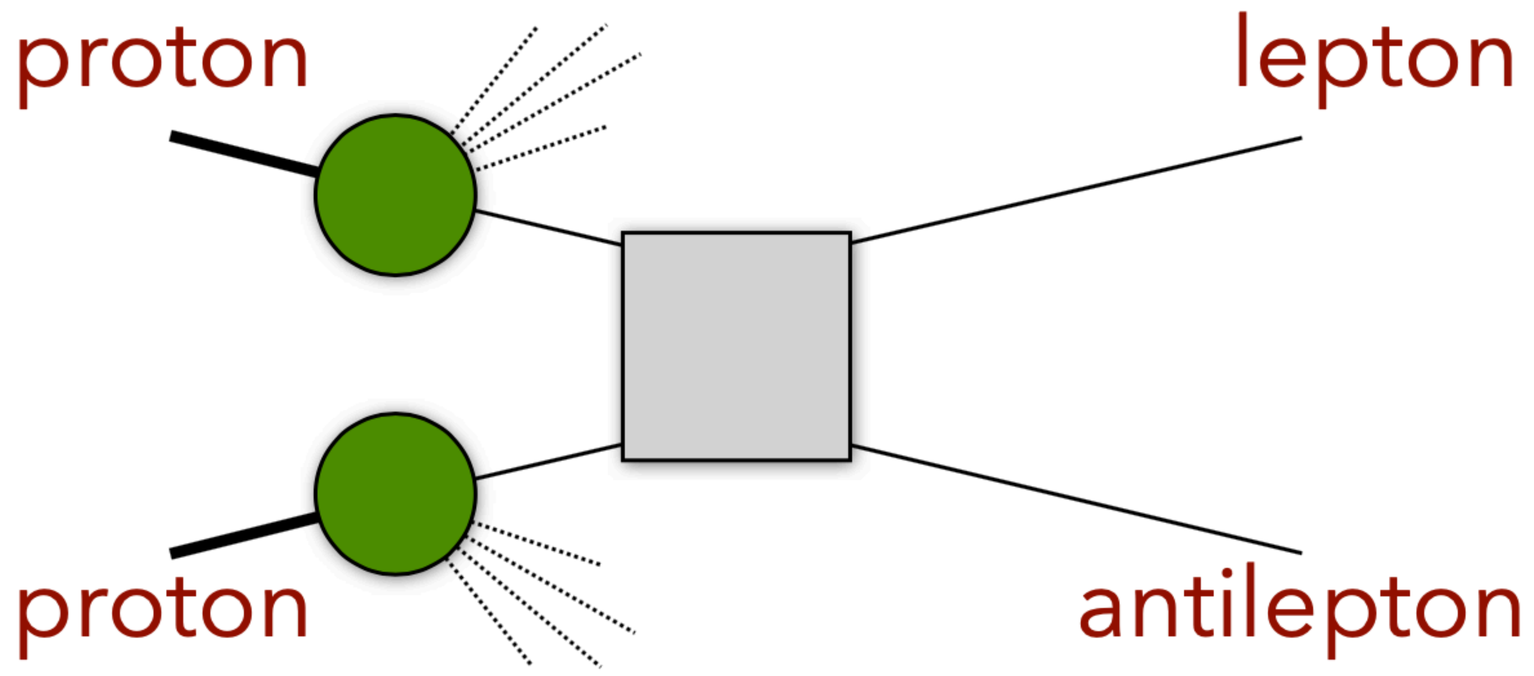
Fragmentation Functions

Factorization

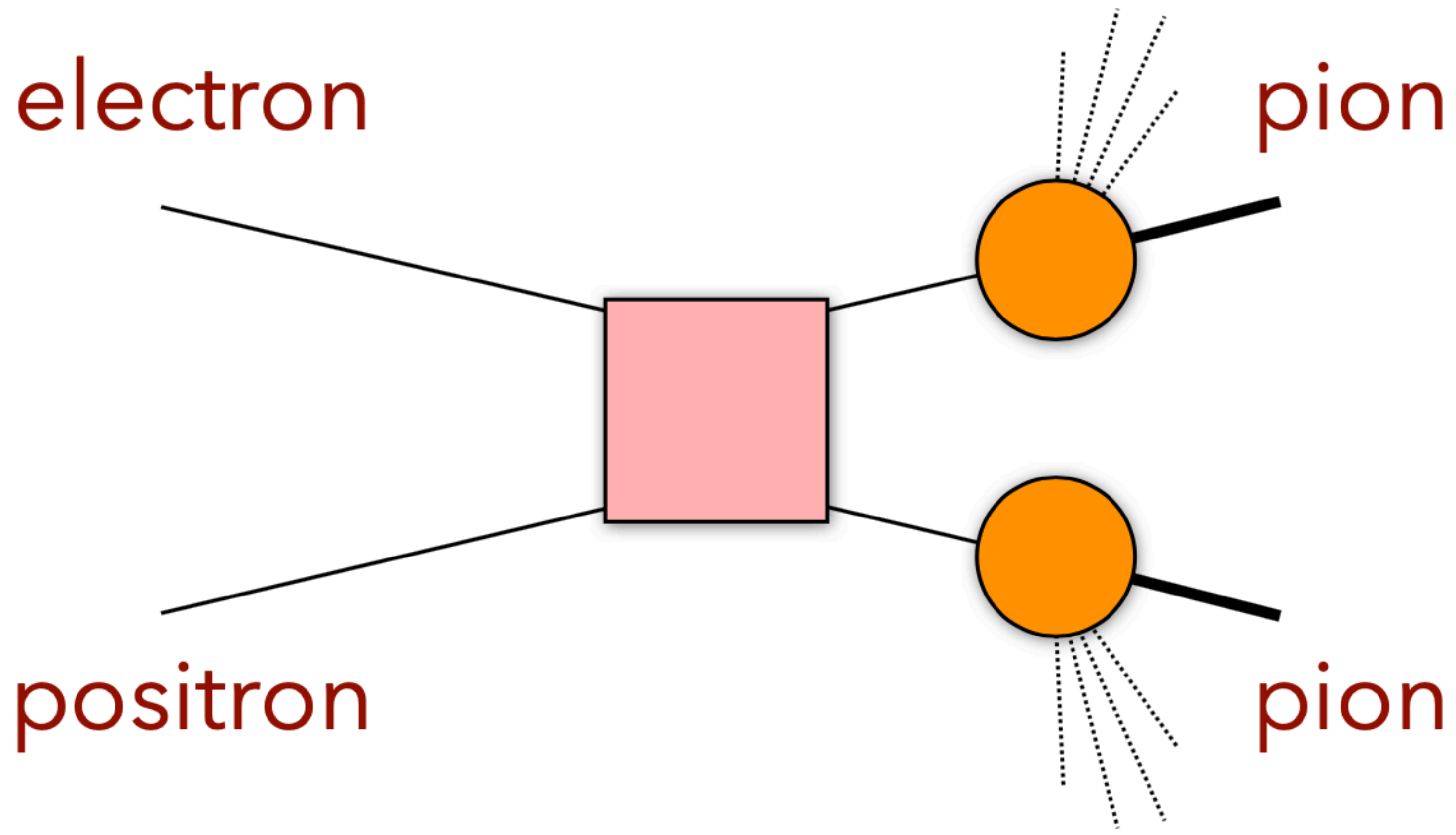
TMD PDFs



semi-inclusive DIS



Drell-Yan



e^-e^+ to pions

TMD FFs

TMD UNIVERSALITY

in these cases, TMD factorization is well understood

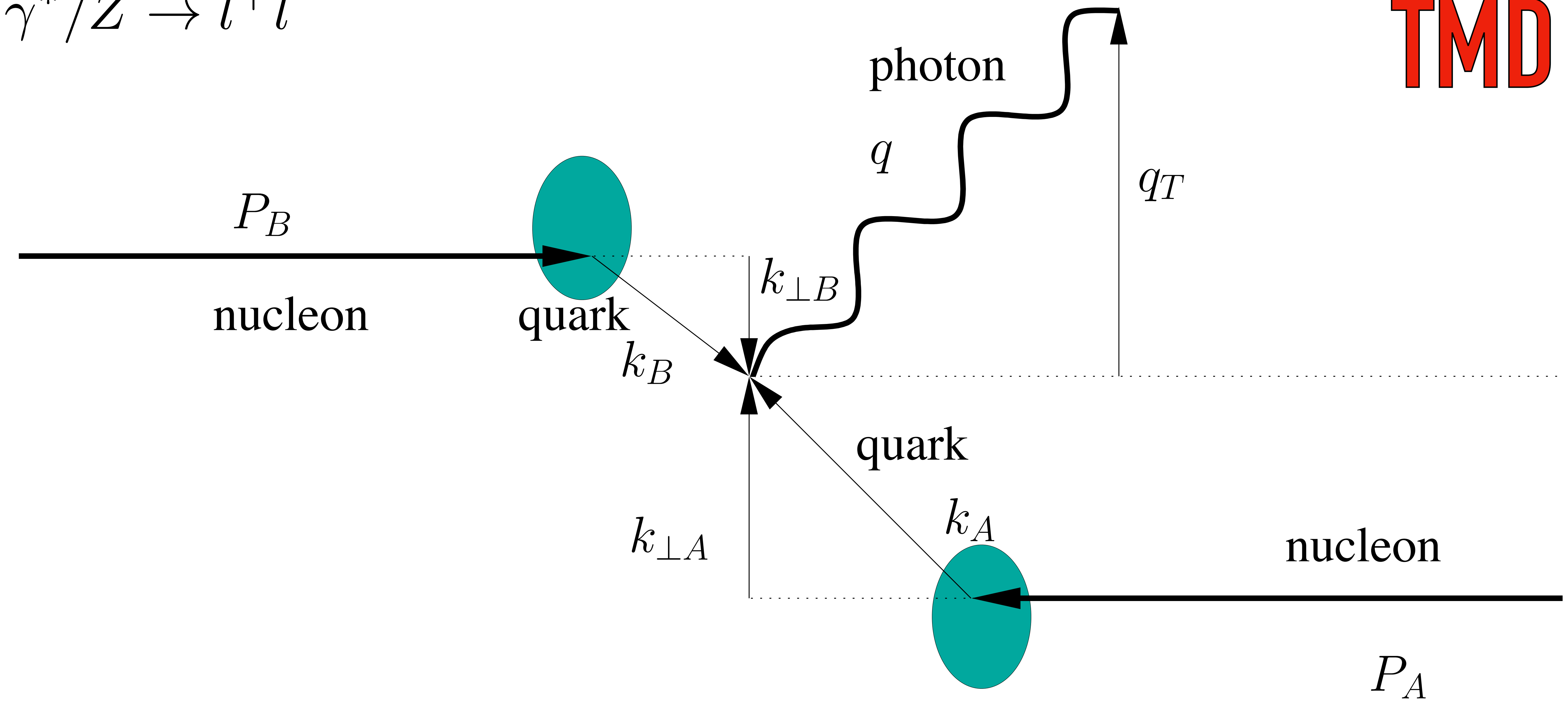
see, e.g., Ji, Ma, Yuan, PRD 71
Collins, "Foundations of Perturbative QCD"
Rogers, Aybat, PRD 83
Echevarria, Idilbi, Scimemi JHEP 1207

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization

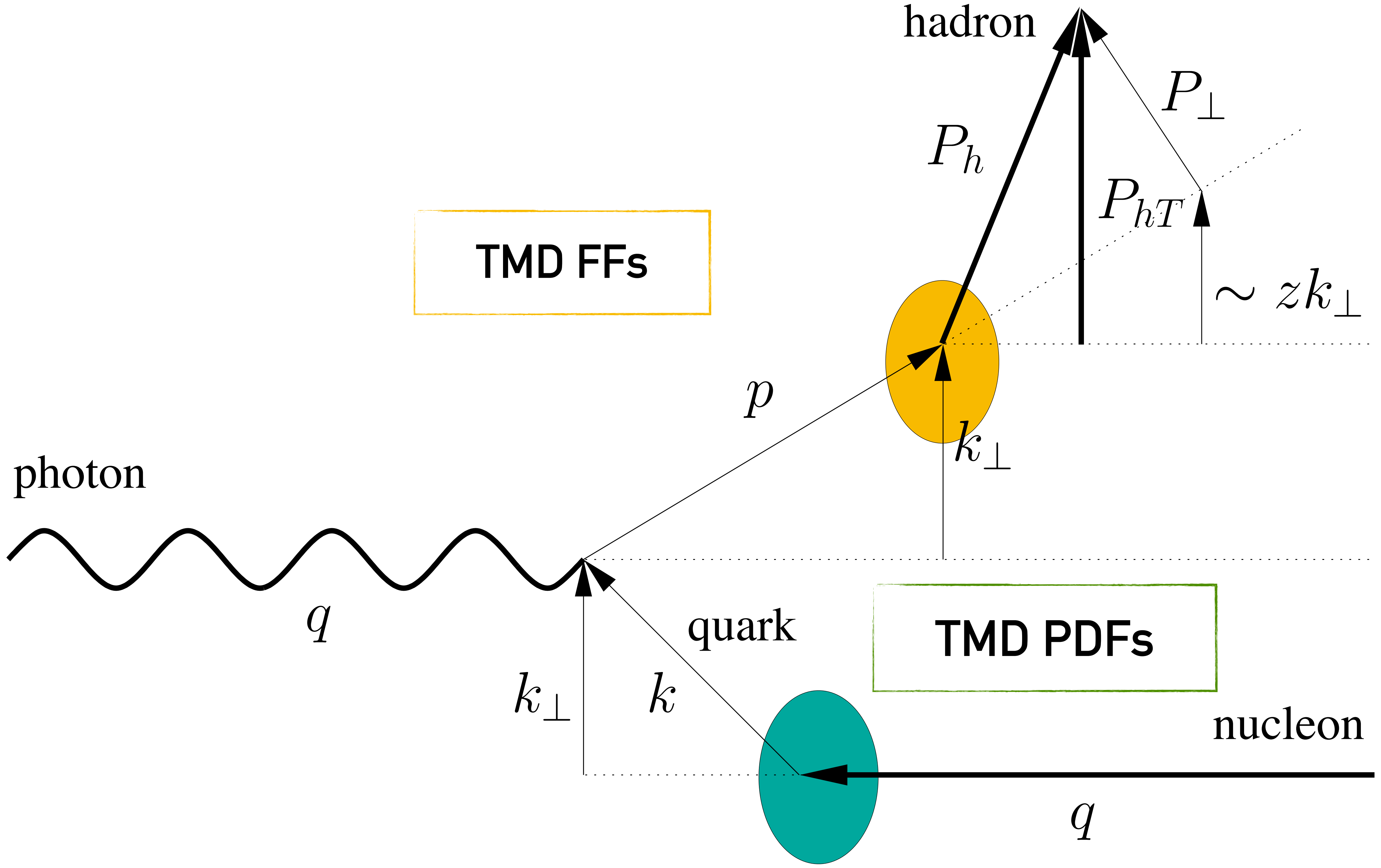


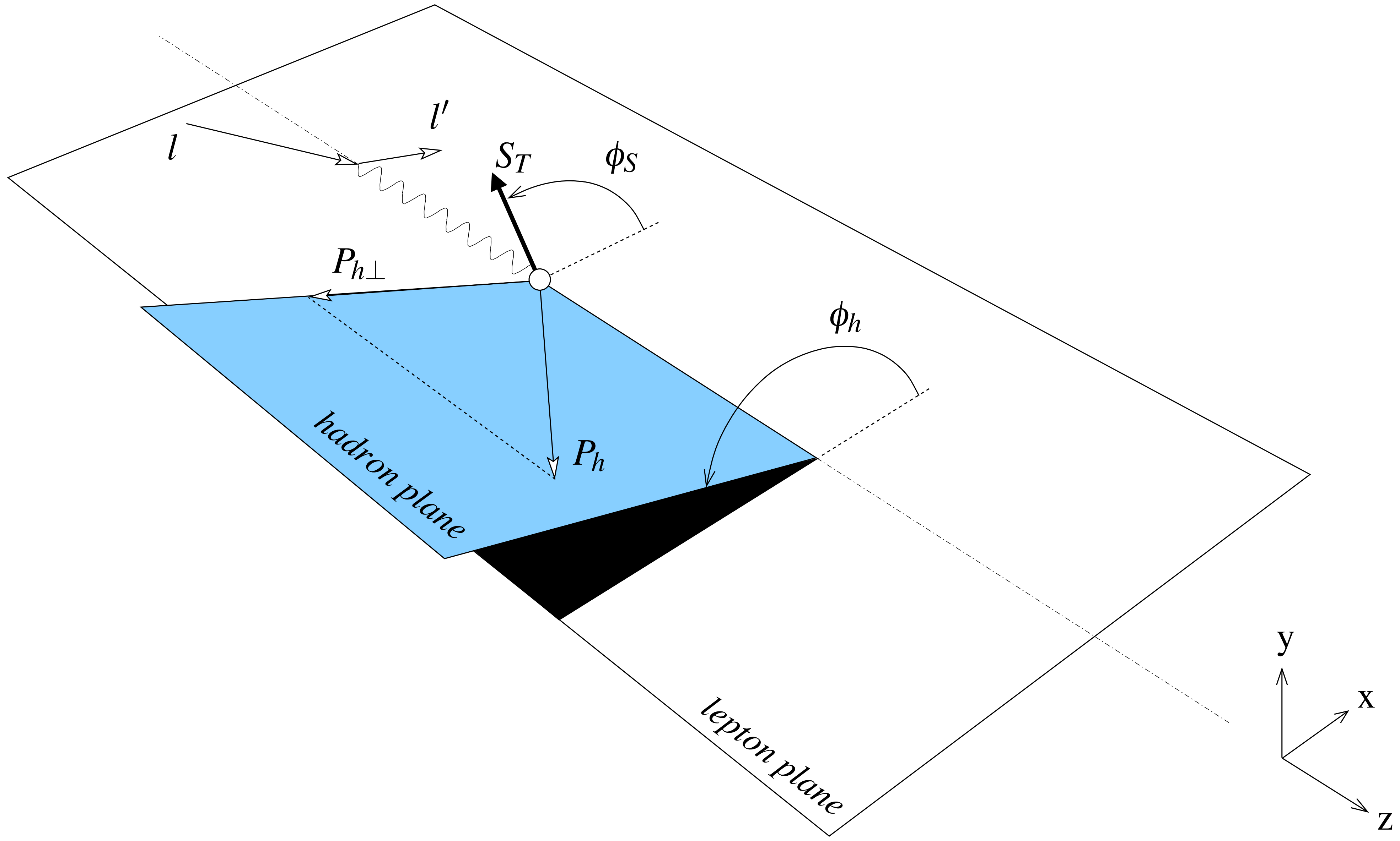
$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1, \mathbf{b}; \mu, \zeta_1) x_2 f_1^{\bar{q}}(x_2, \mathbf{b}; \mu, \zeta_2)$$

SIDIS

Semi-Inclusive Deep Inelastic Scattering

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$





TMD factorization

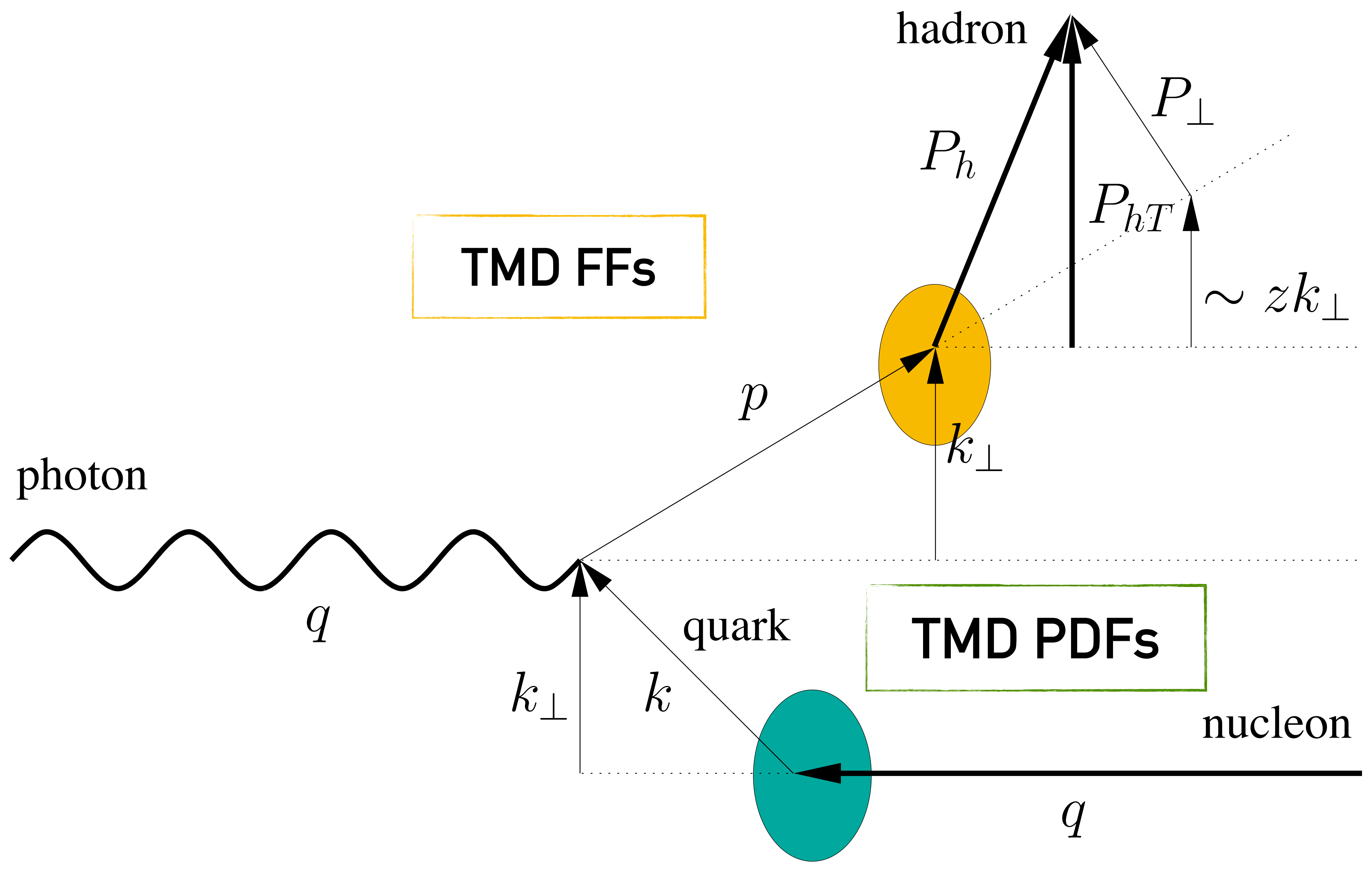
$$P_{hT}^2 \ll Q^2$$

$$\frac{d\sigma}{dq_T} \propto F_{UU,T}(x, z, q_T; Q^2) \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x, \mathbf{b}; \mu) D_1^{q\to h}(z, \mathbf{b}; \mu)$$

SIDIS

Semi-Inclusive Deep Inelastic Scattering

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2)$$
$$\times\, x \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x, \boldsymbol{k}_\perp^2; Q^2)\, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)\, \delta^{(2)}(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$+\, Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)\,.$$

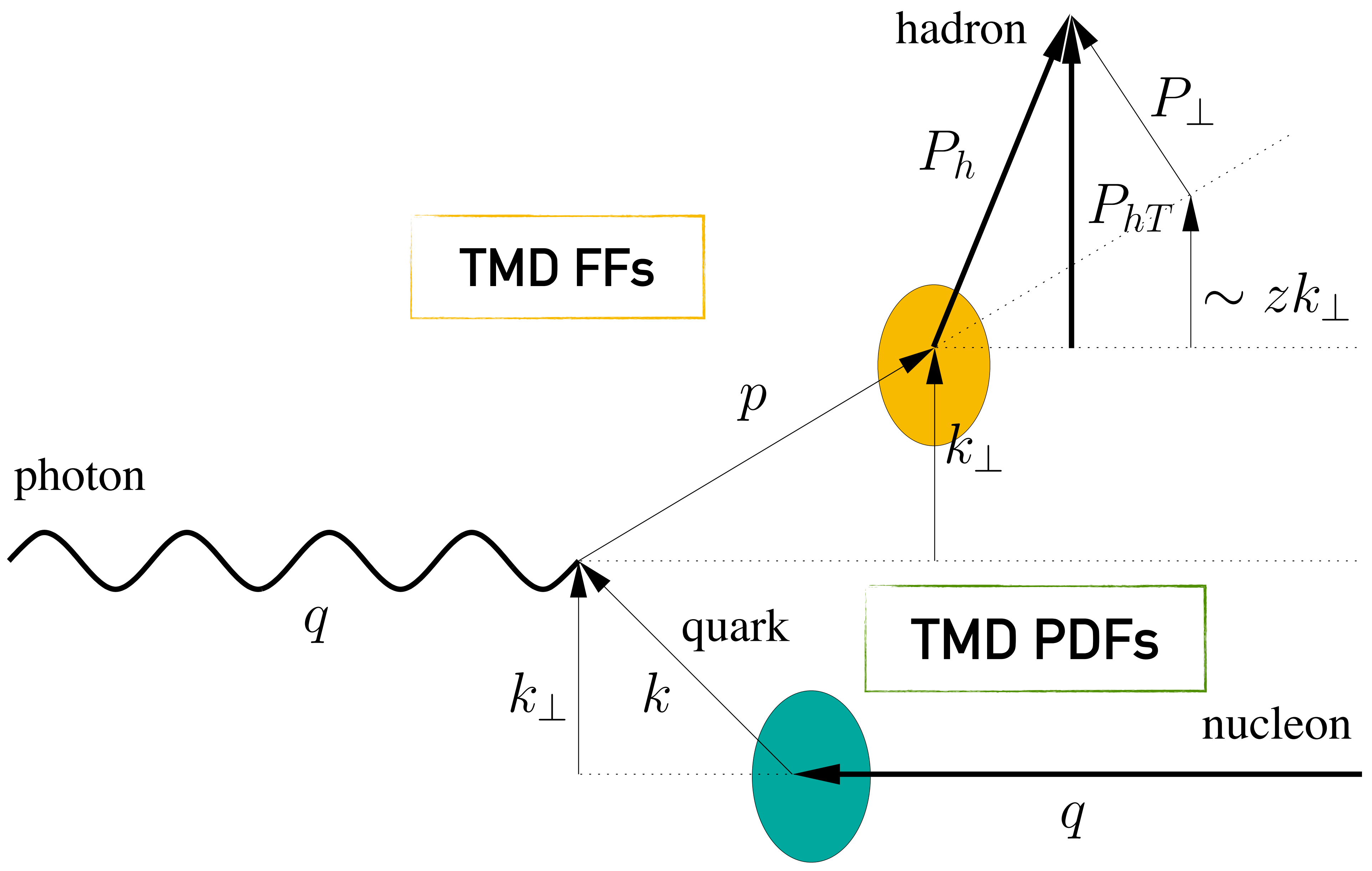
Y term

W term

SIDIS

Semi-Inclusive Deep Inelastic Scattering

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$



- W term dominates in the region where $P_{hT} \ll Q$

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2) \times x \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; Q^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2) \, \delta^{(2)}(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2) \, .$$

