Finding out what you're made of:

Partons on the lattice

Joe Karpie (Columbia University)

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

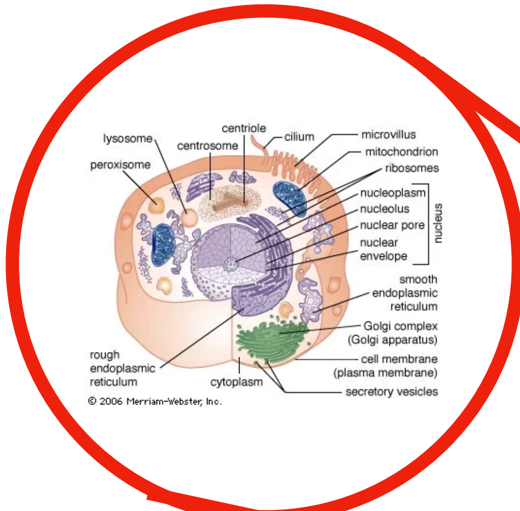
Outline

- **Review Parton Structure**
  - Positivity and phenomenological global analyses
- **Lattice approaches to PDFs**
  - Wilson line operators
  - Inverse problems
  - Techniques for reaching high momentum
- **Numerical results**
  - Lattice analyses of quark and gluon distributions
  - Future of combining lattice and experiment

Digging Deeper and Deeper

- Typical humans in Highline Park (NYC)

Digging Deeper and Deeper



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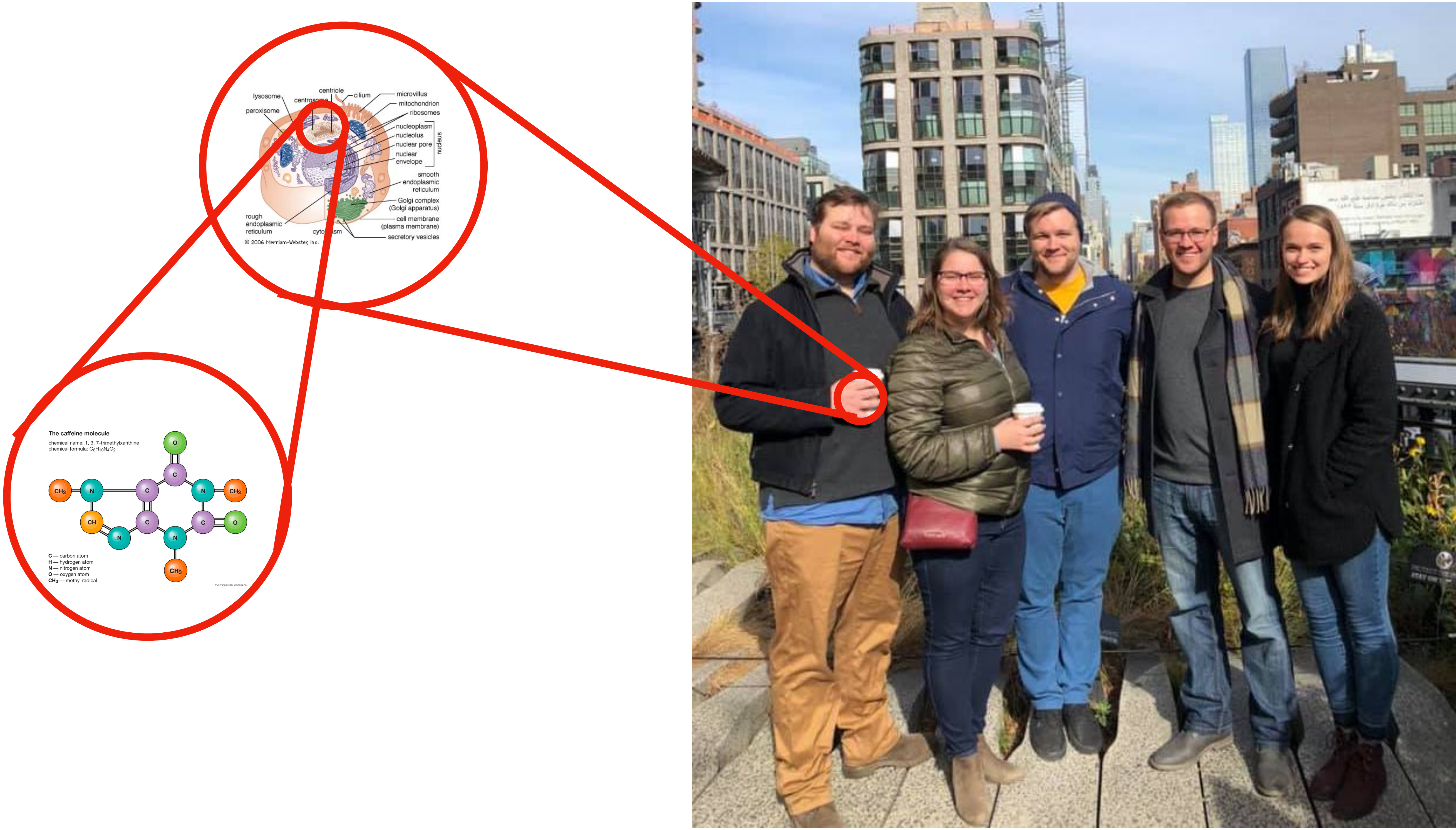

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