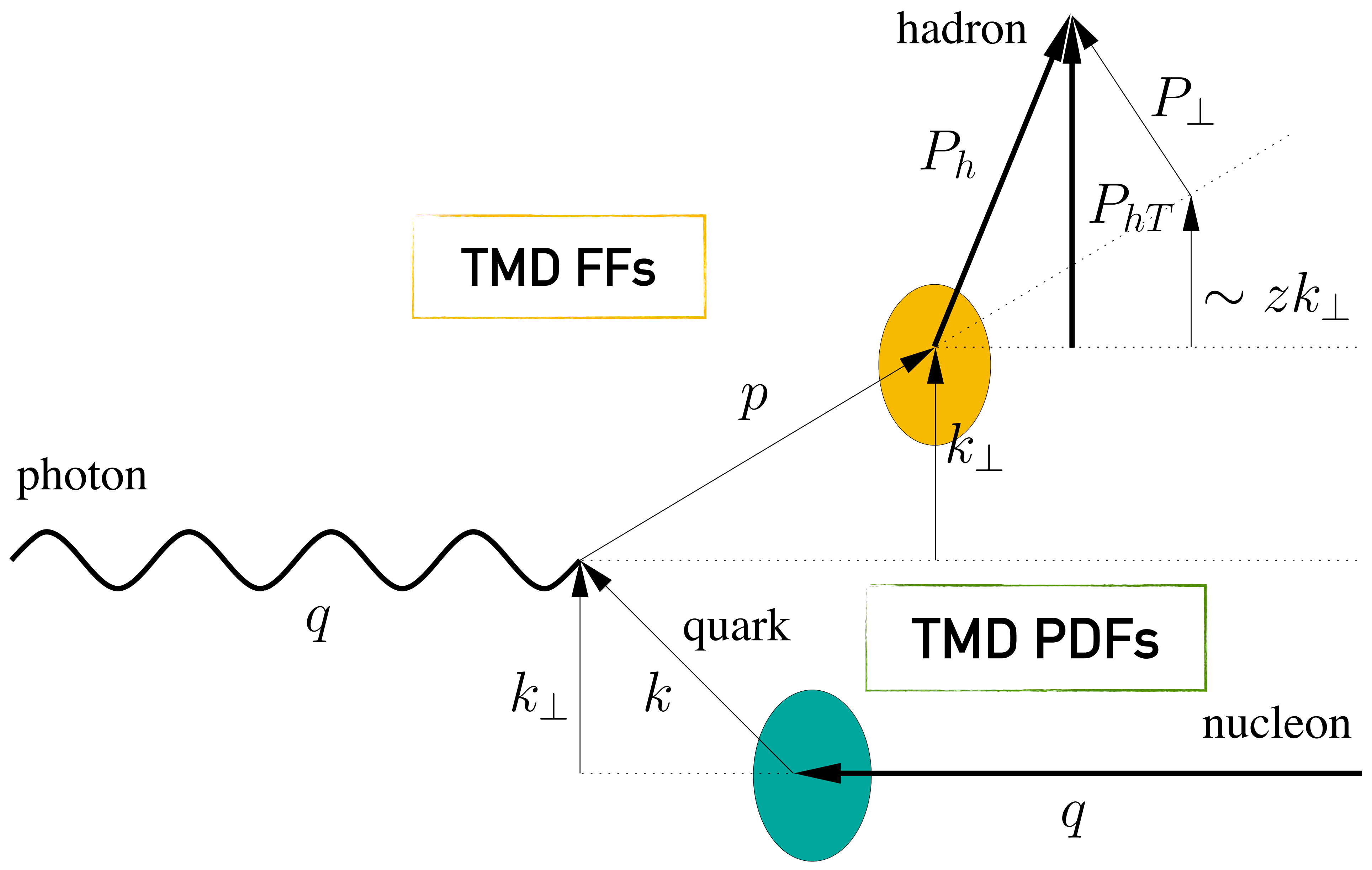
Y term

W term

SIDIS

Semi-Inclusive Deep Inelastic Scattering

$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$



- **W term** dominates in the region where $P_{hT} \ll Q$
- **Y term** not included in the Pavia analyses

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2) \times x \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x, \boldsymbol{k}_\perp^2; Q^2)\, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)\, \delta^{(2)}(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)\,.$$

Y term

W term

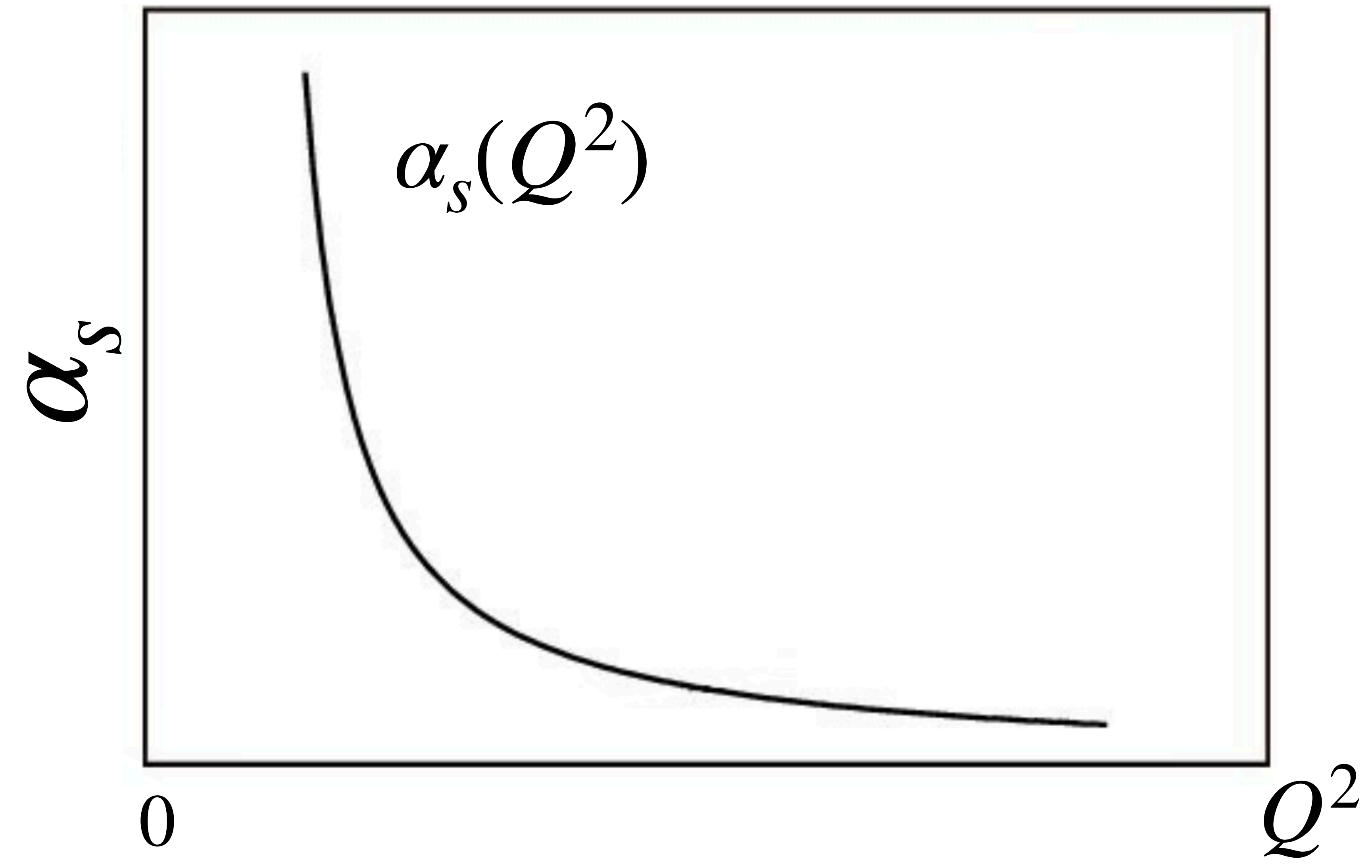
Behavior at large b_T

$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x_1,\mathbf{b}) f_1^{\bar{q}}(x_2,\mathbf{b})$$

integration up to infinity

when b_T becomes large

$$\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1$$



invalidates perturbative calculations

necessary to introduce a prescription

b* prescription and definition of $f_{\rm NP}$

perturbative

$$f(x,b;\mu,\zeta) = \left[\frac{f(x,b;\mu,\zeta)}{f(x,b_*(b);\mu,\zeta)}\right] f(x,b_*(b);\mu,\zeta)$$

non perturbative

$$f_{\rm NP}(x,b,\zeta)$$

fit to data

Non perturbative function depends on the choice of b*-prescription

b* prescription

$$\frac{d\sigma}{dq_T} \propto \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} f_1^q(x_1,\mathbf{b}) f_1^{\bar{q}}(x_2,\mathbf{b})$$

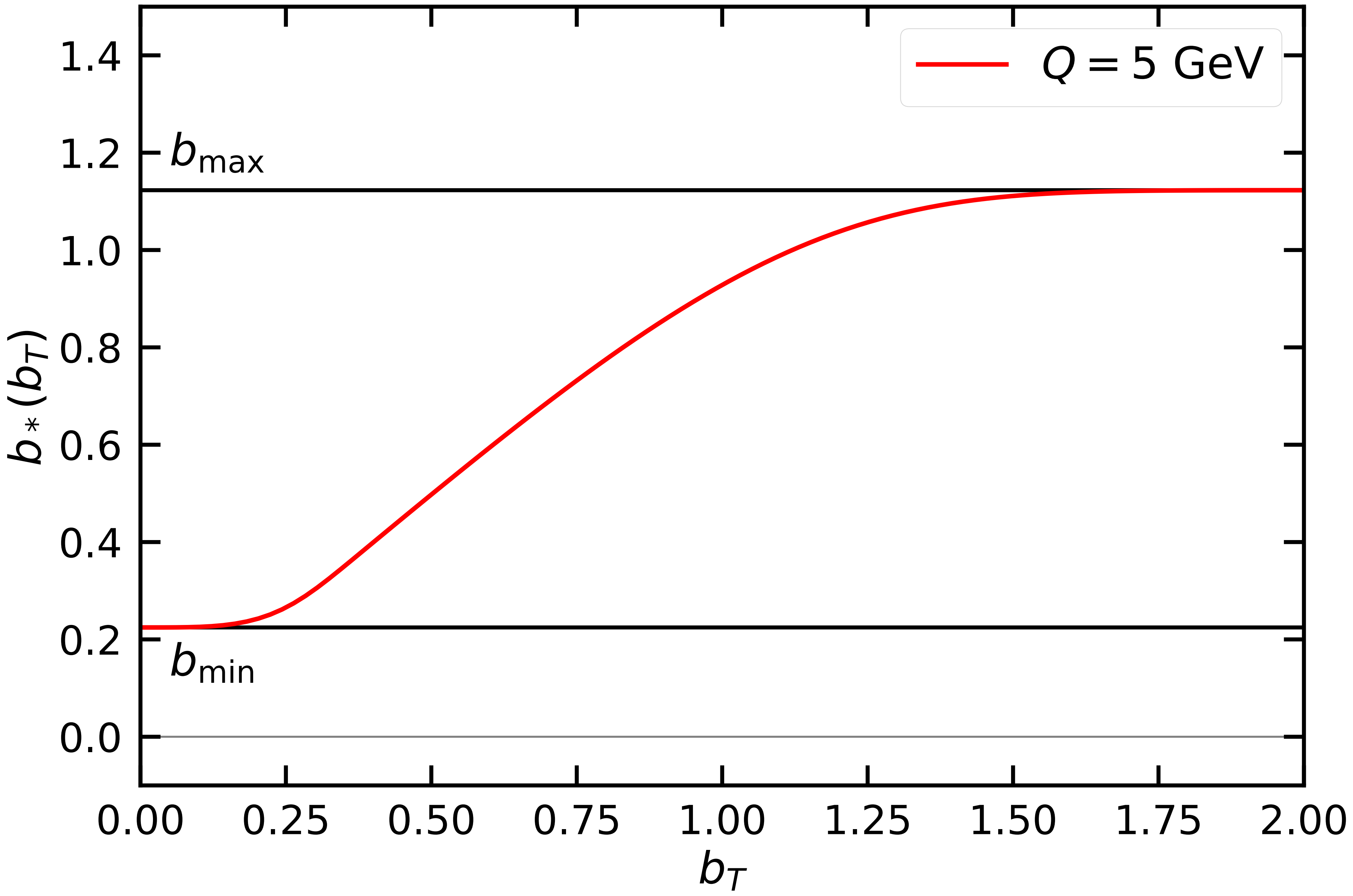
when b_T becomes large

→ $\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1$ → invalidates perturbative calculations $\Rightarrow b_{\max}$

$$b_{\max} = 2e^{-\gamma_E}$$

B-MIN CHOICE $b_{\min} = 2e^{-\gamma_E}/Q$

$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$



TMD PDFs

unpolarized Transverse Momentum Dependent Parton Distribution Functions

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\mathrm{NP}}(x, b)$$

TMD PDFs

unpolarized Transverse Momentum Dependent Parton Distribution Functions

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\rm NP}(x, b)$$

TMD PDFs

unpolarized Transverse Momentum Dependent Parton Distribution Functions

matching to the collinear region

collinear PDFs

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\rm NP}(x,b)$$

perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

TMD PDFs

unpolarized Transverse Momentum Dependent Parton Distribution Functions

matching to the collinear region

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\mathrm{NP}}(x, b)$$

perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

$$L = \ln \frac{Q^2}{\mu_b^2}$$

resummation of large logarithms

TMD PDFs

unpolarized Transverse Momentum Dependent Parton Distribution Functions

matching to the collinear region

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\rm NP}(x, b)$$

perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

non perturbative transverse content

$$L = \ln \frac{Q^2}{\mu_b^2}$$

resummation of large logarithms

parametrized and fitted to data

Logarithmic accuracy

$$\left(\frac{d\sigma}{dq_T}\right) \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1,\mathbf{b};\mu,\zeta_1) x_2 f_1^{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

perturbative expansion in $\alpha_s(\mu)$

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_{q/j} \otimes f^j\right)(x,b_*;\mu_b)$$
$$\times \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$
$$\times f_{\rm NP}(x,b;\zeta)$$

Accuracy	γ_K	γ_F	K	$C_{f/j}$	H
LL	α_s	-	-	1	1
NLL	α_s^2	α_s	α_s	1	1
NLL'	α_s^2	α_s	α_s	α_s	α_s
N²LL	α_s^3	α_s^2	α_s^2	α_s	α_s
N²LL'	α_s^3	α_s^2	α_s^2	α_s^2	α_s^2
N³LL	α_s^4	α_s^3	α_s^3	α_s^2	α_s^2
N³LL'	α_s^4	α_s^3	α_s^3	α_s^3	α_s^3

Logarithmic accuracy

$$\left(\frac{d\sigma}{dq_T}\right) \propto H(Q,\mu) \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} x_1 f_1^q(x_1,\mathbf{b};\mu,\zeta_1) x_2 f_1^{\bar{q}}(x_2,\mathbf{b};\mu,\zeta_2)$$

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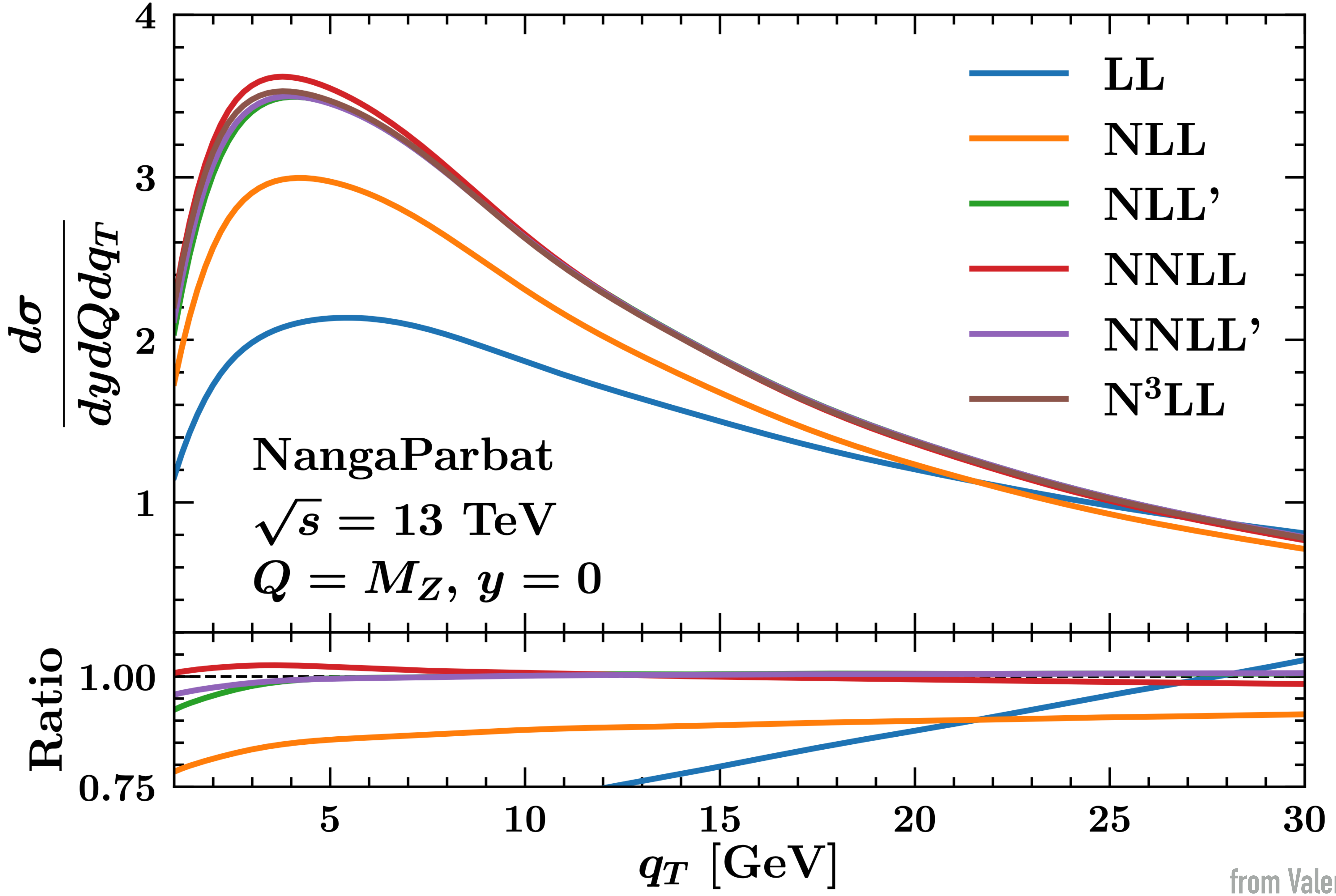
$$\times \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$
$$\times f_{NP}(x,b;\zeta)$$

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL	2	3	4	NNLO

Perturbative convergence



from Valerio Bertone's talk at
https://indico.cern.ch/event/849342/

Recent TMD fits of unpolarized data

	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL′	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

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SV 2019 arXiv:1912.06532	NNLL′	✔	✔	✔	✔	1039	1.06
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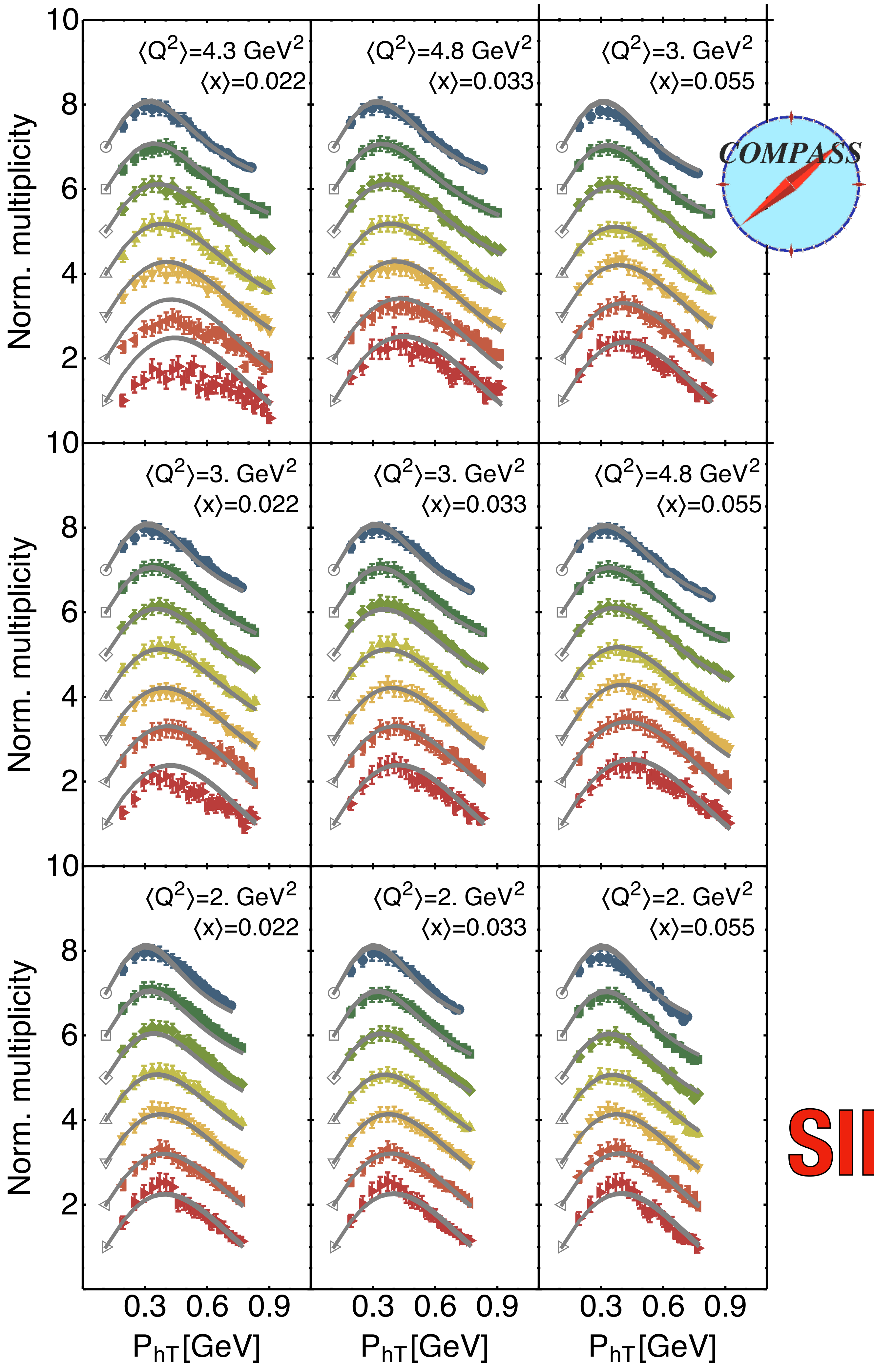
Pavia 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.02

PV17

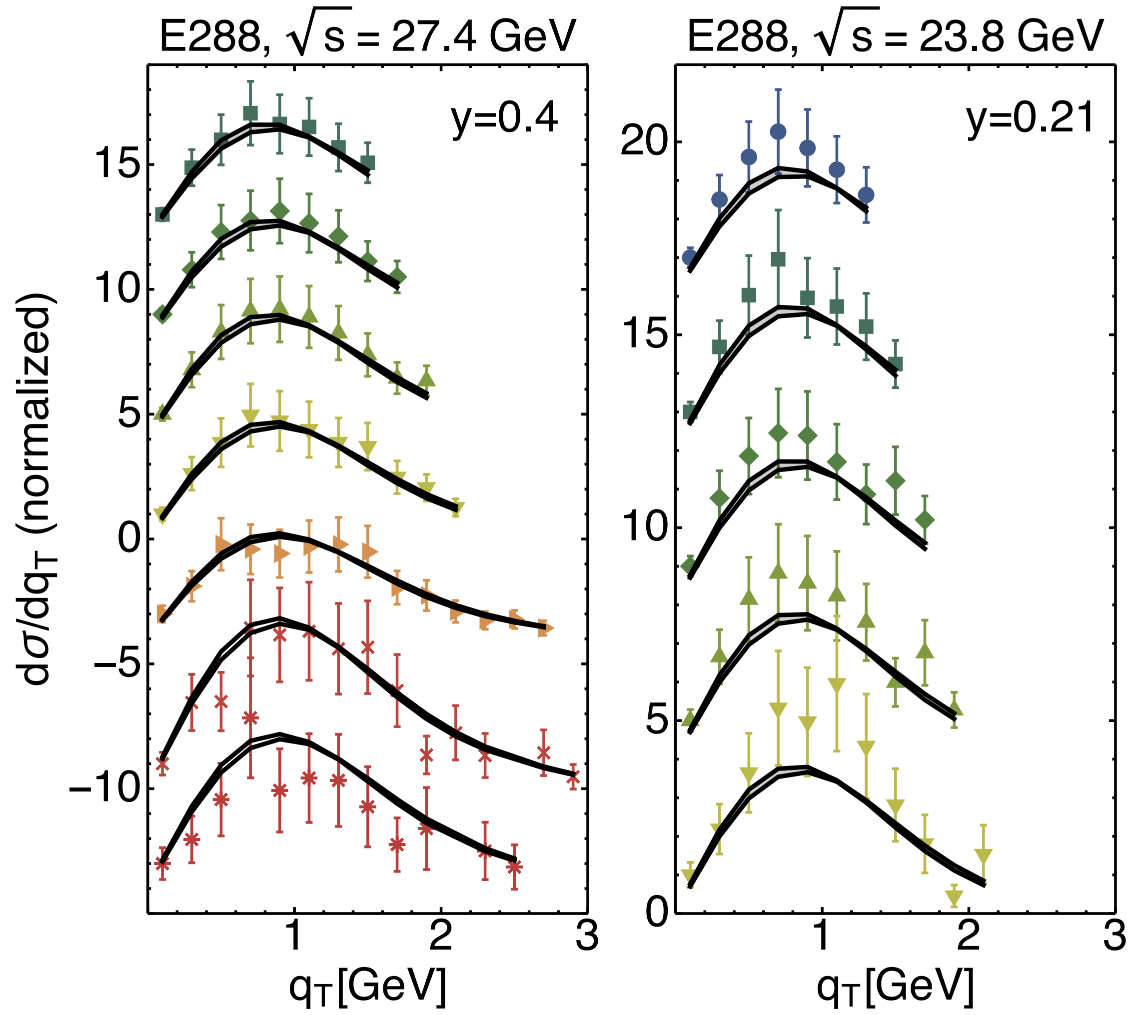
Bacchetta, Delcarro, Pisano, Radici, Signori

arXiv:1703.10157



SIDIS

Fermilab

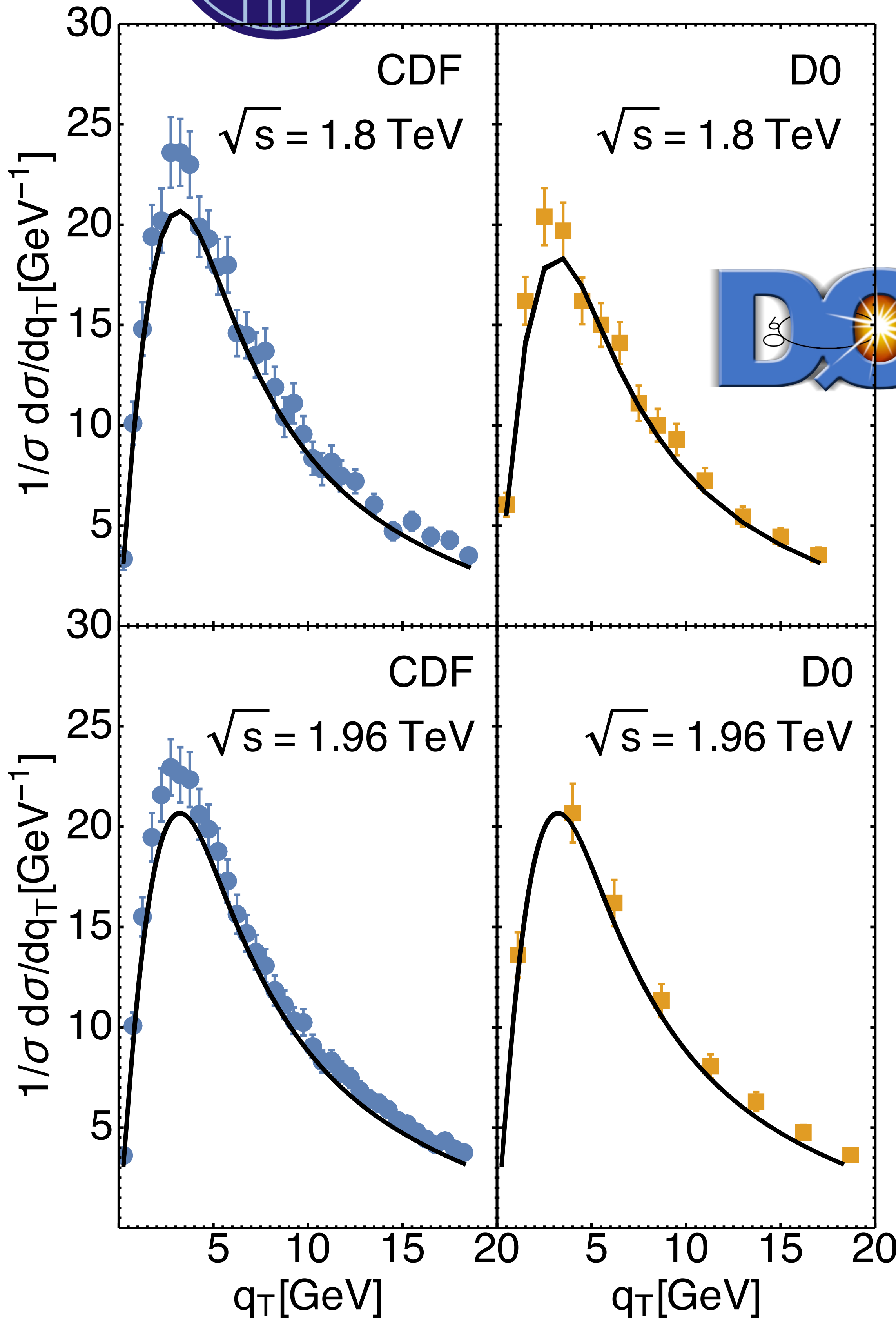


Drell-Yan

global $\chi^2 = 1.55$

NLL

with normalization coefficients



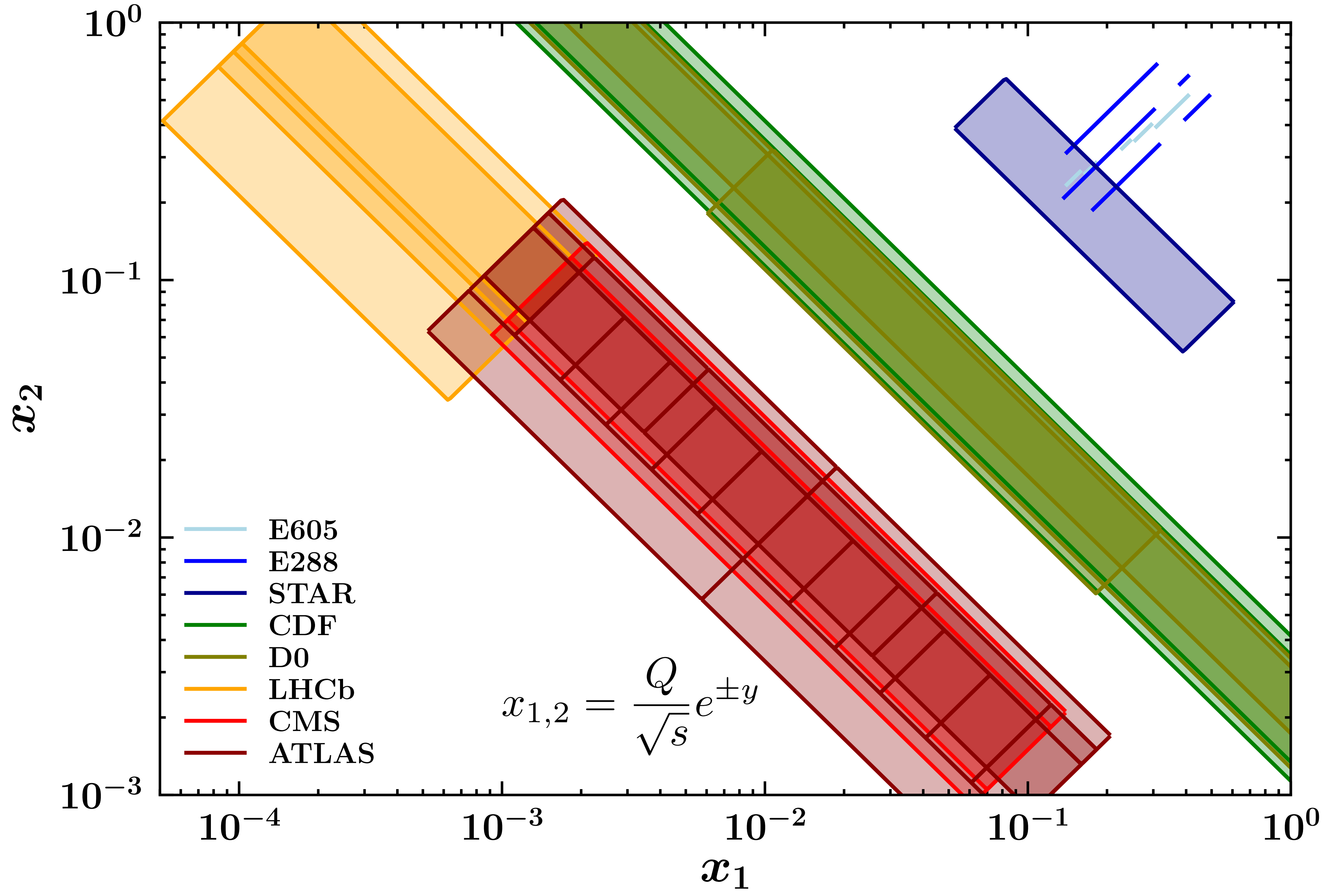
Z production

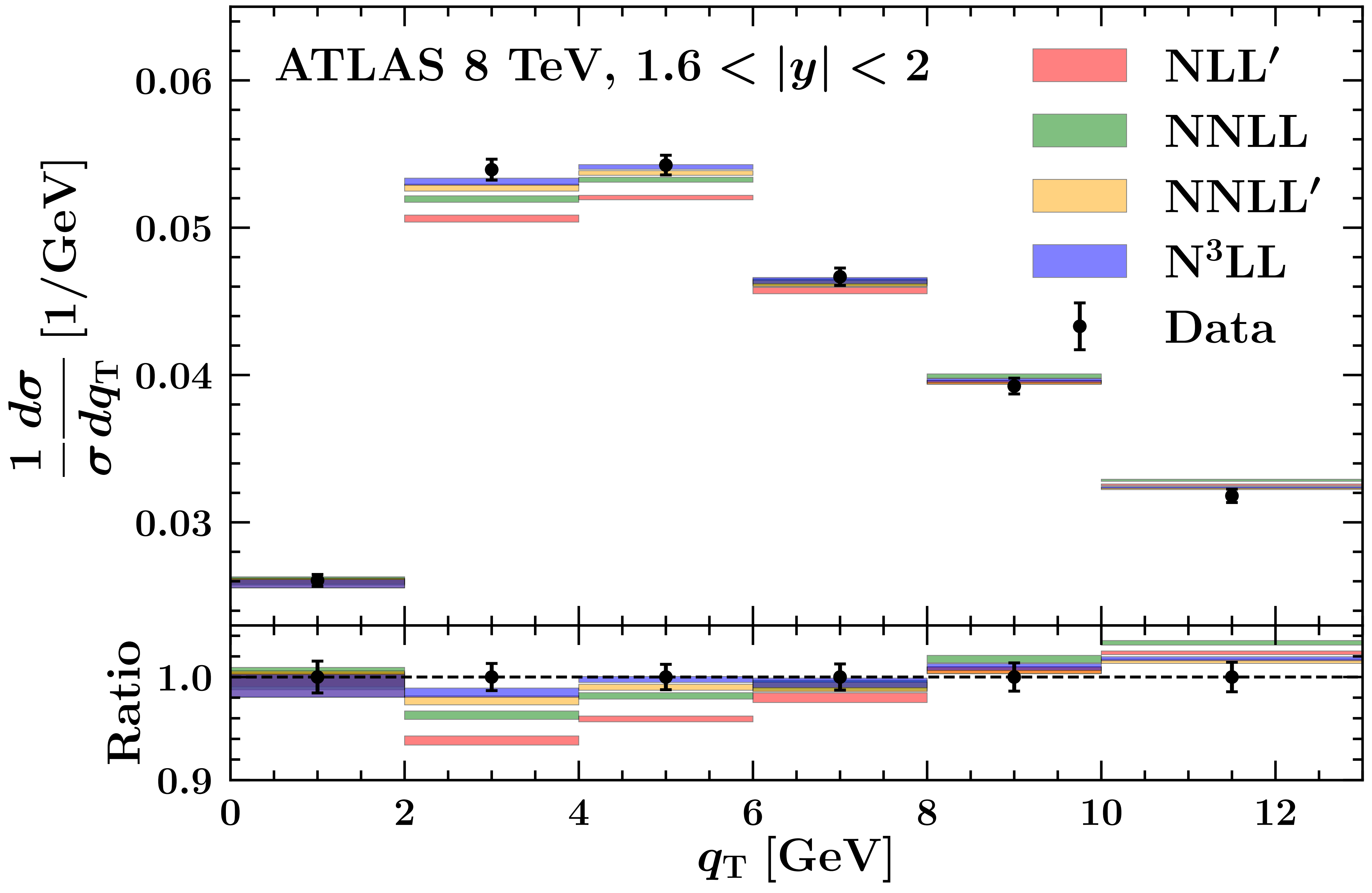
N3LL Drell-Yan fit

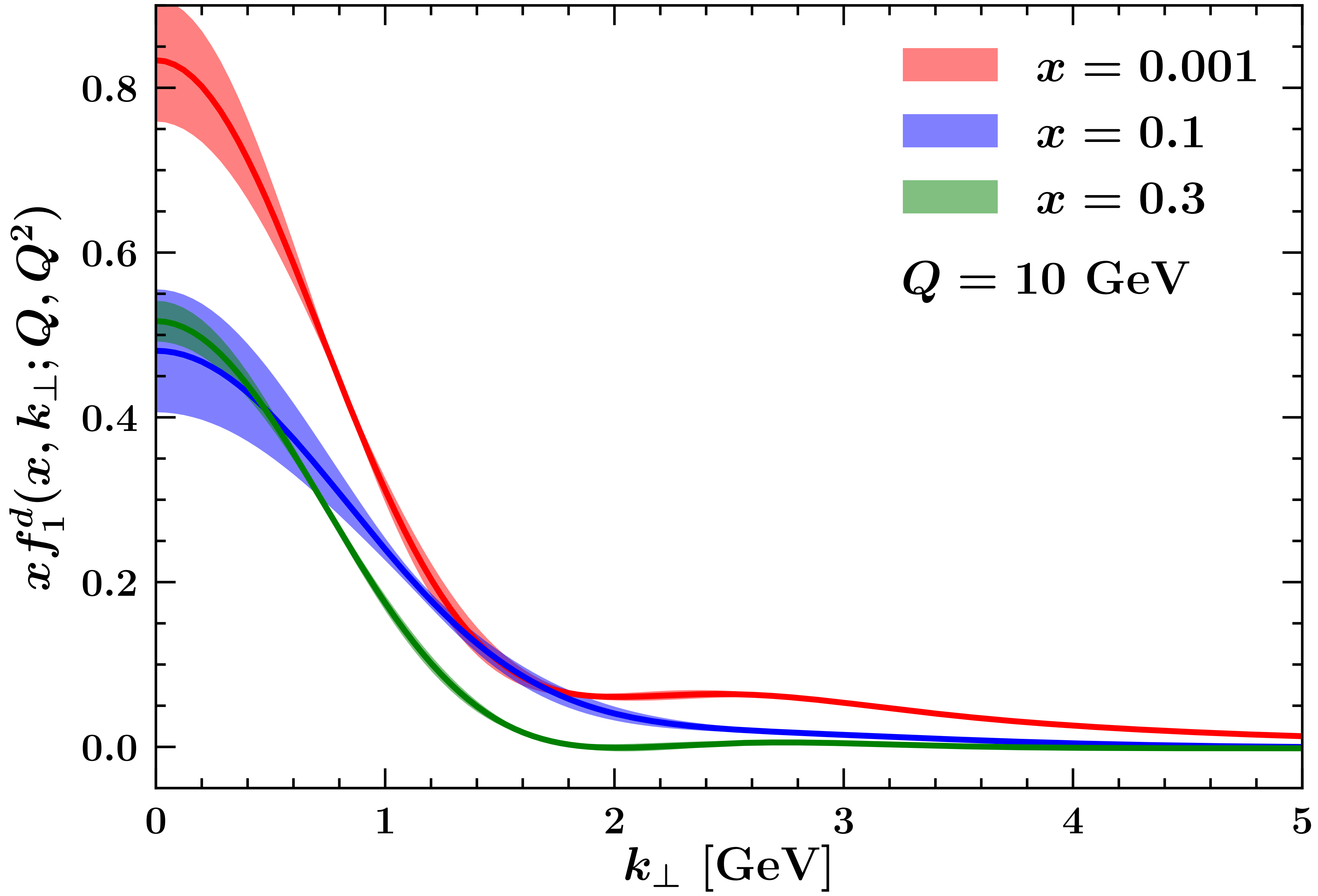
A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, M. Radici
JHEP 07 (2020) 117 e-Print: 1912.07550

NO normalization coefficients

global $\chi^2 = 1.07$







New extraction of unpolarized quark TMDs

in preparation

- **GLOBAL ANALYSIS** of Drell-Yan and Semi-Inclusive DIS data sets → **2031** data points

- Perturbative accuracy: N^3LL^-

- **Normalization** of SIDIS multiplicities beyond NLL

- Extremely good description: $\chi^2/N_{\rm data} \simeq 1$

Global analysis of DY and SIDIS data sets

Cuts on kinematics

$\langle Q \rangle > 1.3$ GeV

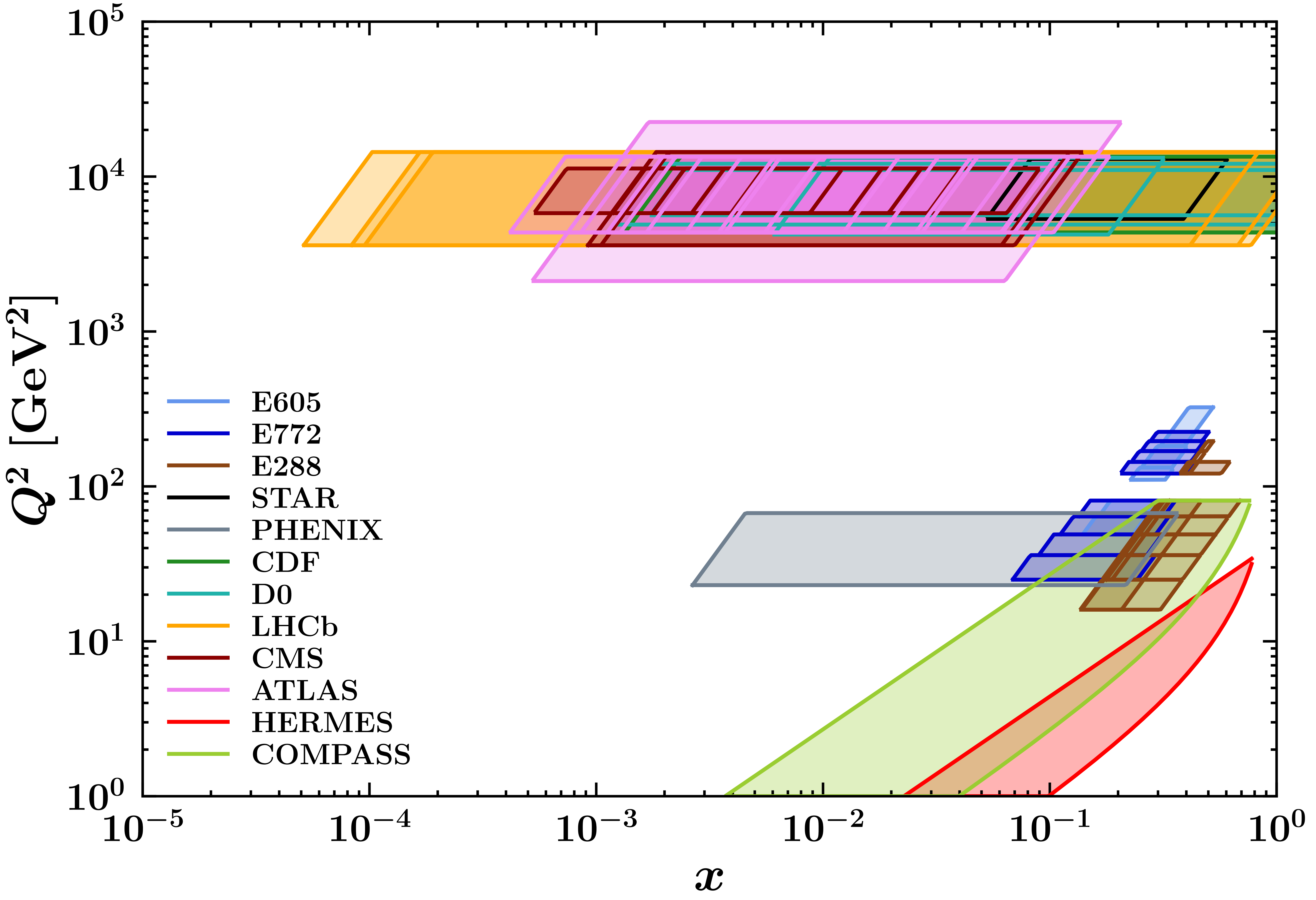
$0.2 < \langle z \rangle < 0.7$

DY

$q_T|_{\max} = 0.2Q$

SIDIS

$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$



Total number of points = 2031

Perturbative accuracy: N^3LL^-

Orders in powers of α_s

Hard factor and matching coefficient → H and C

Ingredients in perturbative Sudakov form factor → K and γ_F, γ_K

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NLO (FF only)
N^3LL	2	3	4	NNLO
N^3LL'	3	3	4	N^3LO

Extraction of unpolarized quark TMDs

What's new?

- **GLOBAL ANALYSIS** of Drell-Yan and Semi-Inclusive DIS data sets → **2031** data points
- Perturbative accuracy: N^3LL^-
- **Normalization** of SIDIS multiplicities beyond NLL

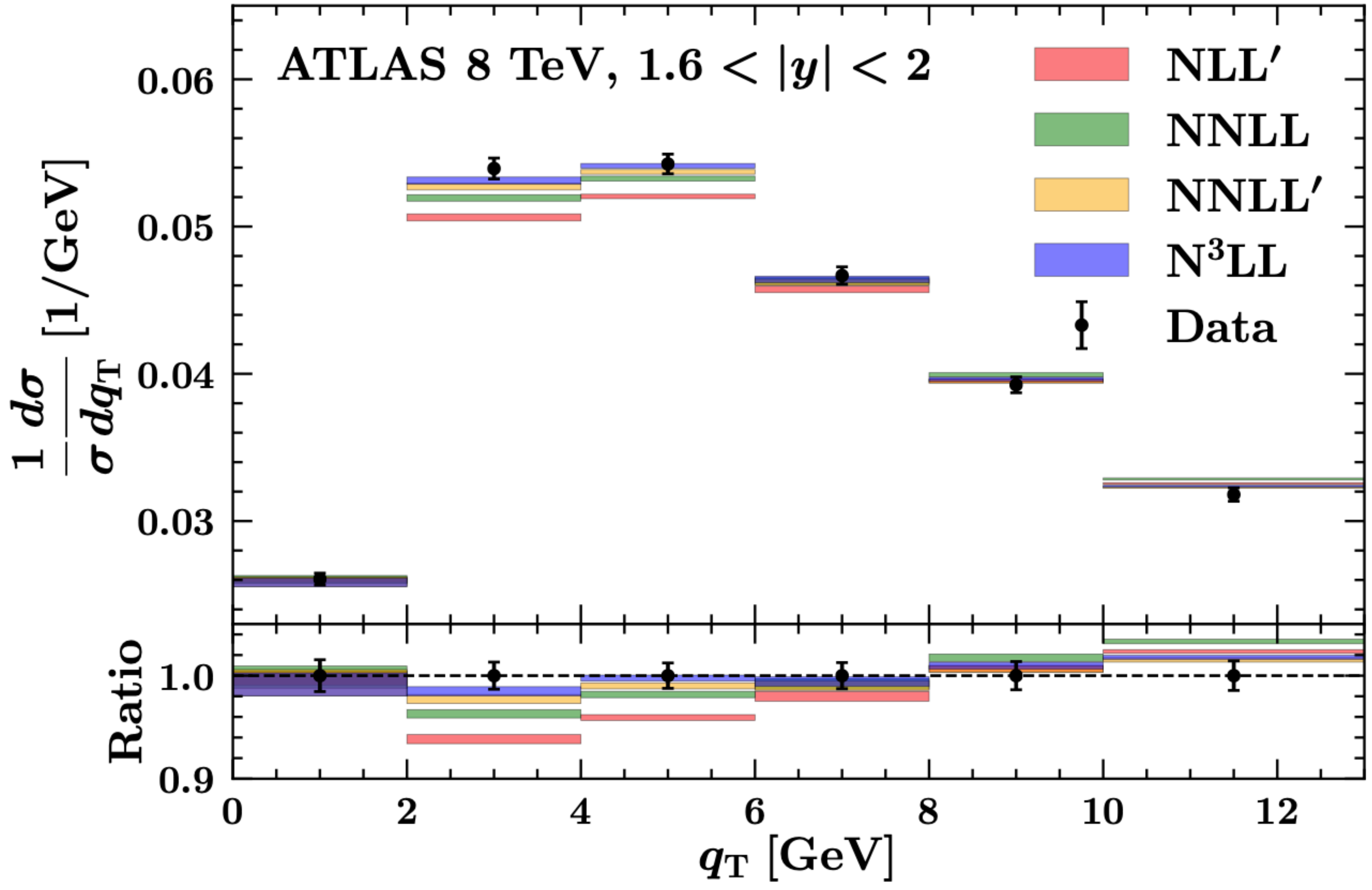
WHY?

- Extremely good description: $\chi^2/N_{\rm data} \simeq 1$

Normalization of SIDIS multiplicities

High-Energy Drell-Yan beyond NLL

$Q \sim 100$ GeV

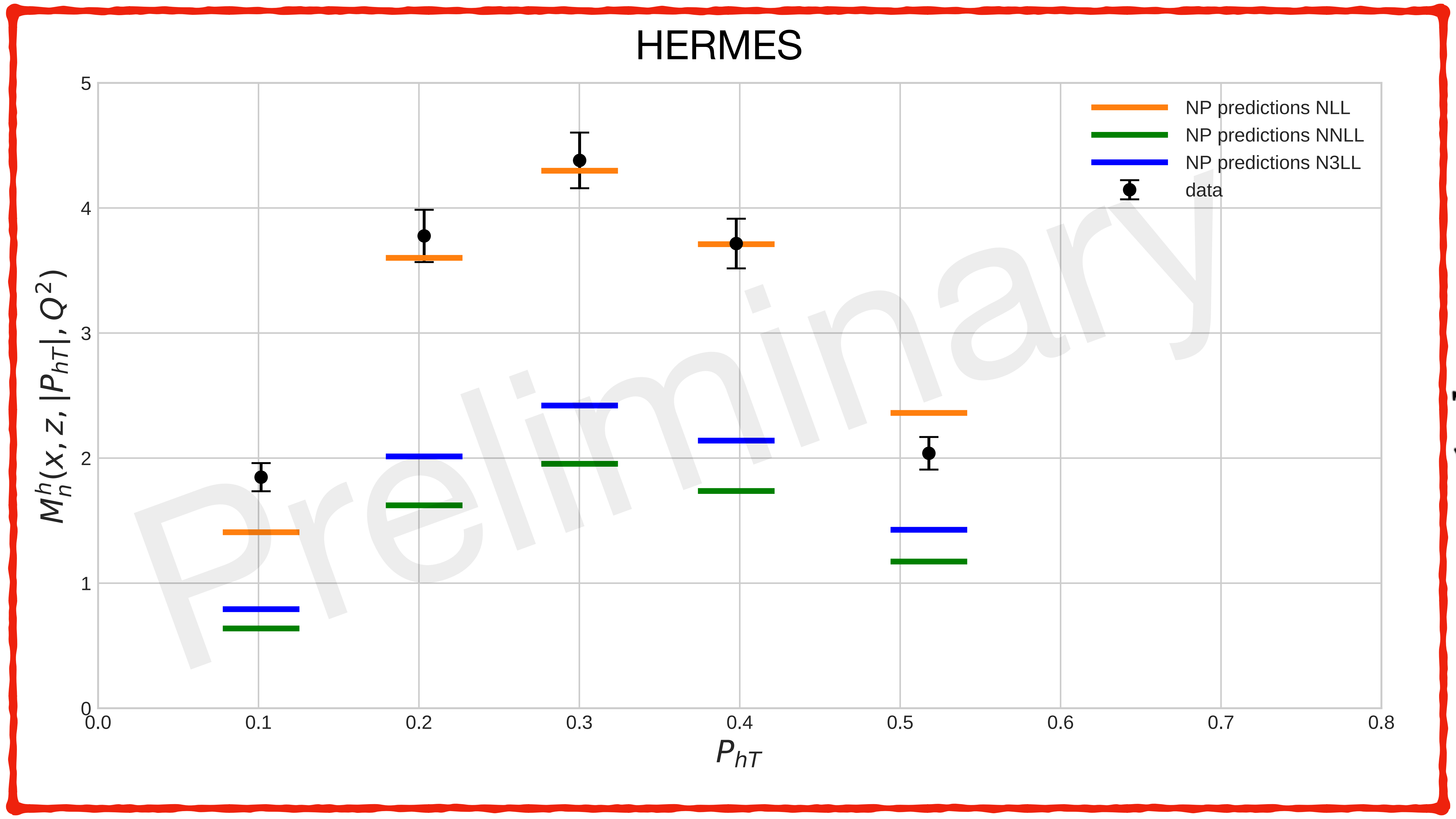


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

Normalization of SIDIS multiplicities

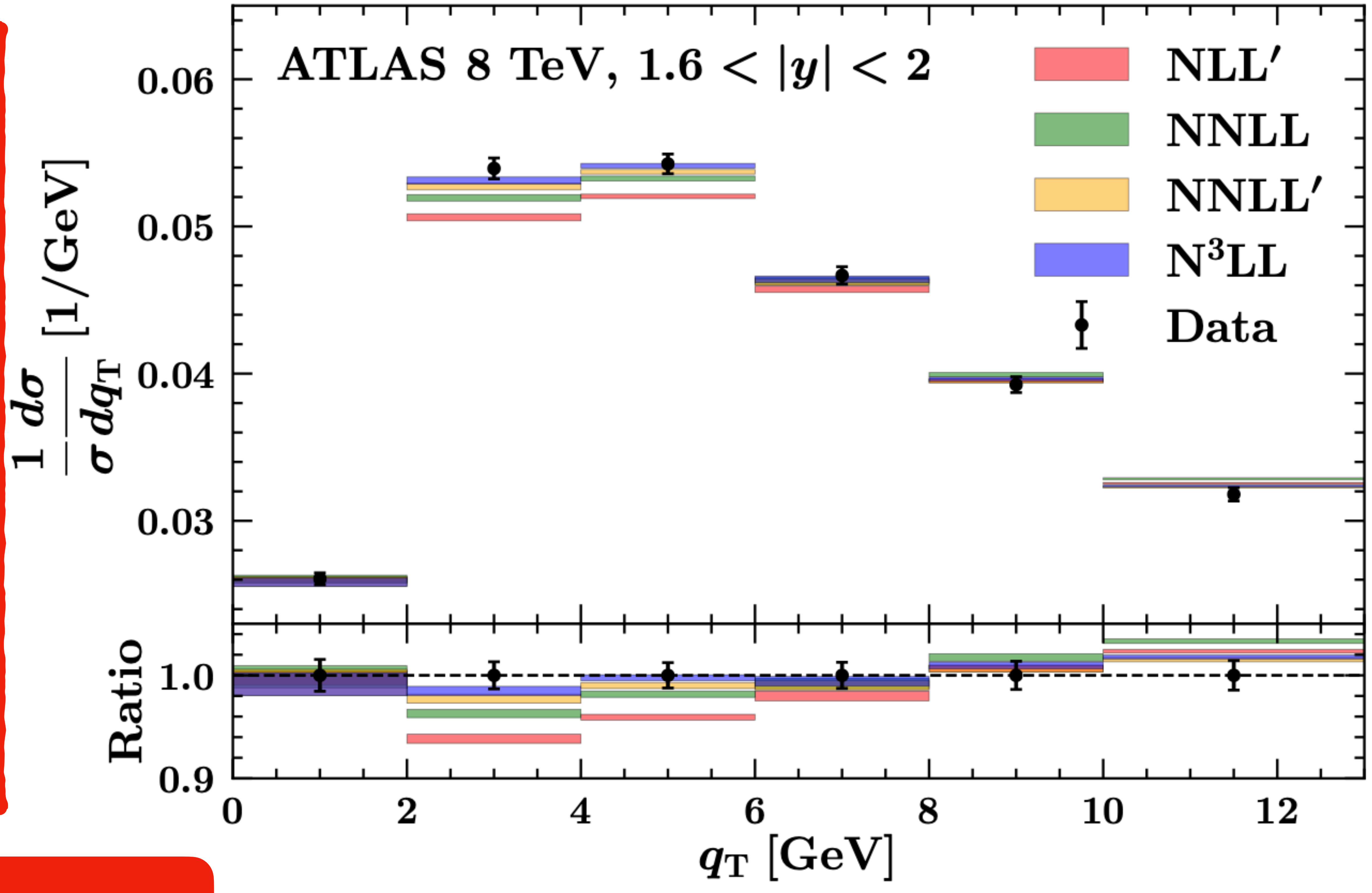
SIDIS multiplicities beyond NLL

$Q \sim 2$ GeV



High-Energy Drell-Yan beyond NLL

$Q \sim 100$ GeV



description considerably worsens at higher orders

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

Source of W term suppression

Hard factor

$$\mathcal{H}_{ab}^{\text{SIDIS}}(Q,Q) = e_a^2\delta_{ab}\left(1+\frac{\alpha_S}{4\pi}C_F\left(-16+\frac{\pi^2}{3}\right)\right)$$

Source of W term suppression

Hard factor

$$\mathcal{H}_{ab}^{\text{SIDIS}}(Q,Q) = e_a^2 \delta_{ab}\left(1 + \frac{\alpha_S}{4\pi} C_F\left(-16 + \frac{\pi^2}{3}\right)\right)$$

- introducing $\mathcal{O}(\alpha_s)$ terms

reduces the structure function to about 60% of its original value.

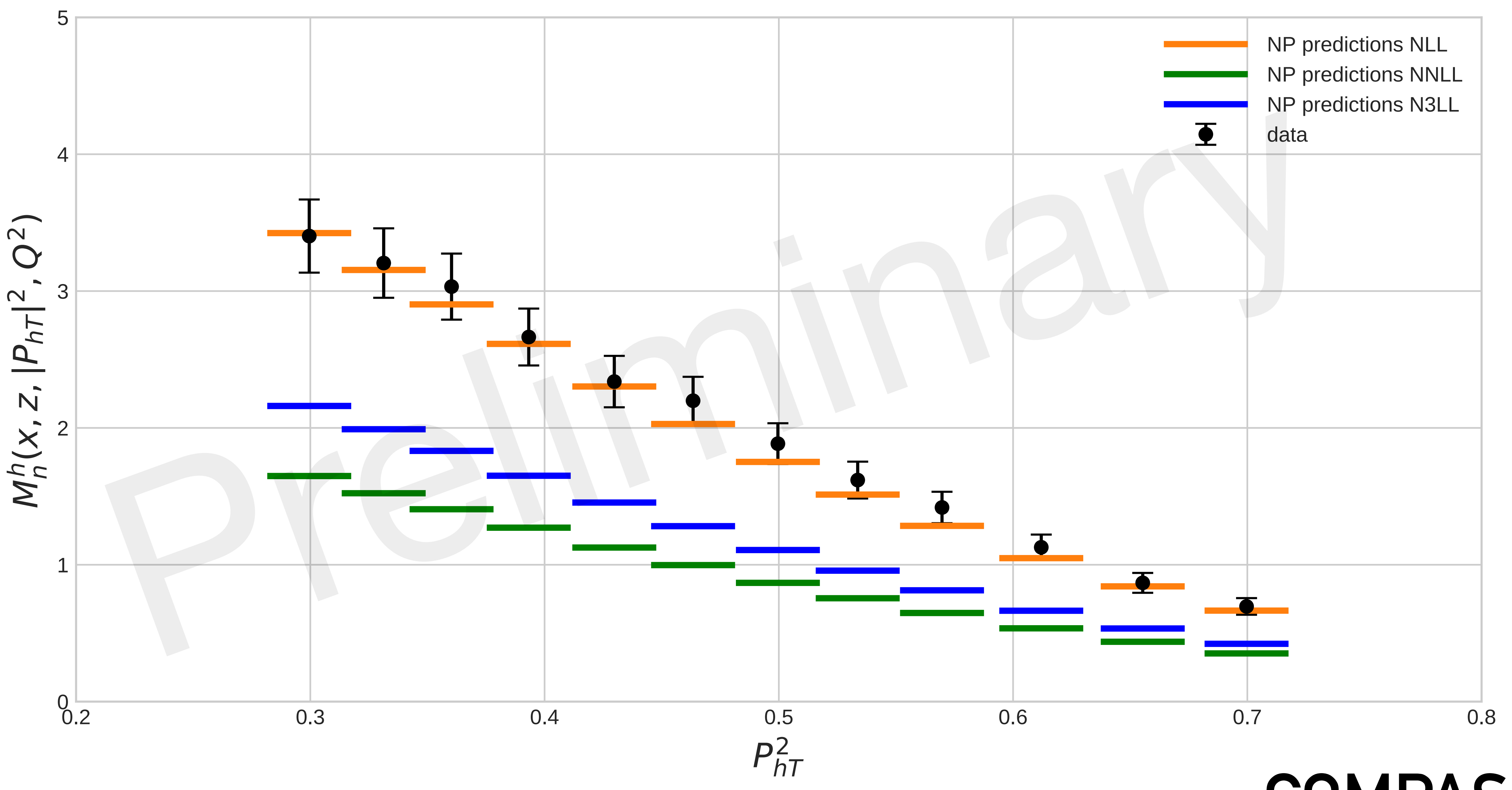
Source of W term suppression

Hard factor

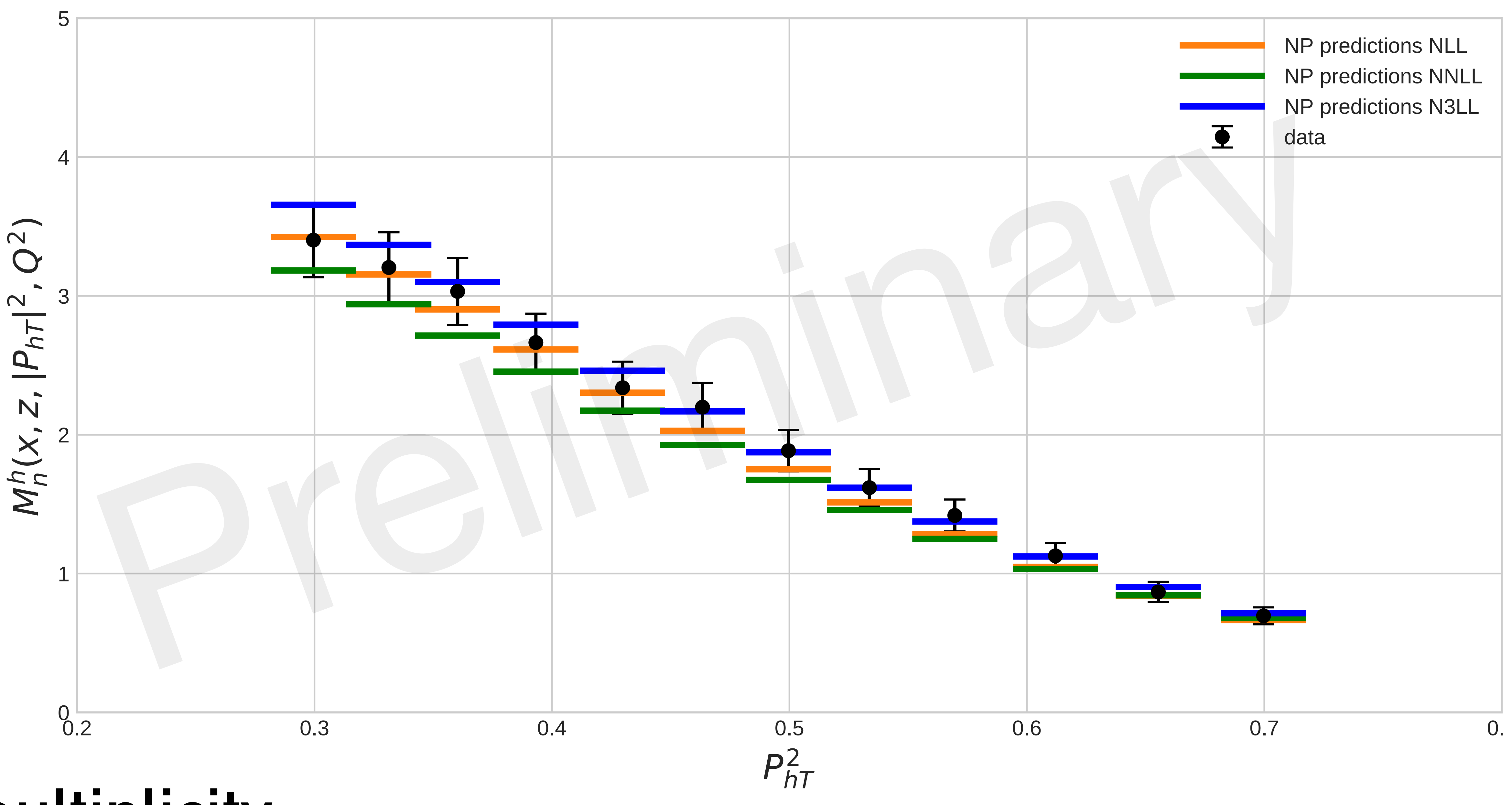
$$\mathcal{H}_{ab}^{\mathrm{SIDIS}}(Q,Q) = e_a^2 \delta_{ab}\left(1 + \boxed{\frac{\alpha_S}{4\pi} C_F\left(-16 + \frac{\pi^2}{3}\right)}\right)$$

- introducing $\mathcal{O}(\alpha_s)$ terms → reduces the structure function to about 60% of its original value.

Full Hard Factor



Hard Factor = 1



COMPASS multiplicity

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

(at low P_{hT})

$$\int W\Big|_{O(\alpha_S)} \sim \frac{d\sigma^h}{dx dQ^2 dz}\Big|_{O(\alpha_S)}$$

integral of the TMD formula

collinear cross section

we would expect

Normalization of SIDIS multiplicities

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$$\int W\Big|_{O(\alpha_S)} \sim \frac{d\sigma^h}{dx\,dQ^2\,dz}\Big|_{O(\alpha_S)}$$

integral of the TMD formula

collinear cross section

we would expect

BUT

Normalization of SIDIS multiplicities

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$$\int W\Big|_{O(\alpha_S)} \sim \frac{d\sigma^h}{dx dQ^2 dz}\Big|_{O(\alpha_S)}$$

integral of the TMD formula

collinear cross section

we would expect

BUT

this is not the case in the experimental regions under consideration

Normalization of SIDIS multiplicities

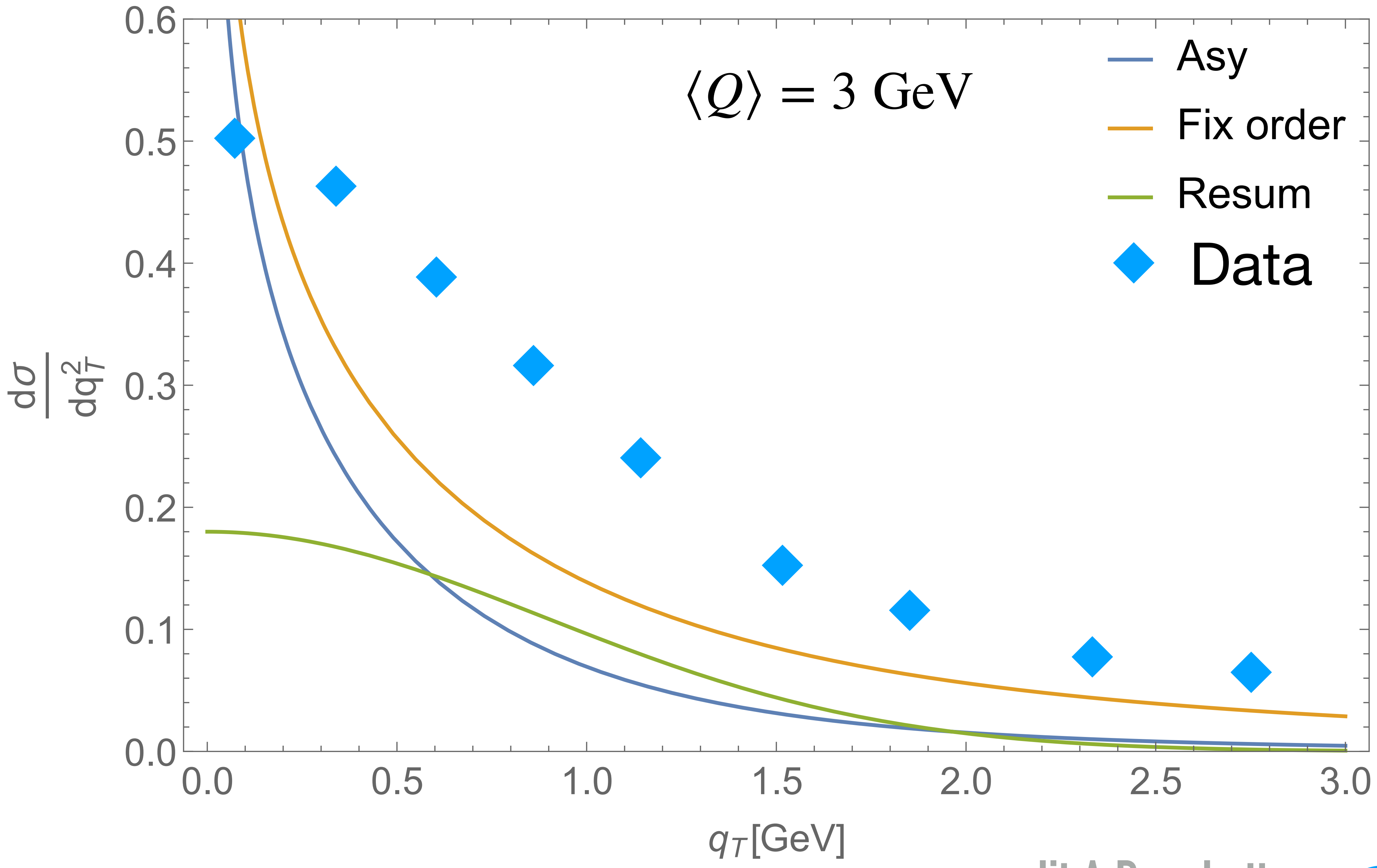
Introduction of a normalization prefactor

(at low P_{hT})

$$\int W \Big|_{O(\alpha_S)} \ll \frac{d\sigma^h}{dx\,dQ^2\,dz}\Big|_{O(\alpha_S)}$$

integral of the TMD formula

collinear cross section



credit A.Bacchetta

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

$$\left.\frac{d\sigma^h}{dx dQ^2 dz}\right|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2}(\delta_{f'f} + \delta_{f'g})\frac{\alpha_S}{\pi}\left\{\left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}\right](x,z,Q)\right\}\Big|_{\text{nonmix.}}$$

$$\int W\Big|_{O(\alpha_S)} = \sigma_0 \sum_{f,f'} \frac{e_f^2}{z^2}(\delta_{f'f} + \delta_{f'g})\frac{\alpha_S}{\pi}\left[D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}\right](x,z,Q)$$

$$\text{PREFACTOR}(x,z,Q) = \frac{\left.\frac{d\sigma^h}{dx dQ^2 dz}\right|_{\text{nonmix.}}}{\int W d^2 q_T}$$

computed a priori, before the fit

- **Depends on the collinear PDFs**
- independent of the fitting parameters

Non-perturbative part of TMDs

TMD PDF

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \, \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

TMD FF

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

NP evolution

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

Non-perturbative part of TMDs

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Gaussians

TMD FF

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Gaussians **weighted Gaussians**

TMD FF

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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Non-perturbative part of TMDs

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$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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Gaussians

weighted Gaussians

TMD FF

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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Non-perturbative part of TMDs

TMD PDF

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

Gaussians

weighted Gaussians

TMD FF

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

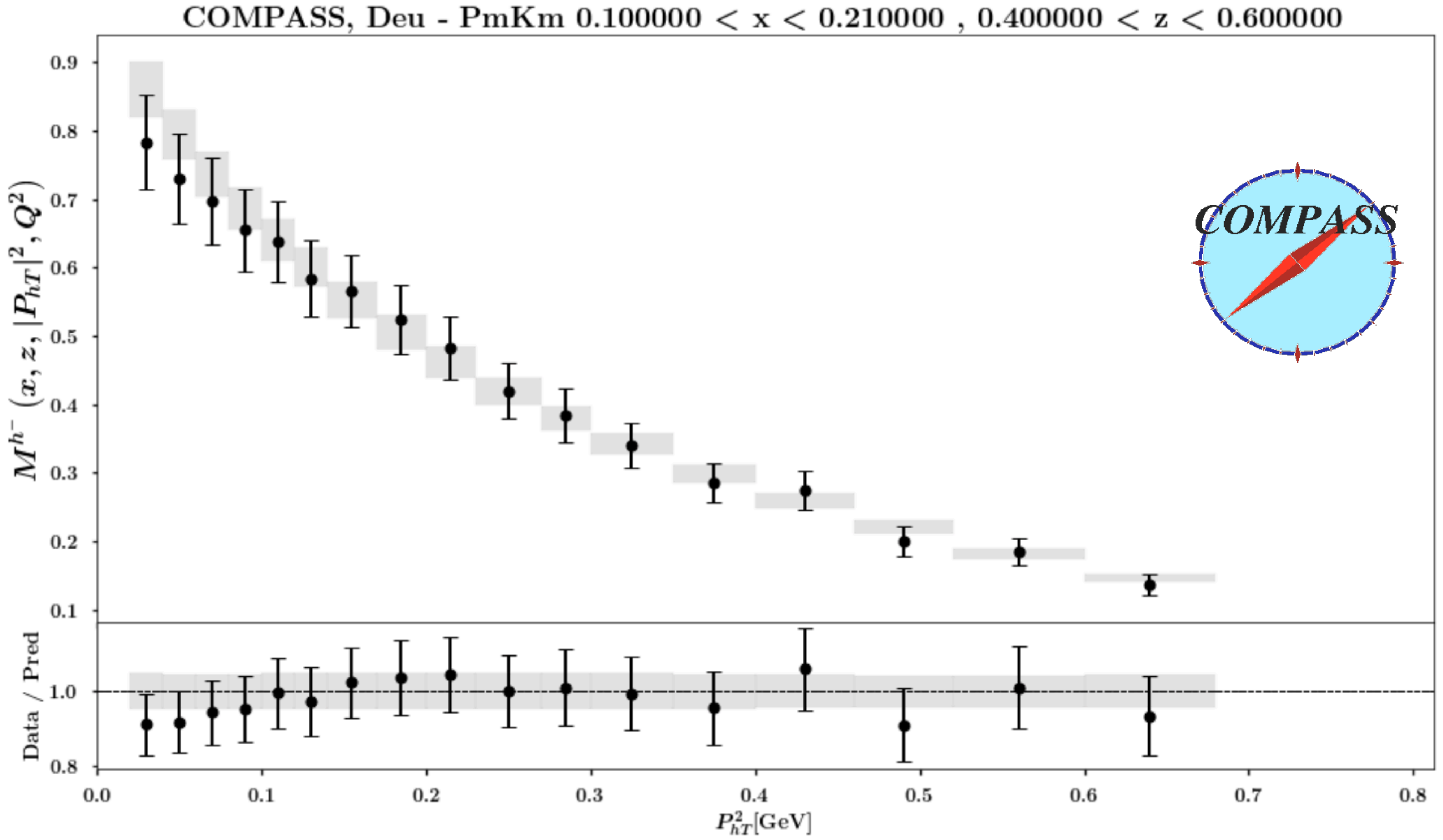
$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

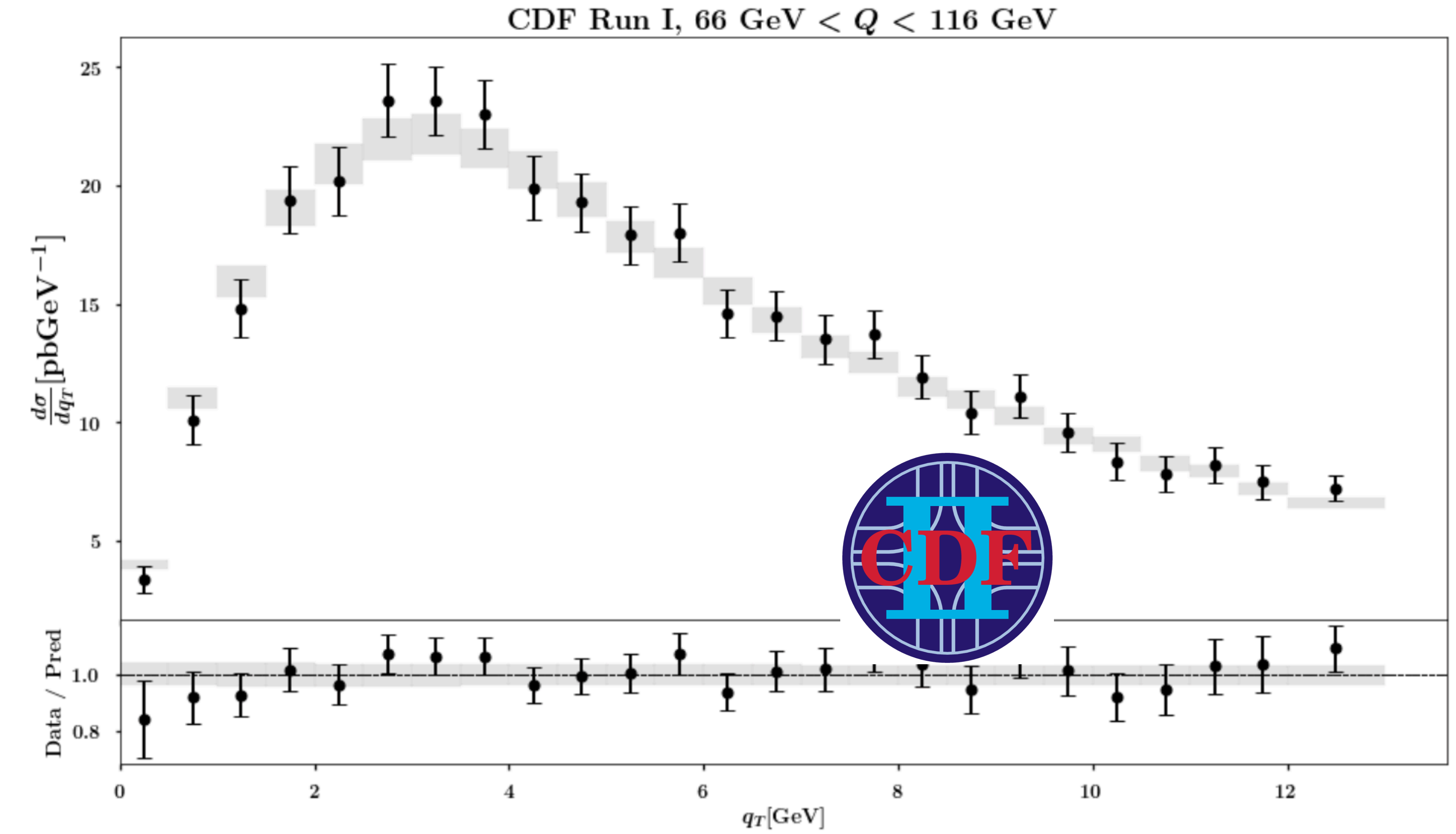
NP evolution

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

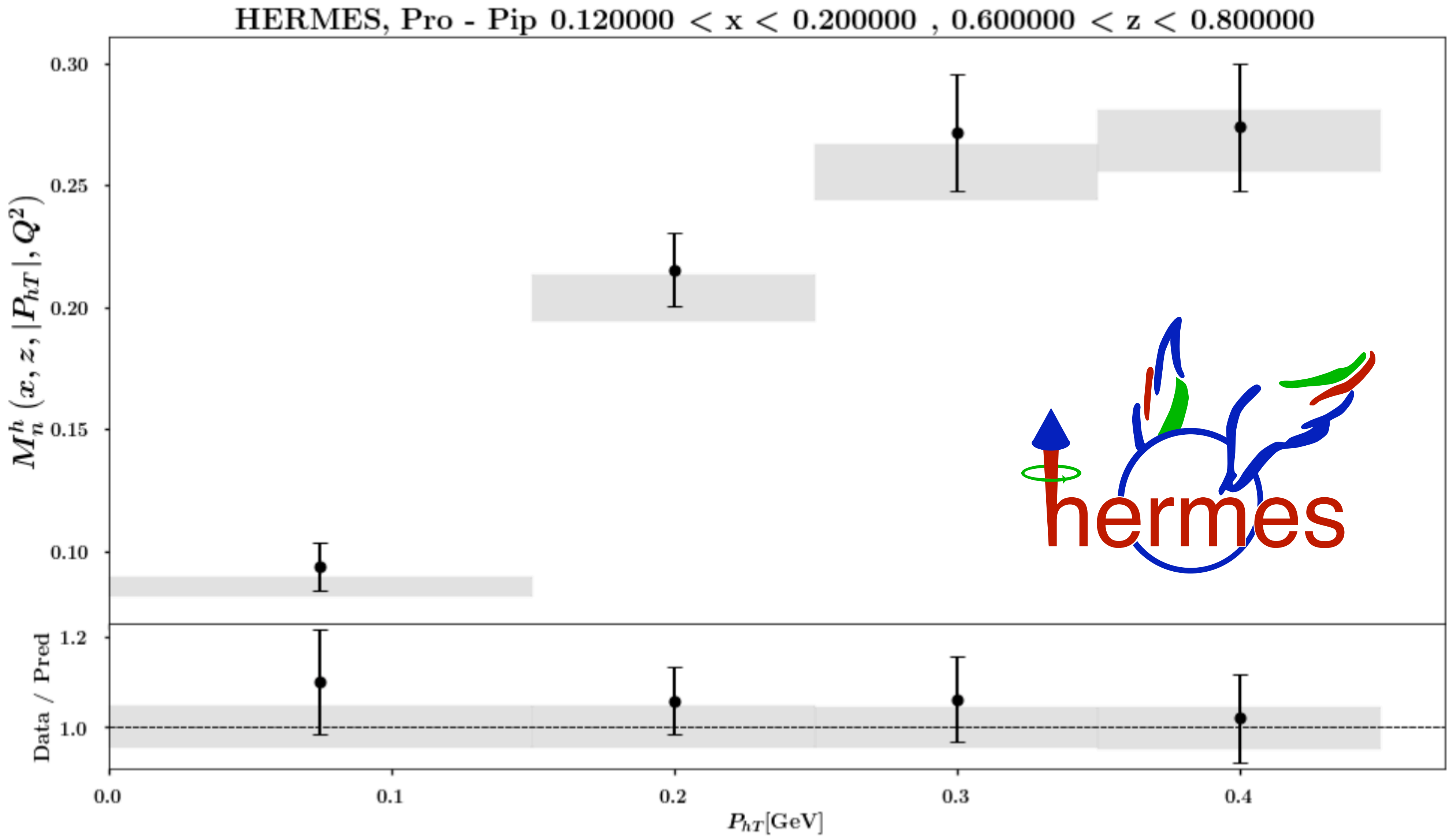
11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters

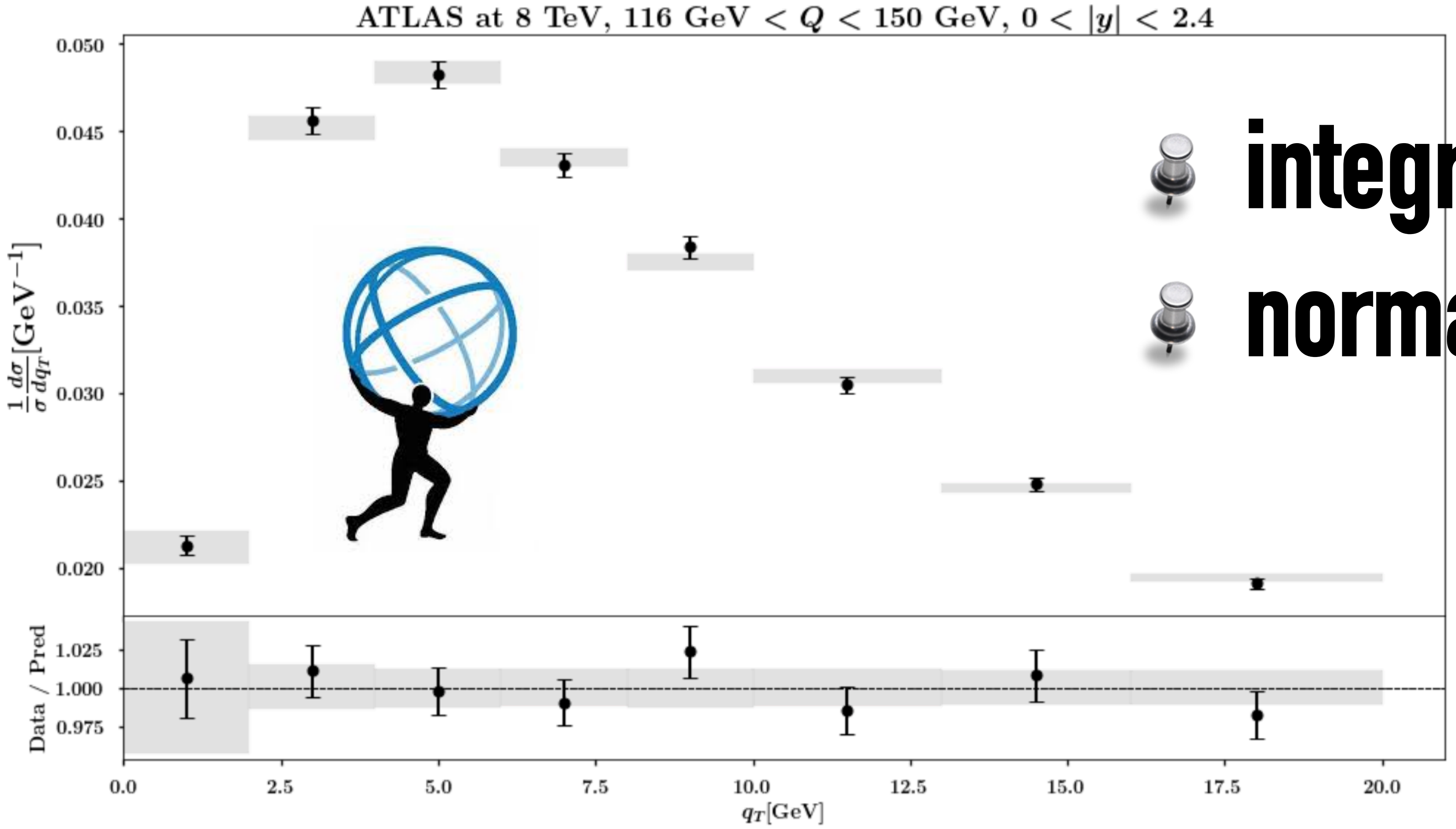
Fit results at N3LL⁻ : comparison with data





$\chi^2 \sim 1$

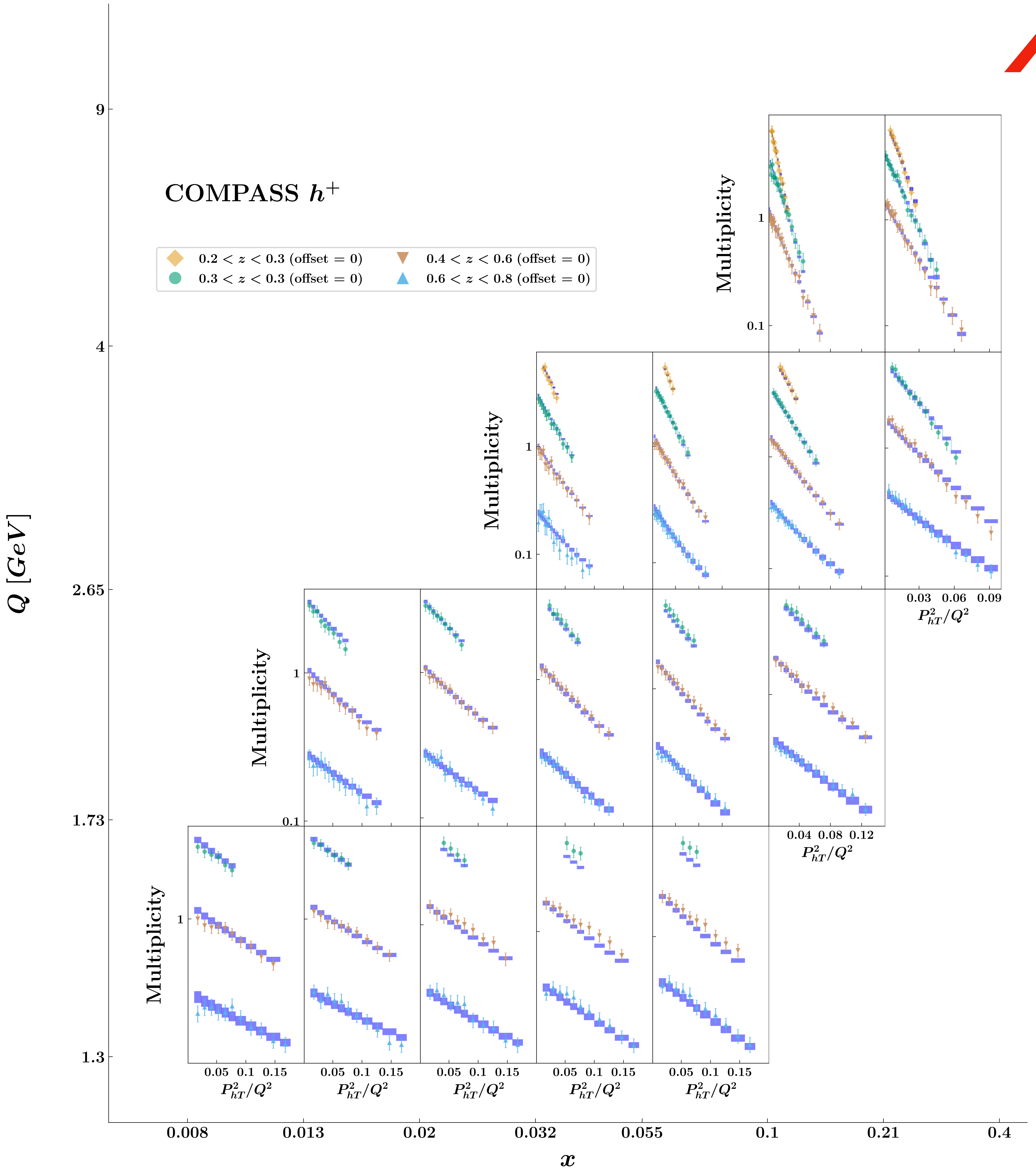


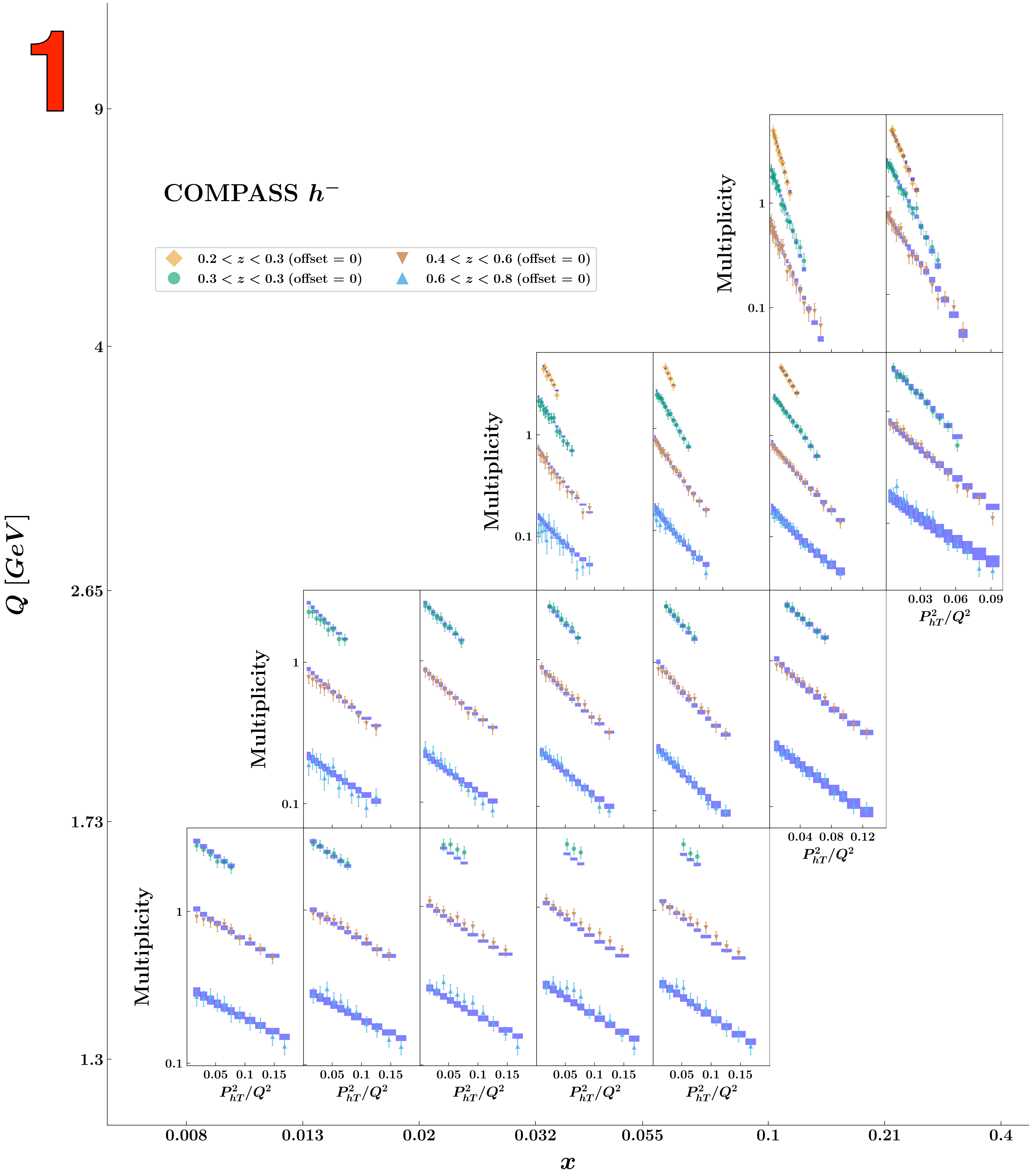


- integration over bins
- normalization for SIDIS

COMPASS data

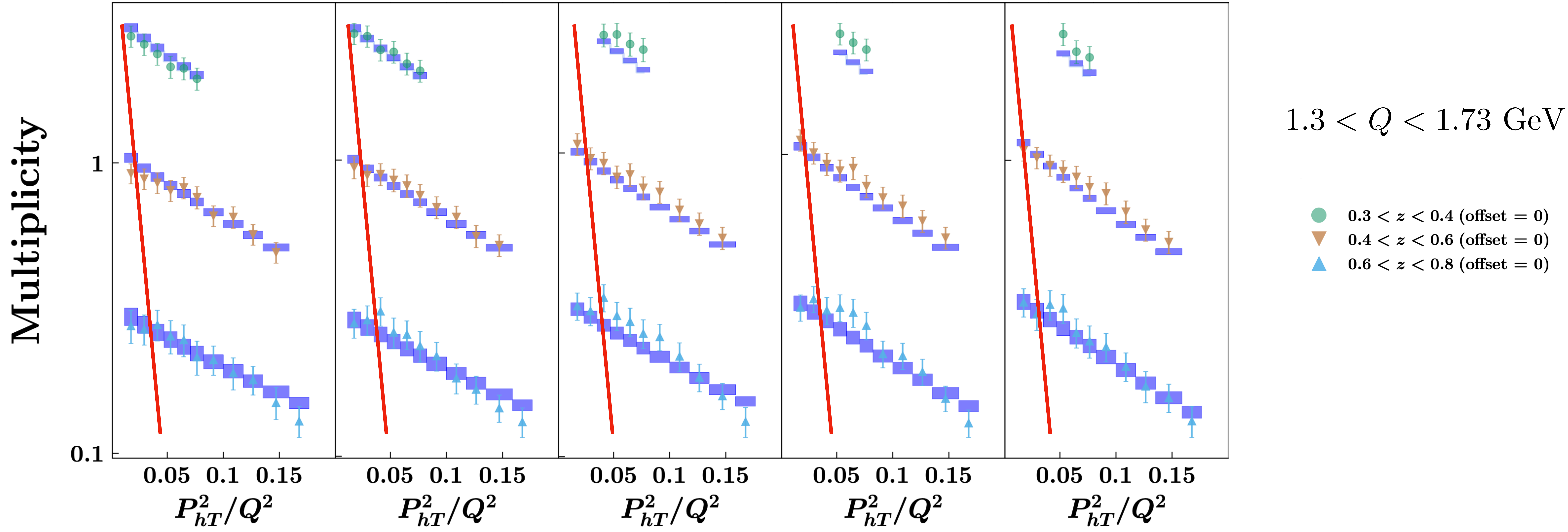
$\chi^2 \sim 1$





SIDIS cut for data selection

COMPASS multiplicities

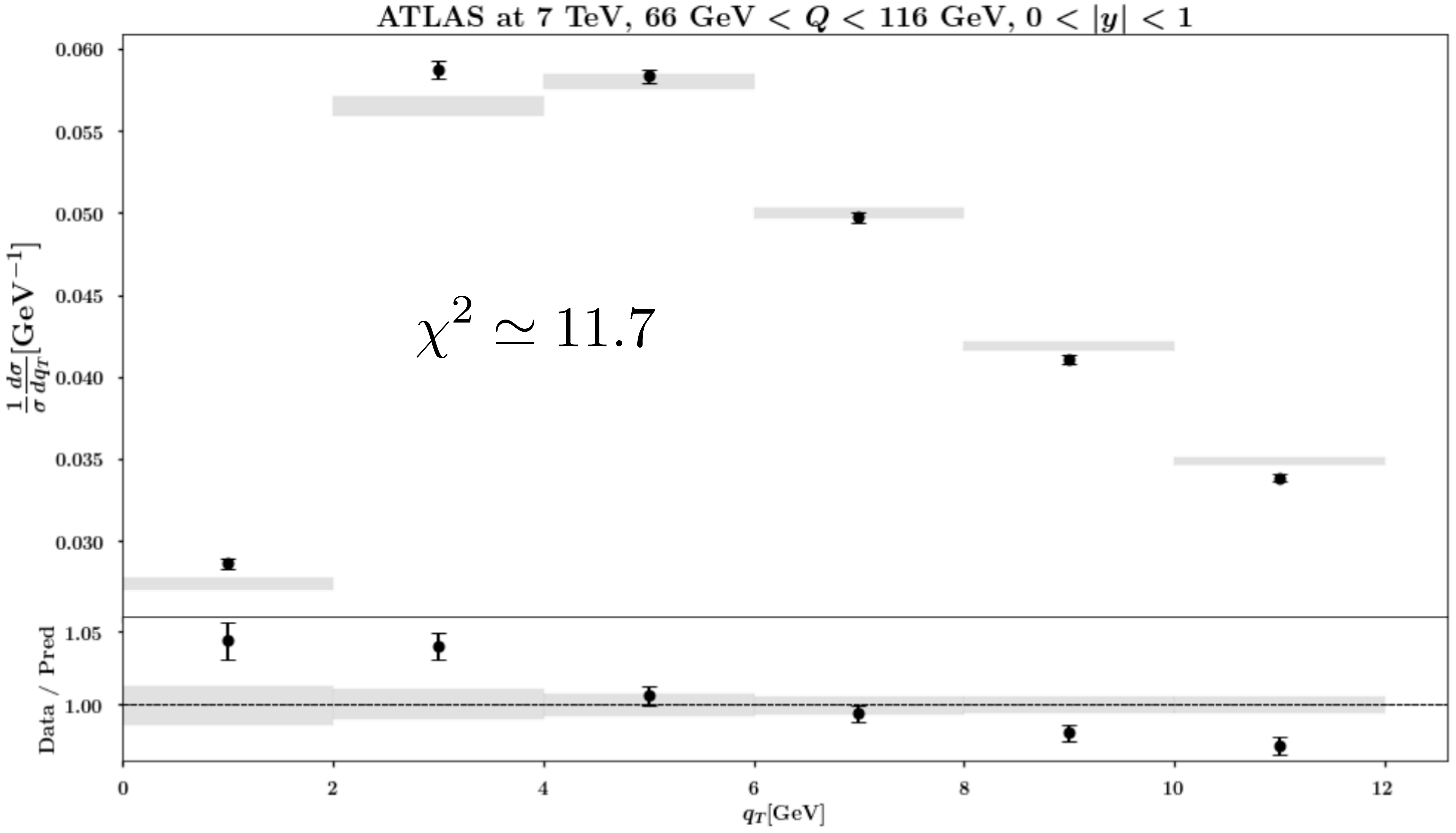


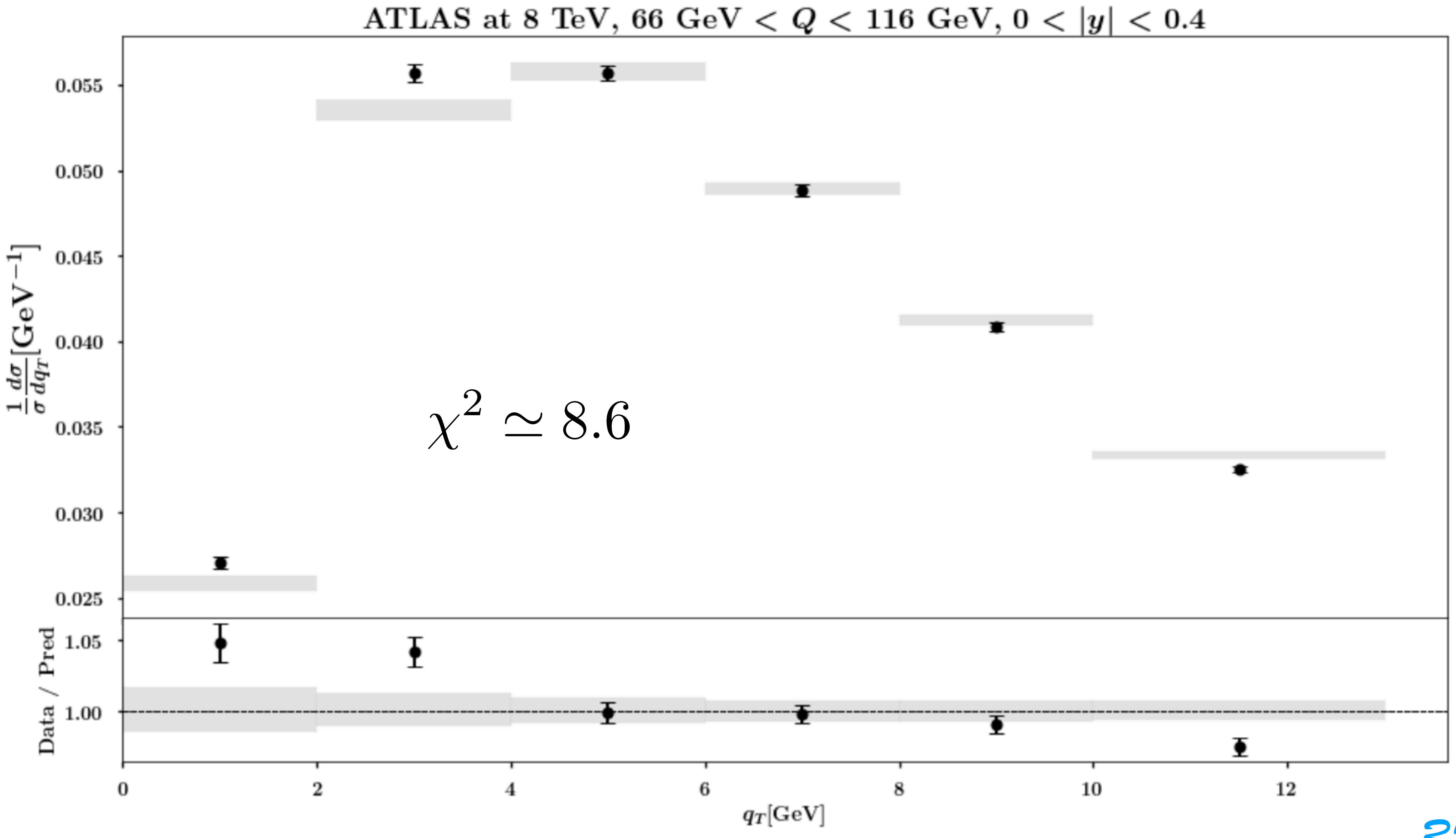
$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

vs

$$\left.\frac{P_{hT}}{zQ}\right|_{\max} = 0.25$$

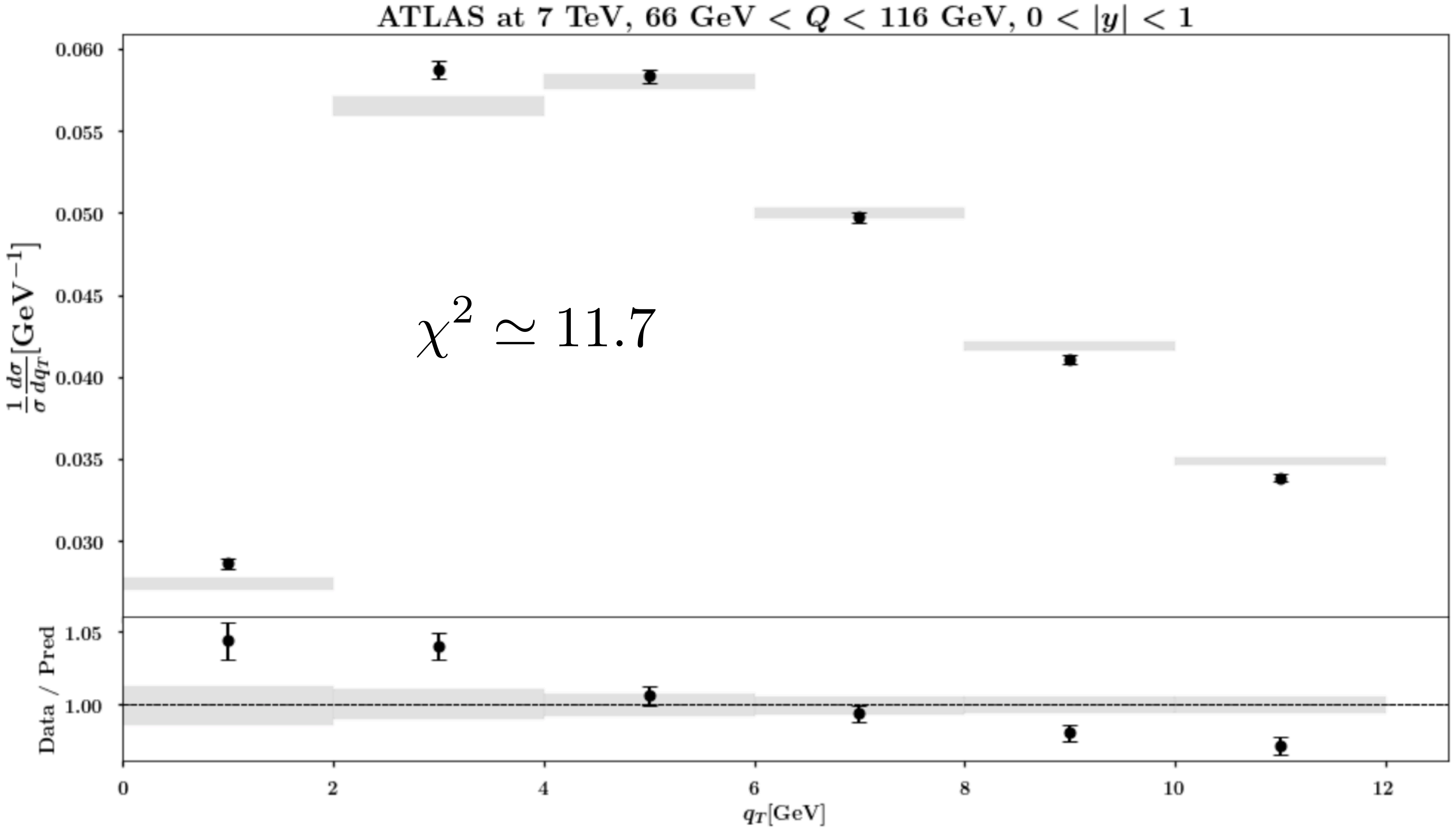
DY description





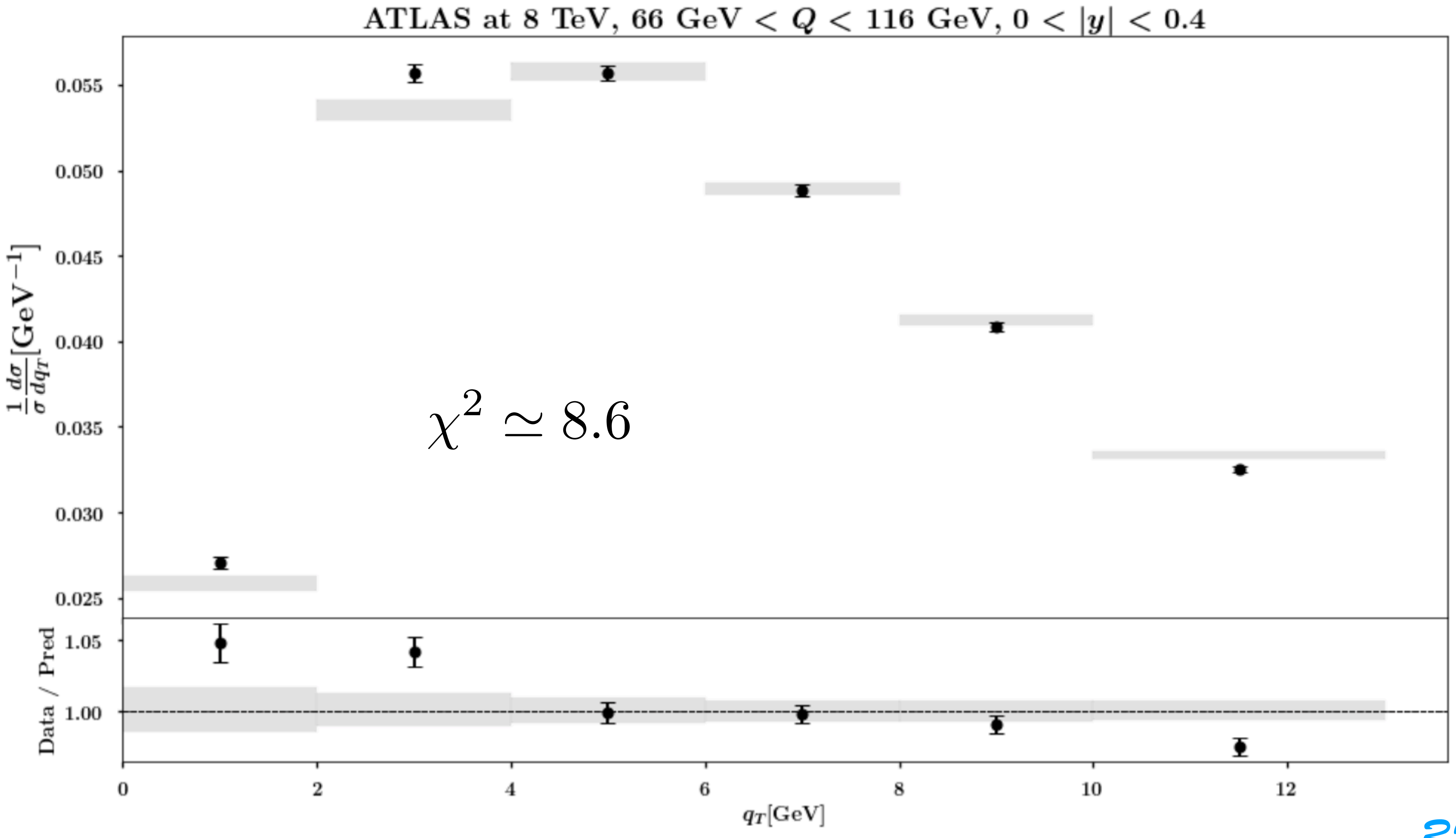
GLOBAL $\chi^2 \sim 1$

DY description



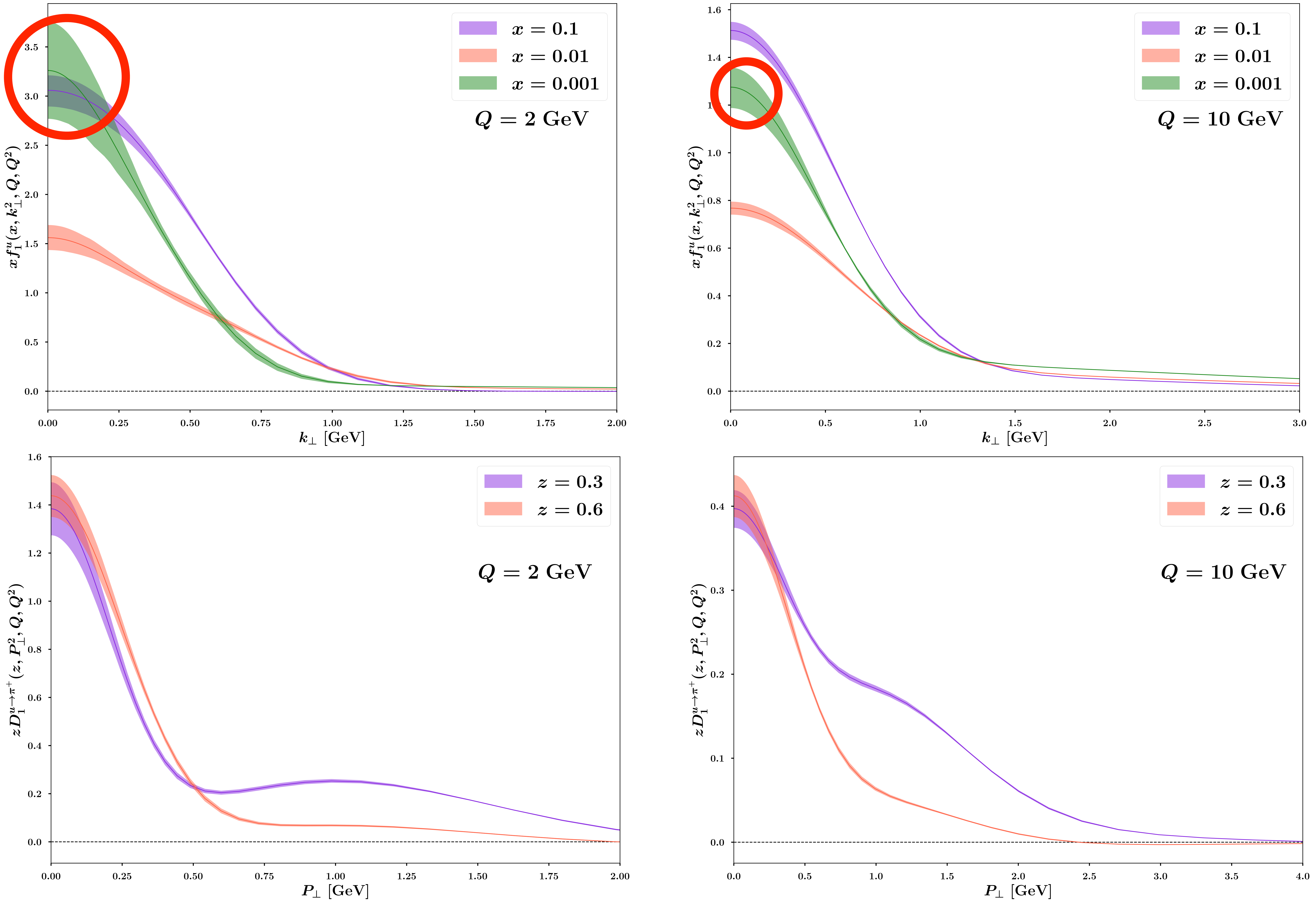
Possible justifications:

- small experimental uncertainties
- approximation of lepton cuts
- effects of the matching between perturbative and non-perturbative physics



GLOBAL $\chi^2 \sim 1$

Fit results



TMD PDFs

TMD FFs

Conclusions

MAP22 GLOBAL FIT – A new extraction of quark TMDs *in preparation*

- **Global analysis** of Drell-Yan and Semi-Inclusive DIS data sets

  2031 data points

- Perturbative accuracy: N^3LL^-

- **Normalization** of SIDIS multiplicities beyond NLL

- Number of parameters: **21**

- Extremely good description: $\chi^2/N_{data} \simeq 1$

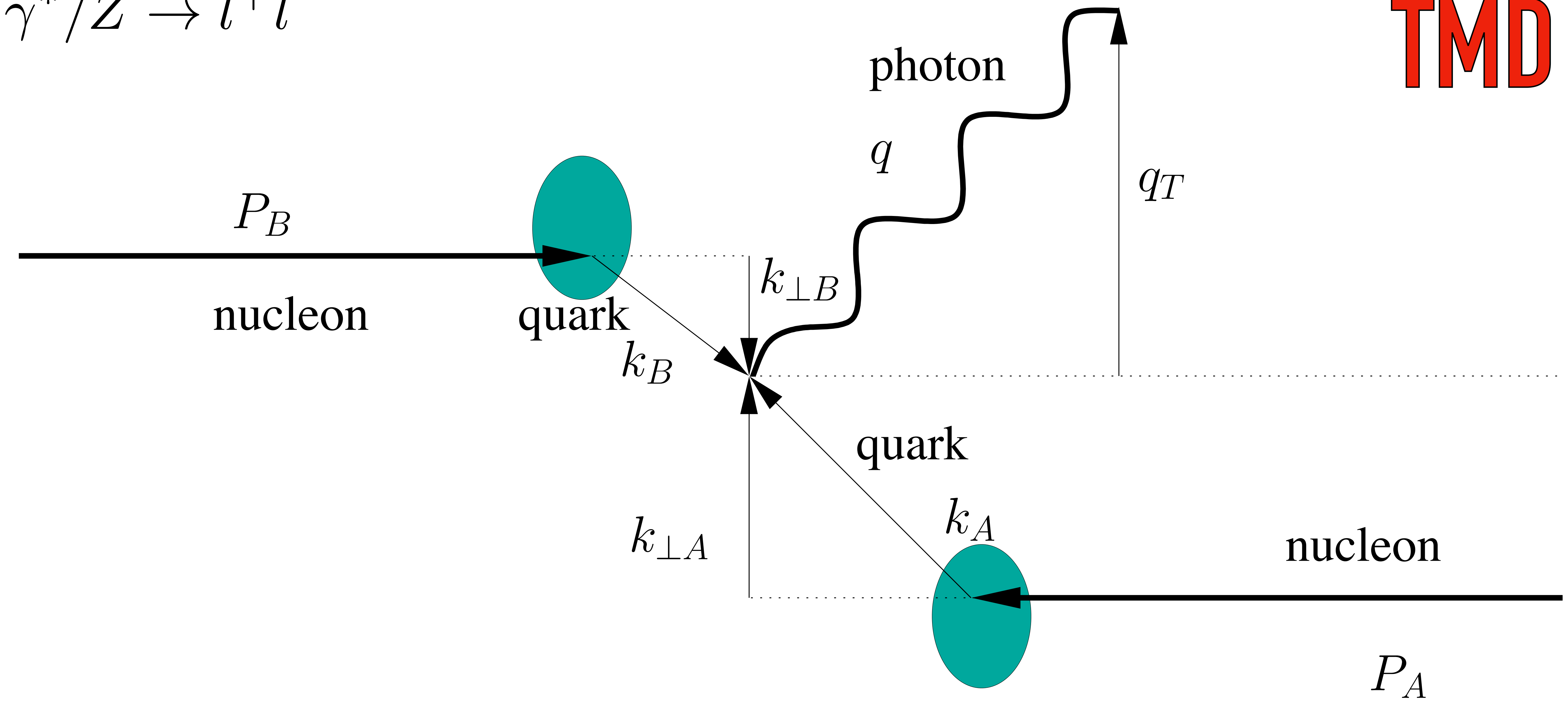
Backup

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization



$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}^{1a}_{UU}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}^2_{\perp A};\mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}^2_{\perp B};\mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$

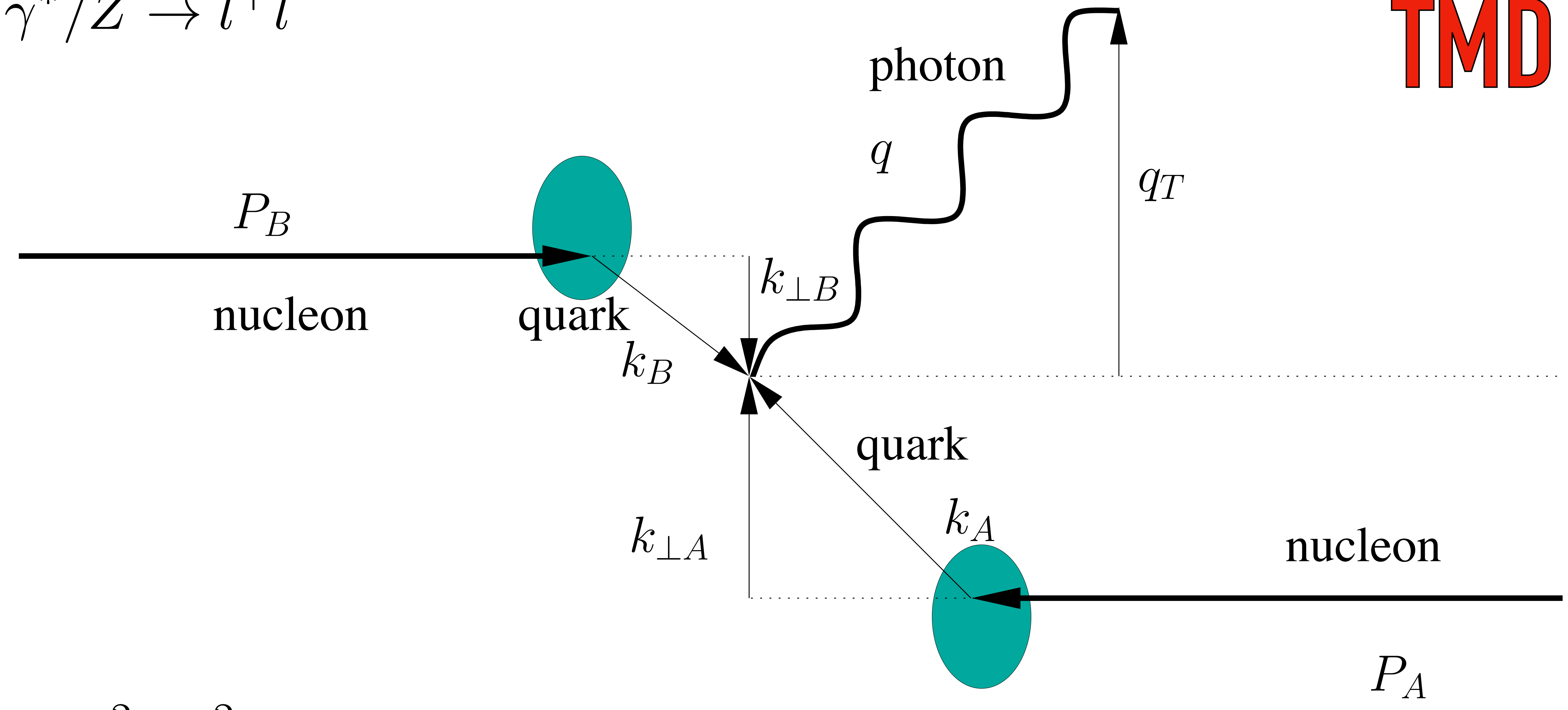
$$+ Y^1_{UU}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization



W term

$$F_{UU}^1(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a\big(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2\big)\, f_1^{\bar a}\big(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2\big)\, \delta^{(2)}\big(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\big)$$

$$+ Y_{UU}^1\big(Q^2, \boldsymbol{q}_T^2\big) + \mathcal{O}\big(M^2/Q^2\big)$$

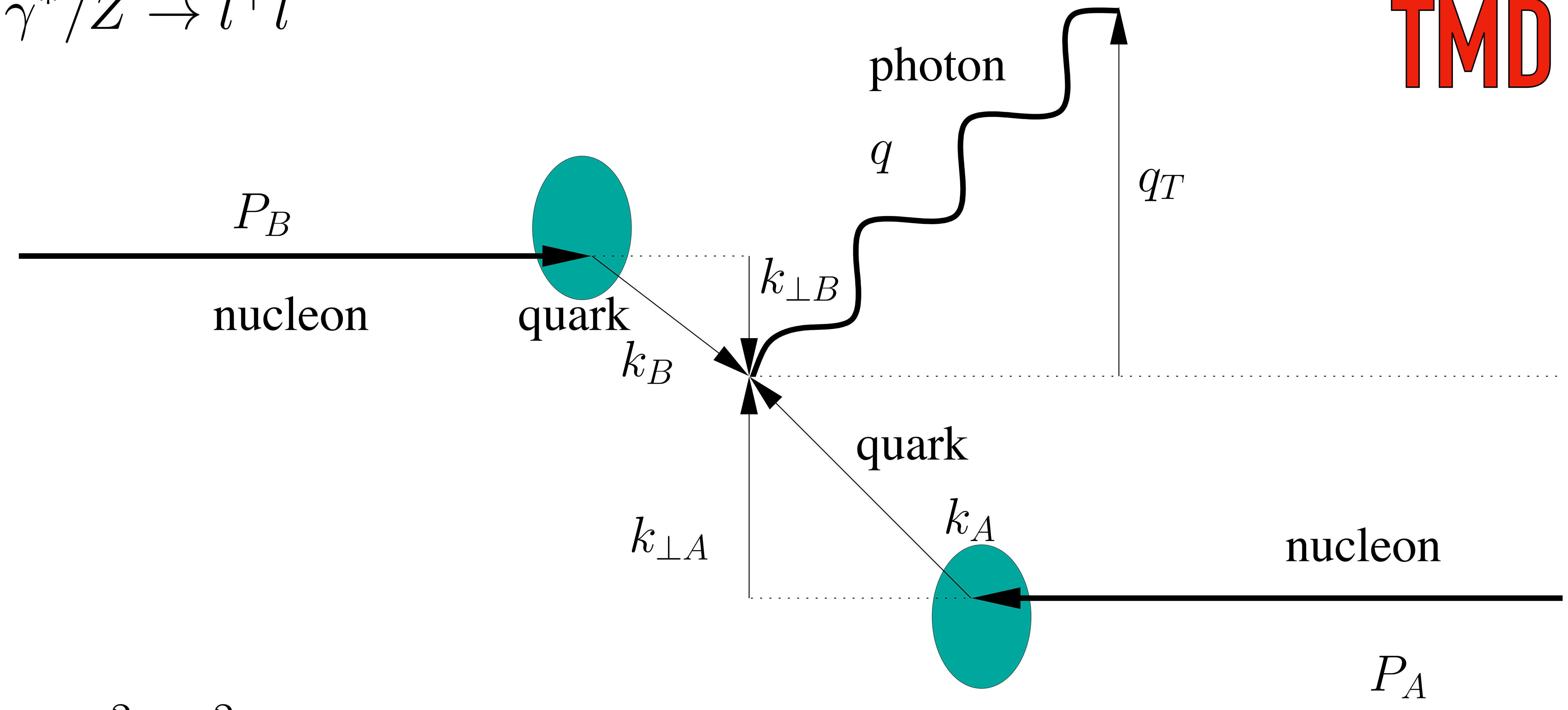
Y term

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization



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$$+ Y_{UU}^1(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Y term

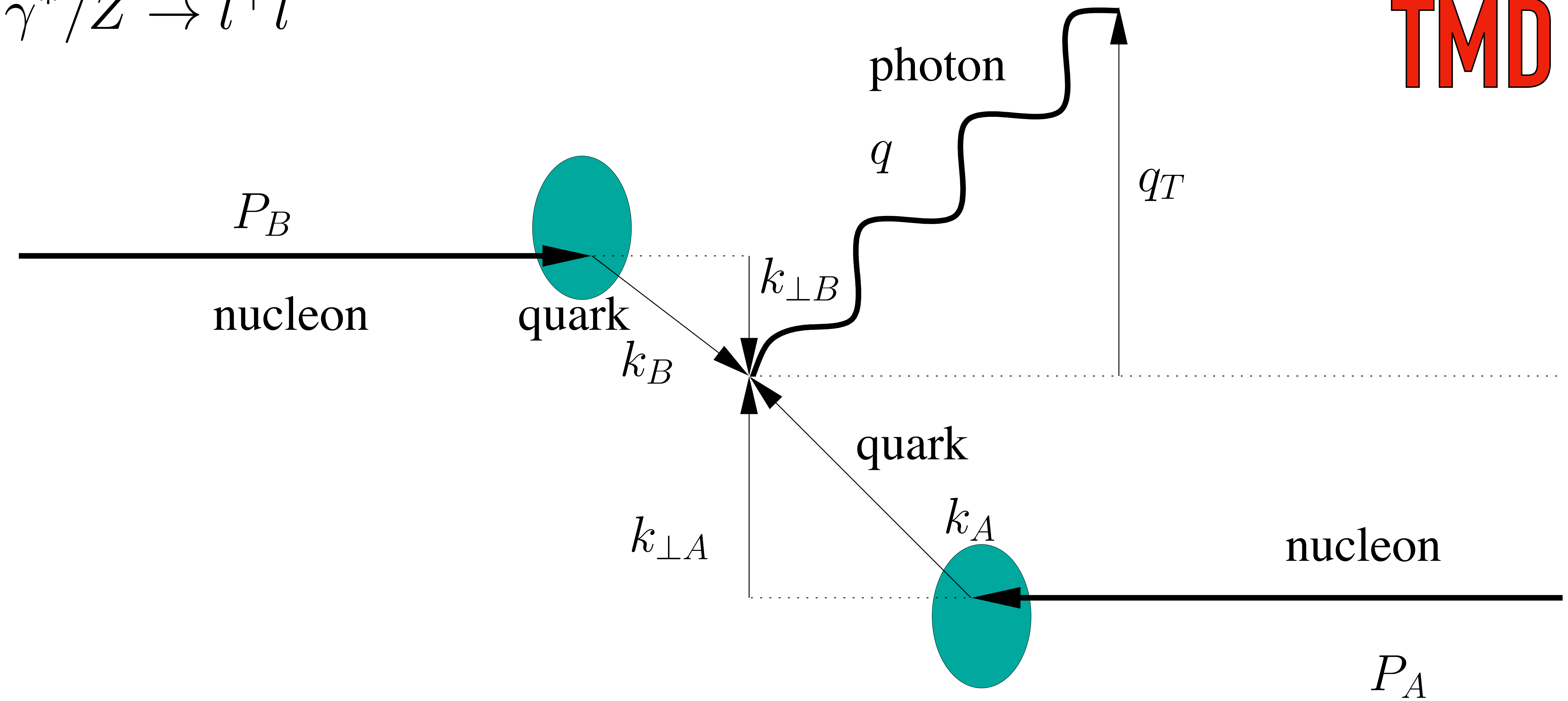
- W term dominates in the region where $q_T \ll Q$

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization



W term

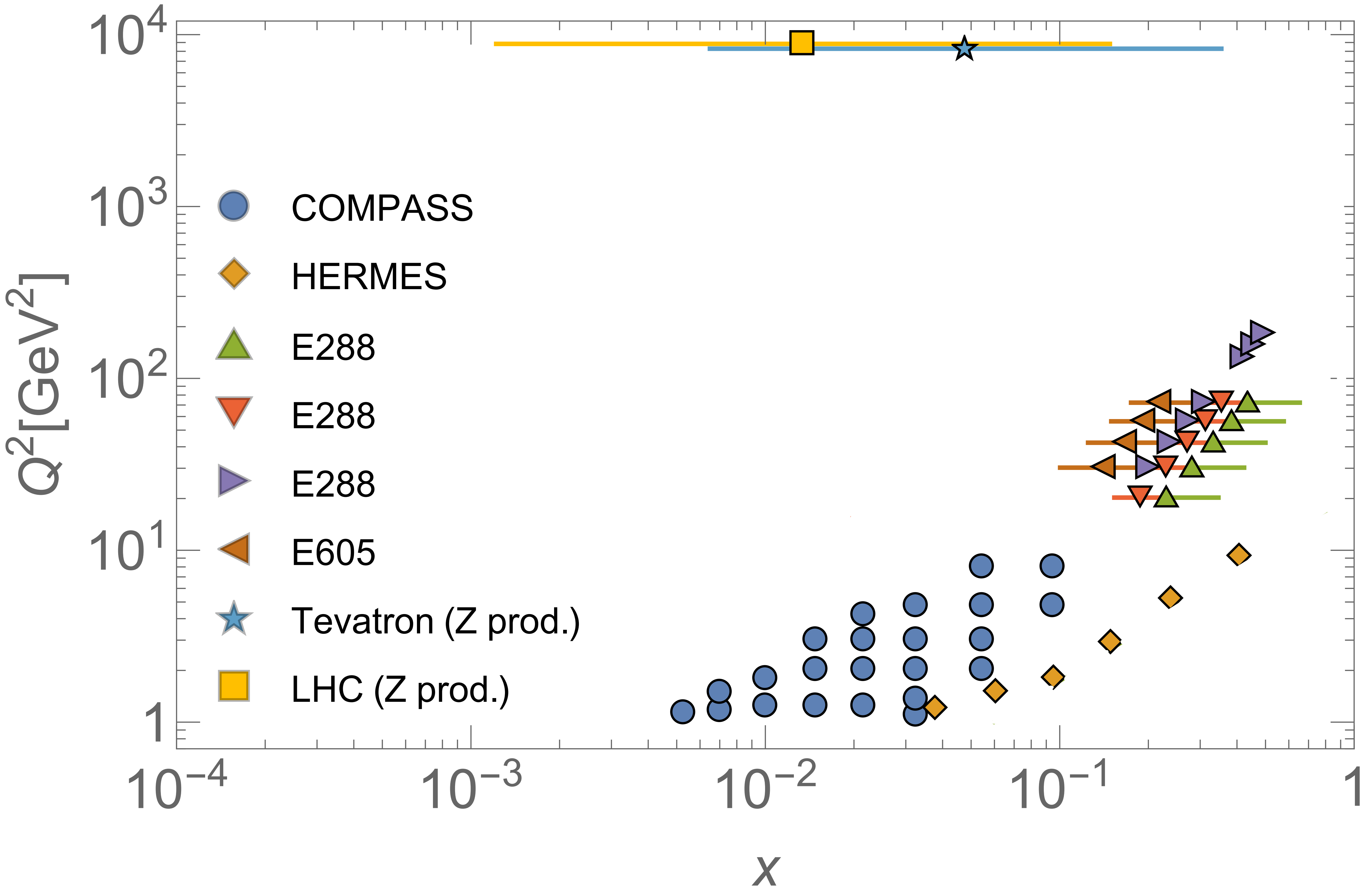
$$F_{UU}^1(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$
$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}_{\perp A}^2;\mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}_{\perp B}^2;\mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$
$$+ Y_{UU}^1(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Y term

- W term dominates in the region where $q_T \ll Q$
- Y term not included in the Pavia analyses

PV17
GLOBAL FIT

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
arXiv:1703.10157



cuts

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < \mathrm{Min}[0.2\ Q, 0.7\ Qz] + 0.5\ \mathrm{GeV}$

Total number of points:
8059

PV17 non perturbative functions

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
arXiv:1703.10157

$$f_{1\mathrm{NP}}^{a}(x,\mathbf{k}_\perp^2) = \frac{1}{\pi}\frac{\left(1+\lambda\mathbf{k}_\perp^2\right)}{g_{1a}+\lambda g_{1a}^2} e^{-\frac{\mathbf{k}_\perp^2}{g_{1a}}}$$

11 free parameters

$$D_{1\mathrm{NP}}^{a\to h}(z,\mathbf{P}_\perp^2) = \frac{1}{\pi}\frac{1}{g_{3a\to h}+\left(\lambda_F/z^2\right)g_{4a\to h}^2}\left(e^{-\frac{\mathbf{P}_\perp^2}{g_{3a\to h}}}+\lambda_F\frac{\mathbf{P}_\perp^2}{z^2}\,e^{-\frac{\mathbf{P}_\perp^2}{g_{4a\to h}}}\right)$$

x-dependence

$$g_1(x) = N_1\frac{(1-x)^{\alpha}\,x^{\sigma}}{(1-\hat{x})^{\alpha}\,\hat{x}^{\sigma}}$$

$$g_{3,4}(z) = N_{3,4}\frac{(z^{\beta}+\delta)\,(1-z)^{\gamma}}{(\hat{z}^{\beta}+\delta)\,(1-\hat{z})^{\gamma}}$$

non-perturbative Sudakov factor

$$g_K(b_T) = -g_2 b_T^2/2$$

PV19 non perturbative function

A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, M. Radici
JHEP 07 (2020) 117 e-Print: 1912.07550

$$f_{\mathrm{NP}}(x,b,\zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b^2}{4}} + \lambda \exp\left(-g_{1,B}(x)\frac{b^2}{4}\right)\right]$$

$$\times \quad \exp\left[-\left(g_2 + g_{2B} b^2\right) \log\left(\frac{\zeta}{Q_0^2}\right)\frac{b^2}{4}\right]$$

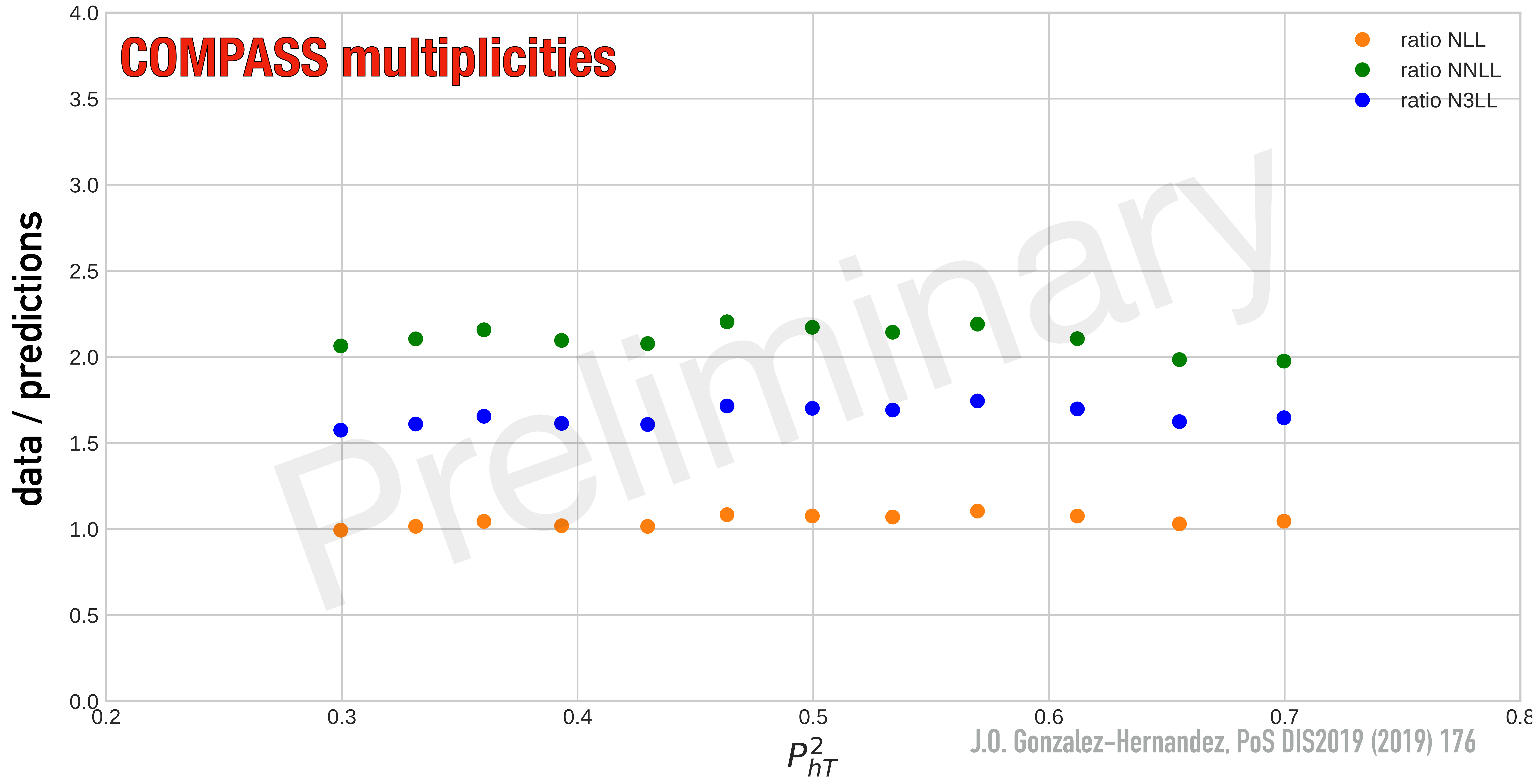
x-dependence

$$g_1(x) = \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1,B}(x) = \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

9 parameters

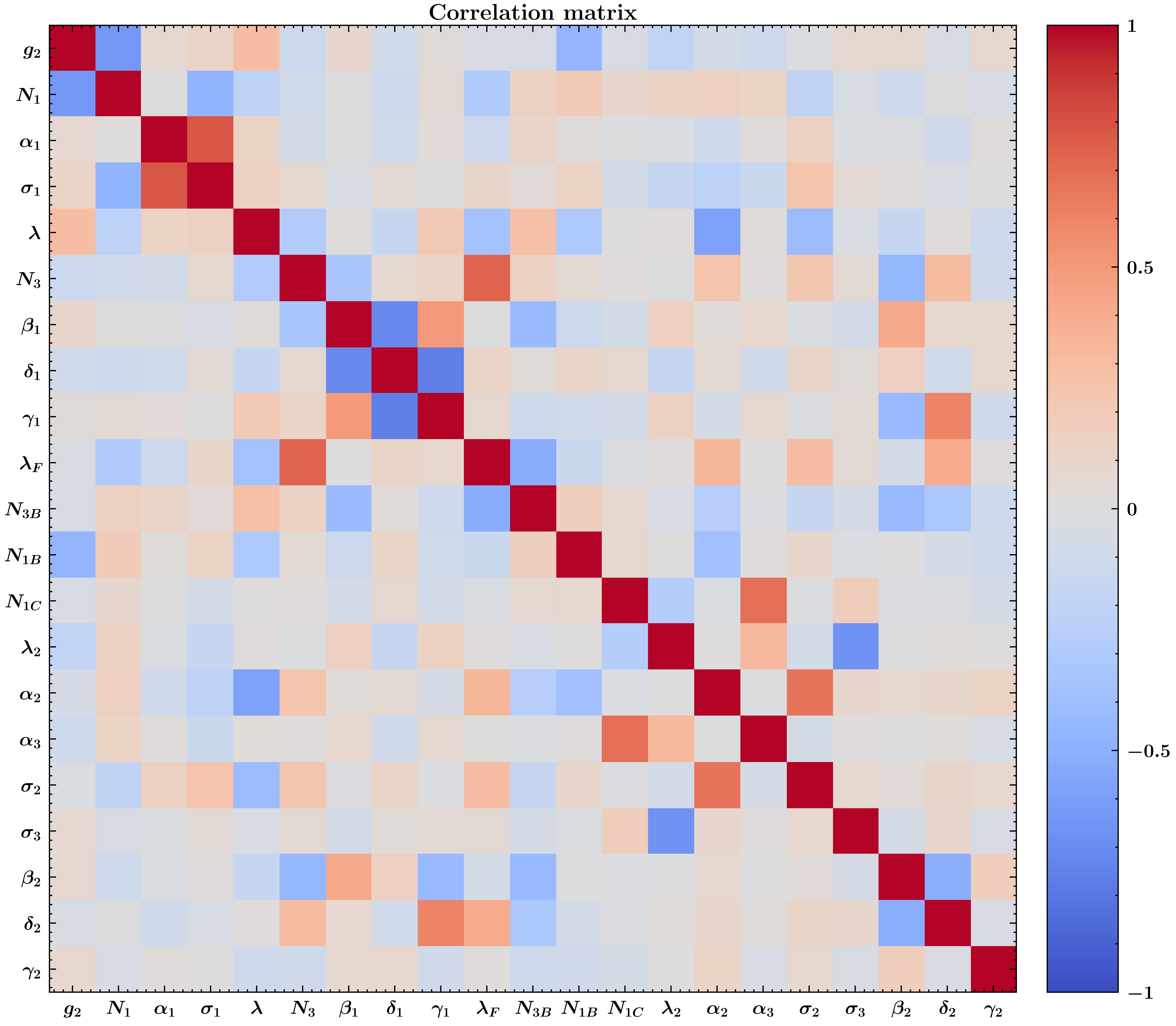
Normalization of SIDIS multiplicities



The discrepancy amounts to an almost **constant factor**

Fit results: correlation matrix

250 Montecarlo replicas



- **21 parameters**
- ideal situation: red diagonal, gray off-diagonal

Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{unc}} \pm \sigma^{(1)}_{i,\mathrm{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\mathrm{corr}}$$

uncorrelated

correlated

additive

multiplicative

central value

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i)\, V^{-1}_{ij}\, (m_j - t_j)$$

predictions

$$\sigma^{(l)}_{i,\mathrm{corr}} \equiv \delta^{(l)}_{i,\mathrm{corr}} m_i$$

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\mathrm{add}} \delta^{(l)}_{j,\mathrm{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\mathrm{mult}} \delta^{(l)}_{j,\mathrm{mult}} \right) m_i m_j$$

Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{unc}} \pm \sigma^{(1)}_{i,\mathrm{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\mathrm{corr}}$$

uncorrelated

correlated

additive multiplicative

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i)\, V^{-1}_{ij}\, (m_j - t_j)$$

predictions

covariance matrix

D'Agostini bias

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\mathrm{add}} \delta^{(l)}_{j,\mathrm{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\mathrm{mult}} \delta^{(l)}_{j,\mathrm{mult}} \right) m_i m_j$$

Experimental uncertainties

$$m_i \pm \sigma_{i,\mathrm{unc}} \pm \sigma_{i,\mathrm{corr}}^{(1)} \pm \cdots \pm \sigma_{i,\mathrm{corr}}^{(k)}$$

uncorrelated

correlated

additive multiplicative

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i)\, V_{ij}^{-1}\, (m_j - t_j)$$

predictions

t_0 prescription

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \sum_{l=1}^{k_a} \delta_{i,\mathrm{add}}^{(l)} \delta_{j,\mathrm{add}}^{(l)} m_i m_j + \sum_{l=1}^{k_m} \delta_{i,\mathrm{mult}}^{(l)} \delta_{j,\mathrm{mult}}^{(l)} t_i^{(0)} t_j^{(0)}$$

Chisquare

$$d_i = \sum_{\alpha=1}^{k} \lambda_\alpha \sigma_{i,\mathrm{corr}}^{(\alpha)}$$

shift

$$\bar{t}_i = t_i + d_i$$

shifted prediction

$$\frac{\partial \chi^2}{\partial \lambda_\alpha} = 0$$

nuisance parameters

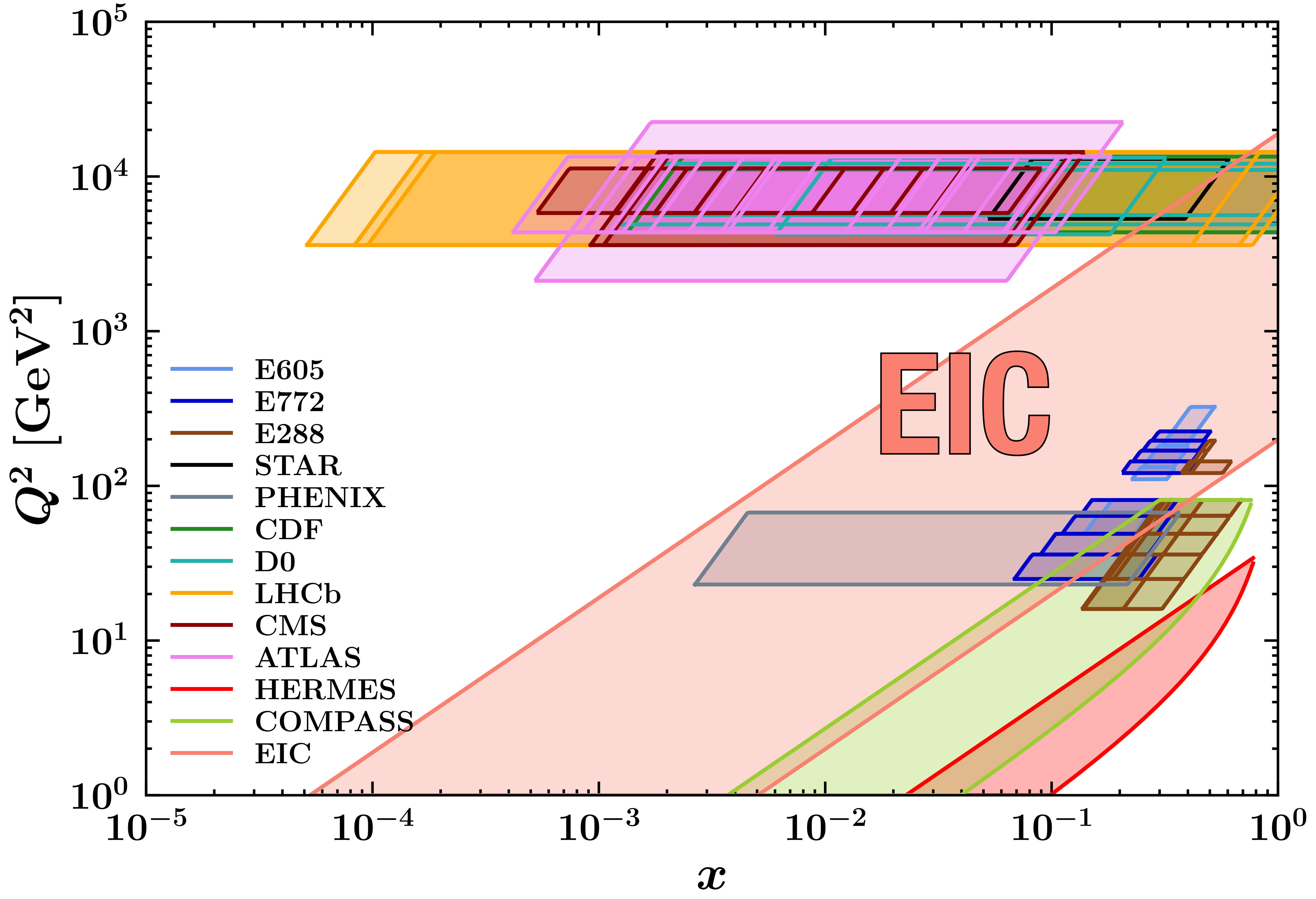
$$\chi^2 = \sum_{i=1}^{n} \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^{k} \lambda_\alpha^2$$

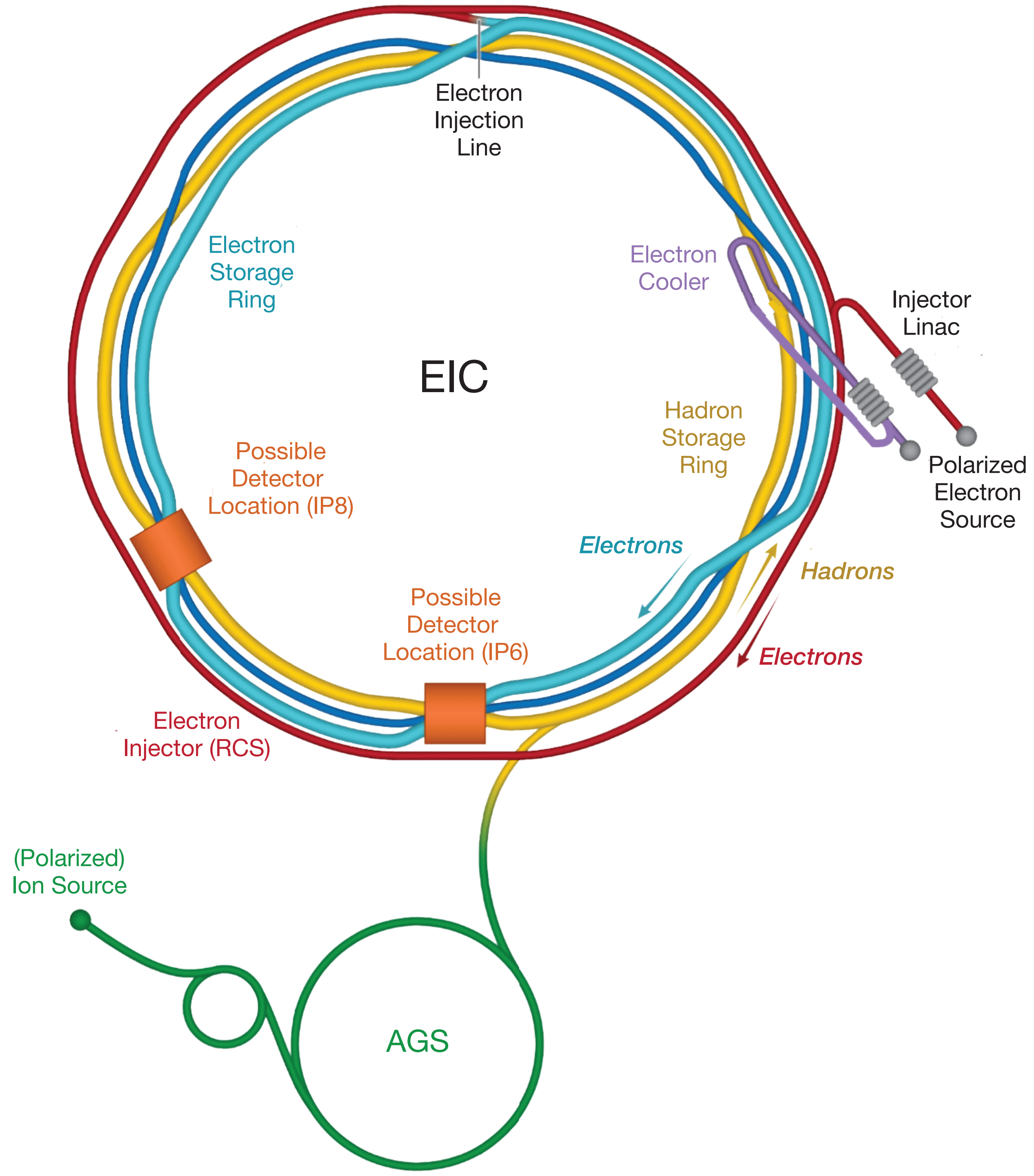
uncorrelated contribution

penalty term

Outlook

- What happens to TMDs
  once we include EIC data?

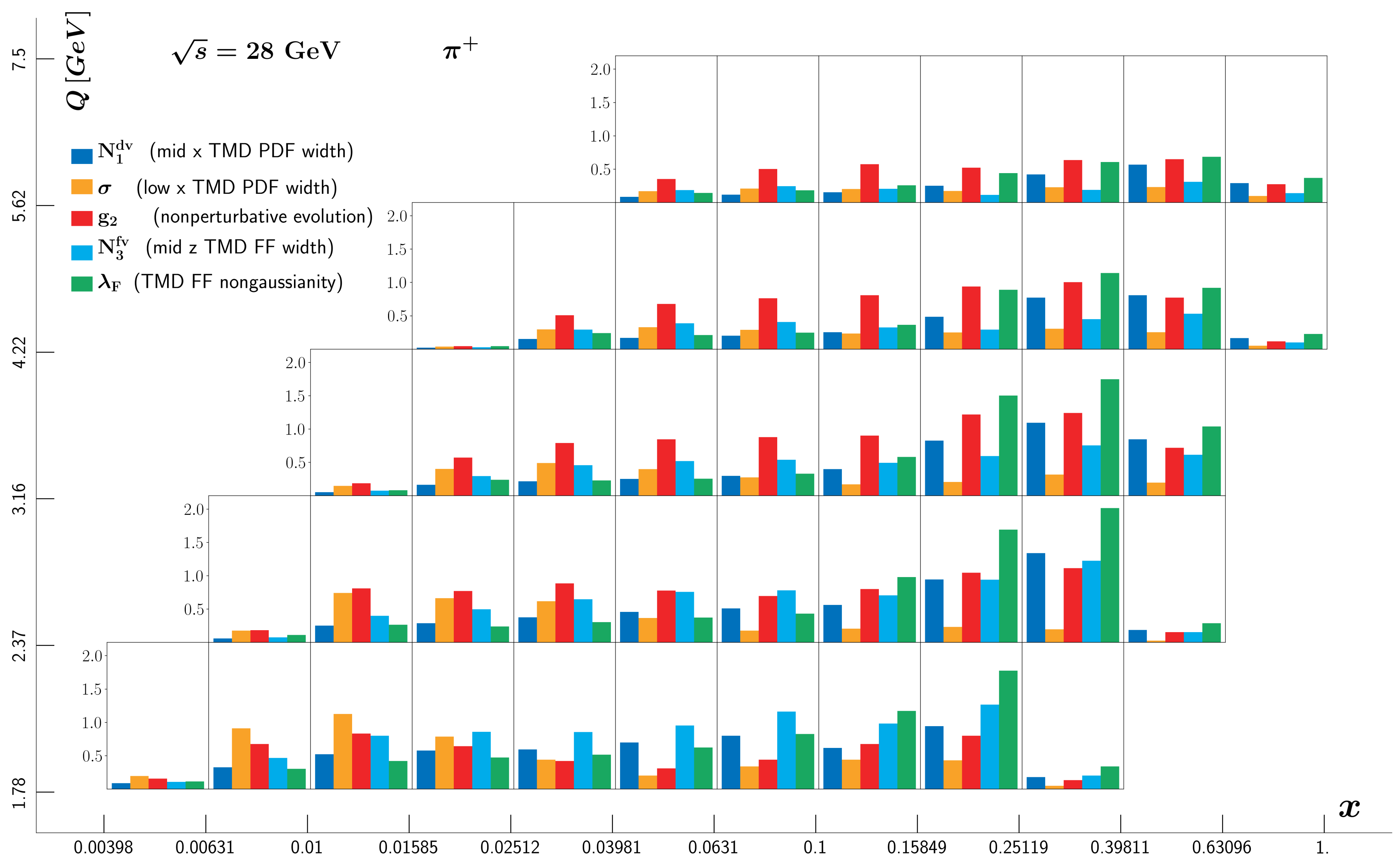




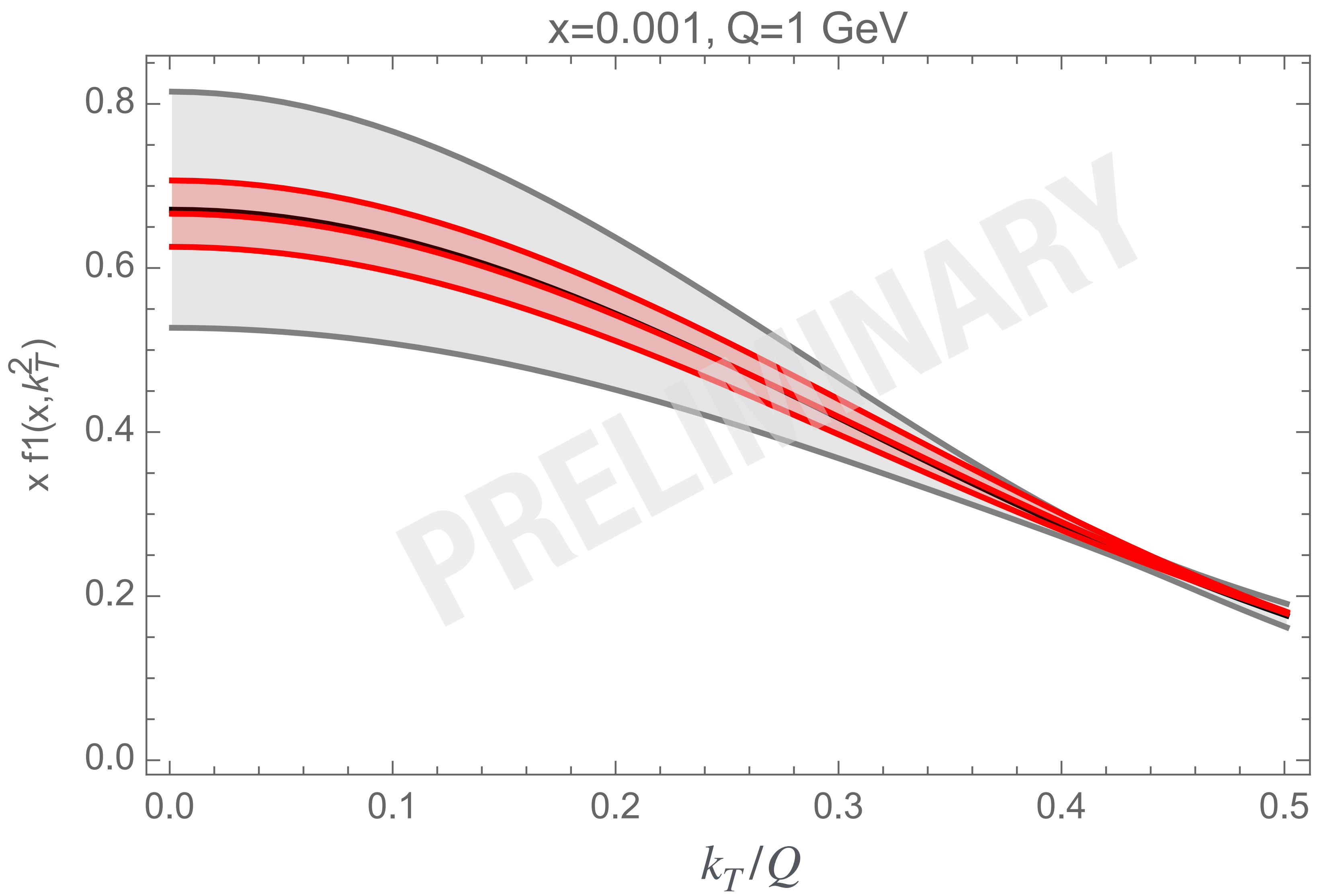
Electron-Ion Collider
to be built at Brookhaven National Lab

A few tools to estimate EIC impact

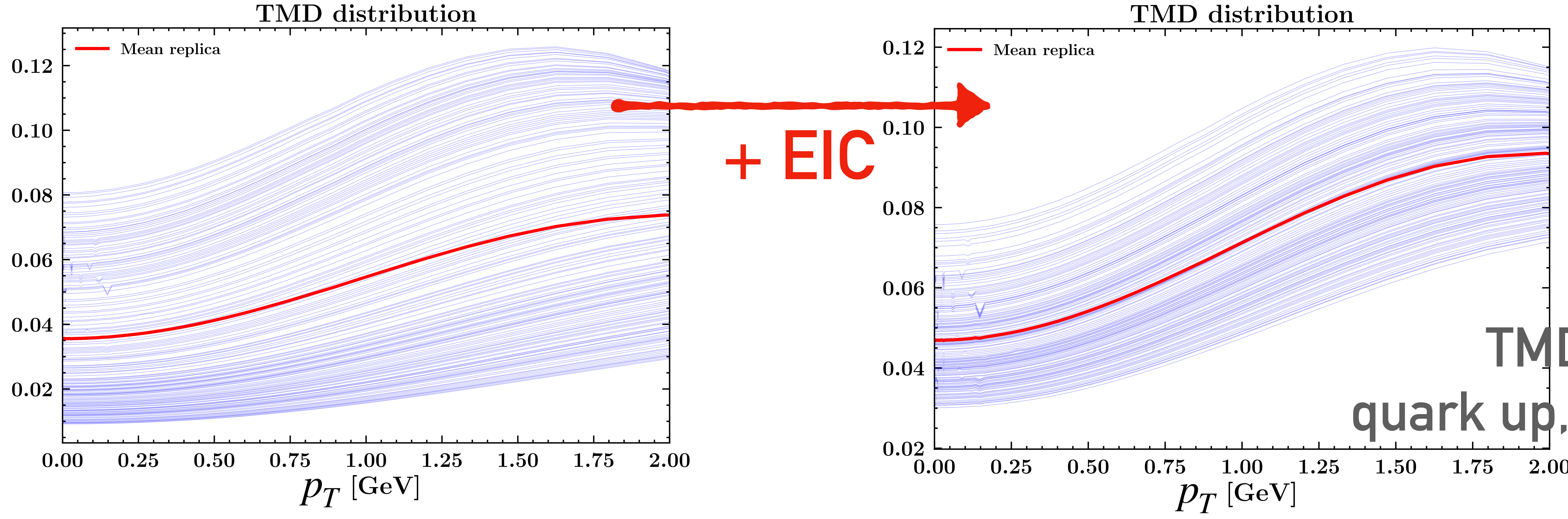
- sensitivity coefficients



- reweighing



- new fit with EIC pseudo data



TMD FFs
quark up, $Q = 2$ GeV

Impact studies starting point

EIC pseudodata

PV17 TMDs

predictions using global fit of Pavia 2017

we took the average **kinematic variables** of each point and the **relative uncertainty** on the observable

$$F_{UU,T}(x, z, q_T; Q^2)$$

Bacchetta, Delcarro, Pisano, Radici, Signori
arXiv:1703.10157

EIC impact studies

SENSITIVITY COEFFICIENTS

from E. Aschenauer, I. Borsa, G. Lucero, A. S. Nunes, R. Sassot
arXiv:2007.08300

$F_{UU,T}(x, z, q_T; Q^2)$ — observable

distribution — TMD parameters

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \Delta \mathcal{O} \Delta f_i}$$

experimental uncertainty
(from pseudodata)

$$\xi \equiv \frac{\delta \mathcal{O}}{\Delta \mathcal{O}}$$

~~theoretical uncertainty~~

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}[f_i^{(k)}]$$

EIC impact studies

SENSITIVITY COEFFICIENTS

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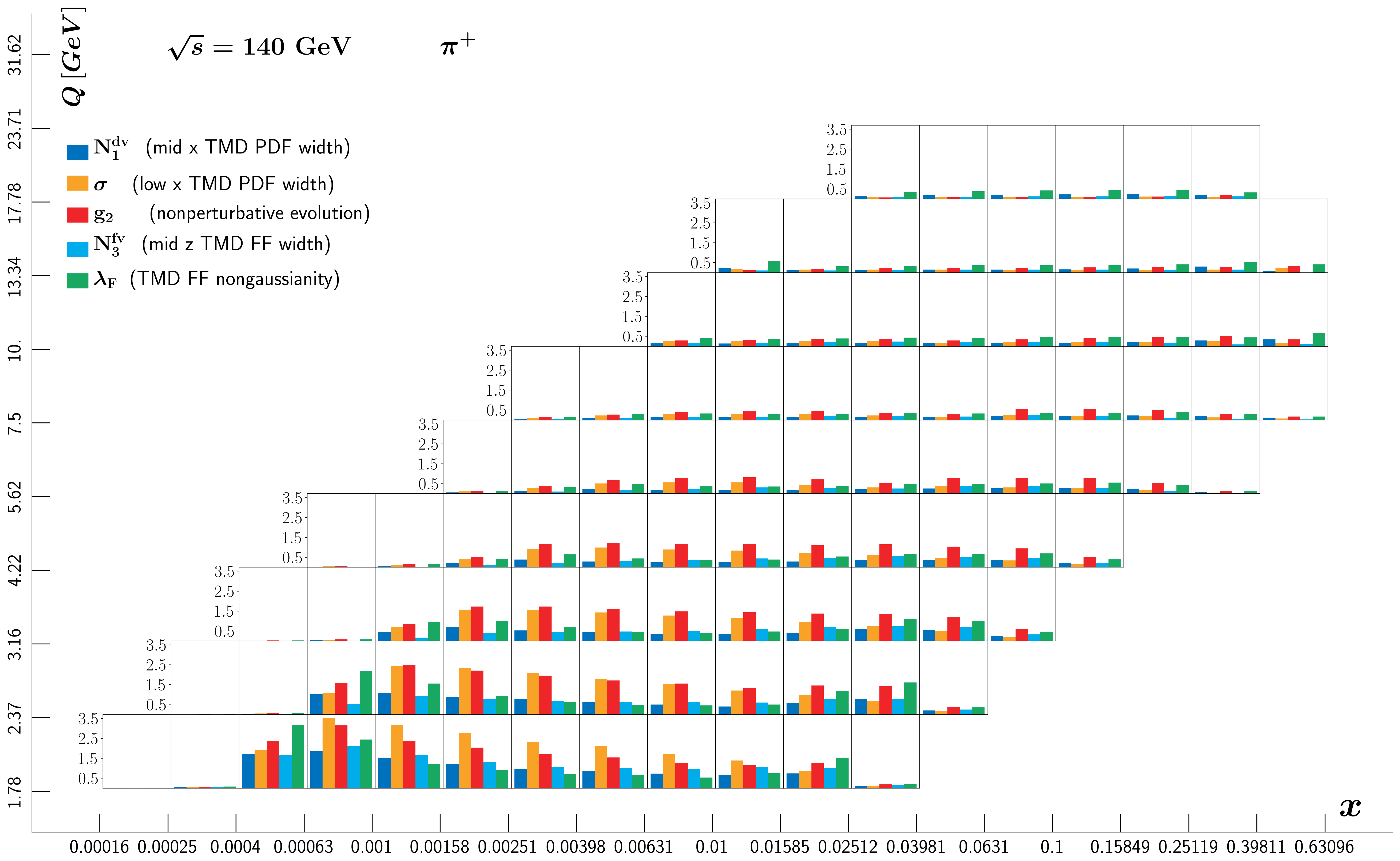
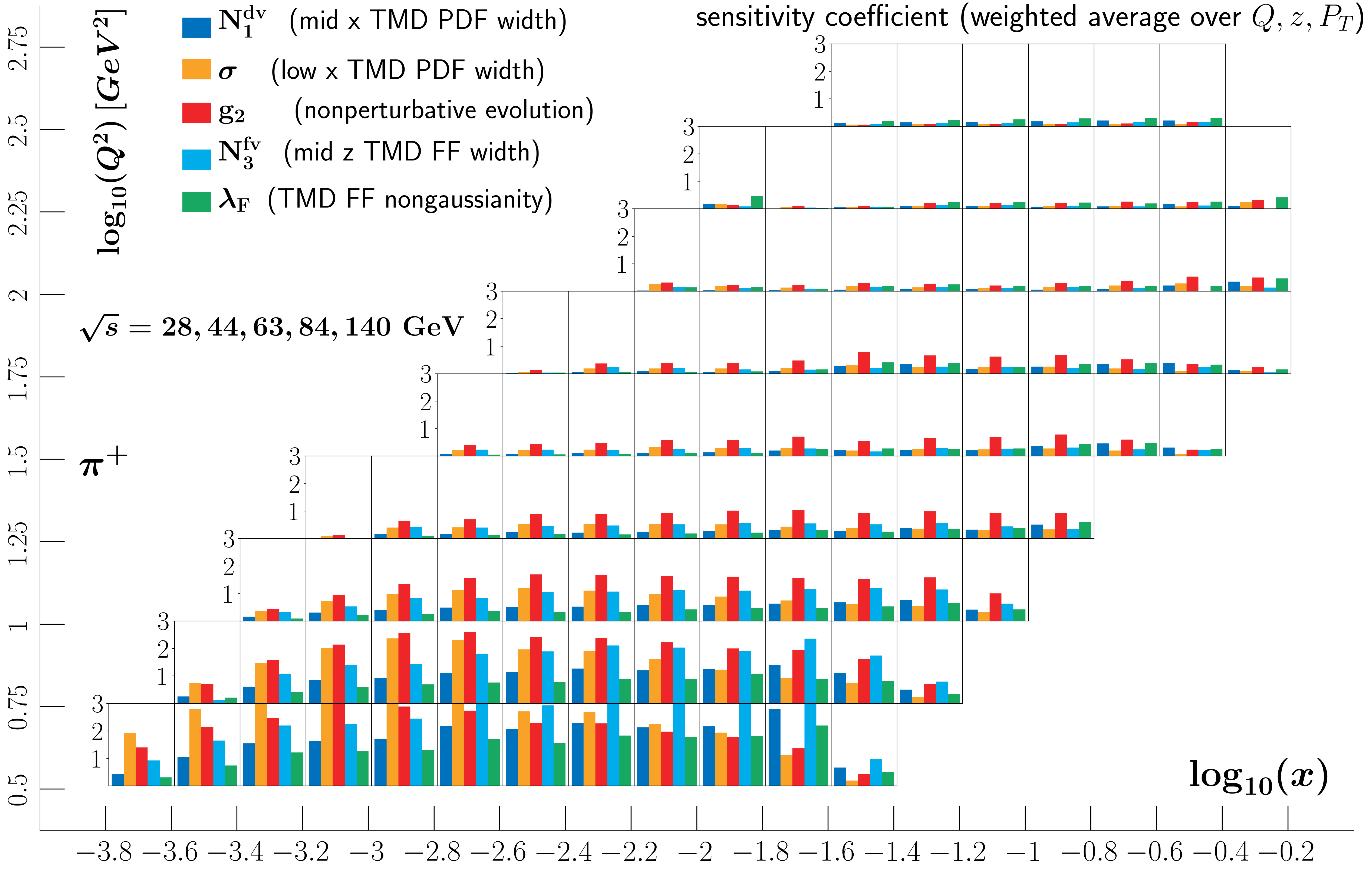


EIC impact studies

SENSITIVITY COEFFICIENTS

$$S[f_i, \mathcal{O}] = \frac{\langle \mathcal{O} \cdot f_i \rangle - \langle \mathcal{O} \rangle \langle f_i \rangle}{\xi \Delta \mathcal{O} \Delta f_i}$$



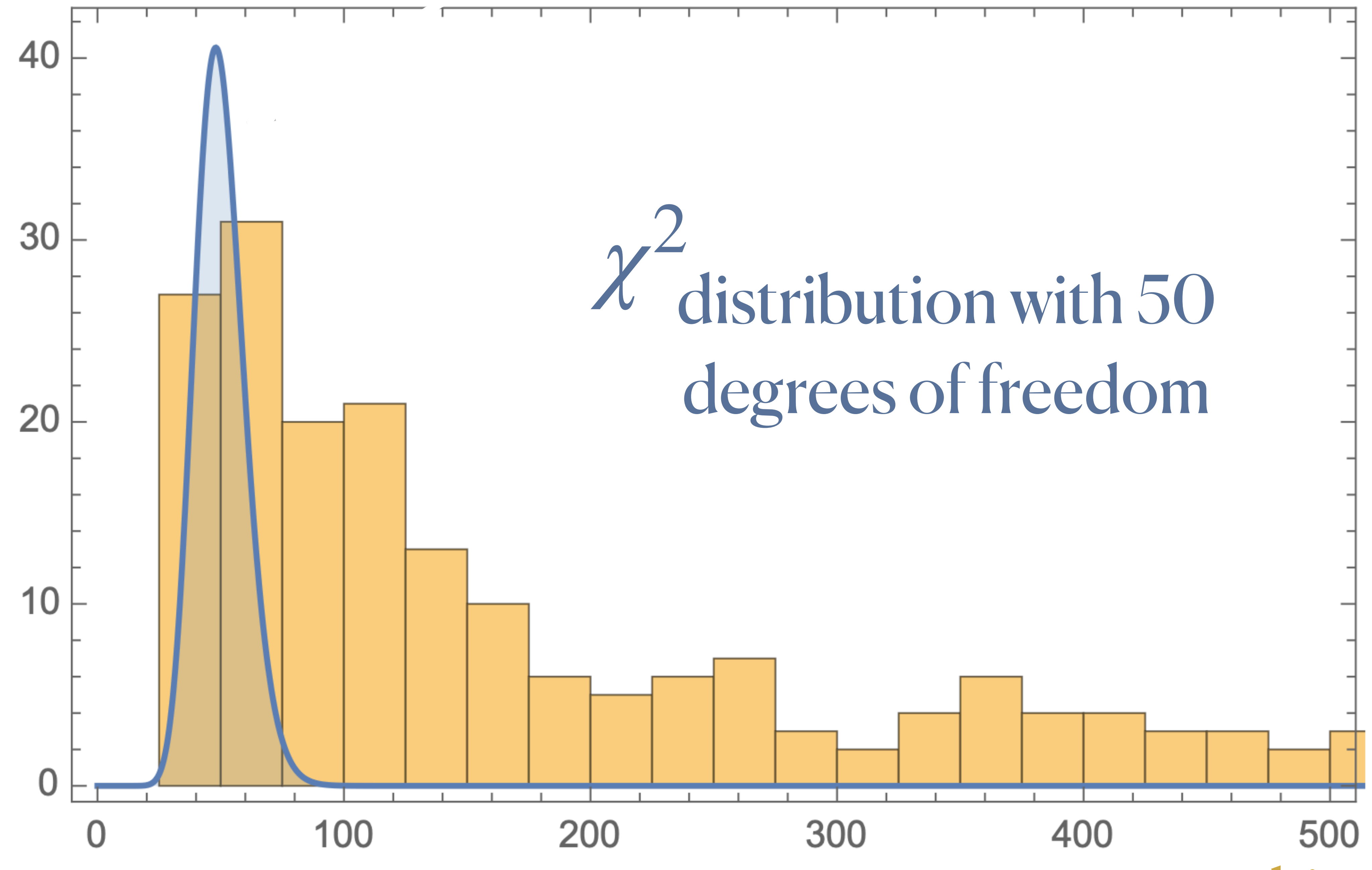
EIC impact studies

REWEIGHING

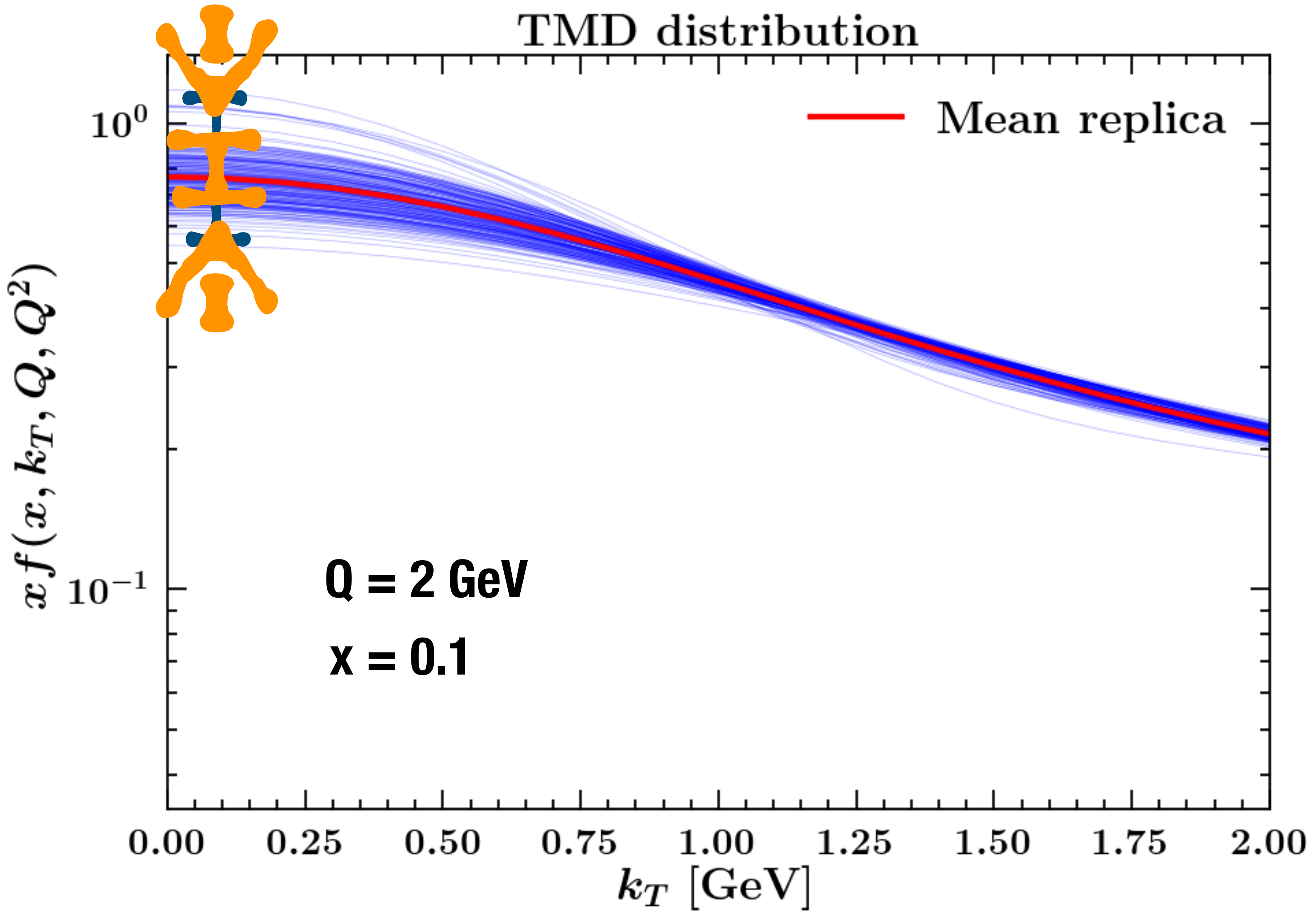
from NNPDF Collaboration
arXiv:1108.1758

$$w_k \propto \mathcal{P}(f_k|\chi_k) \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

200 replicas are compared with pseudodata



histogram of χ^2 distribution of 200 replicas



with n= n.of points

too few replicas survive

FIT NECESSARY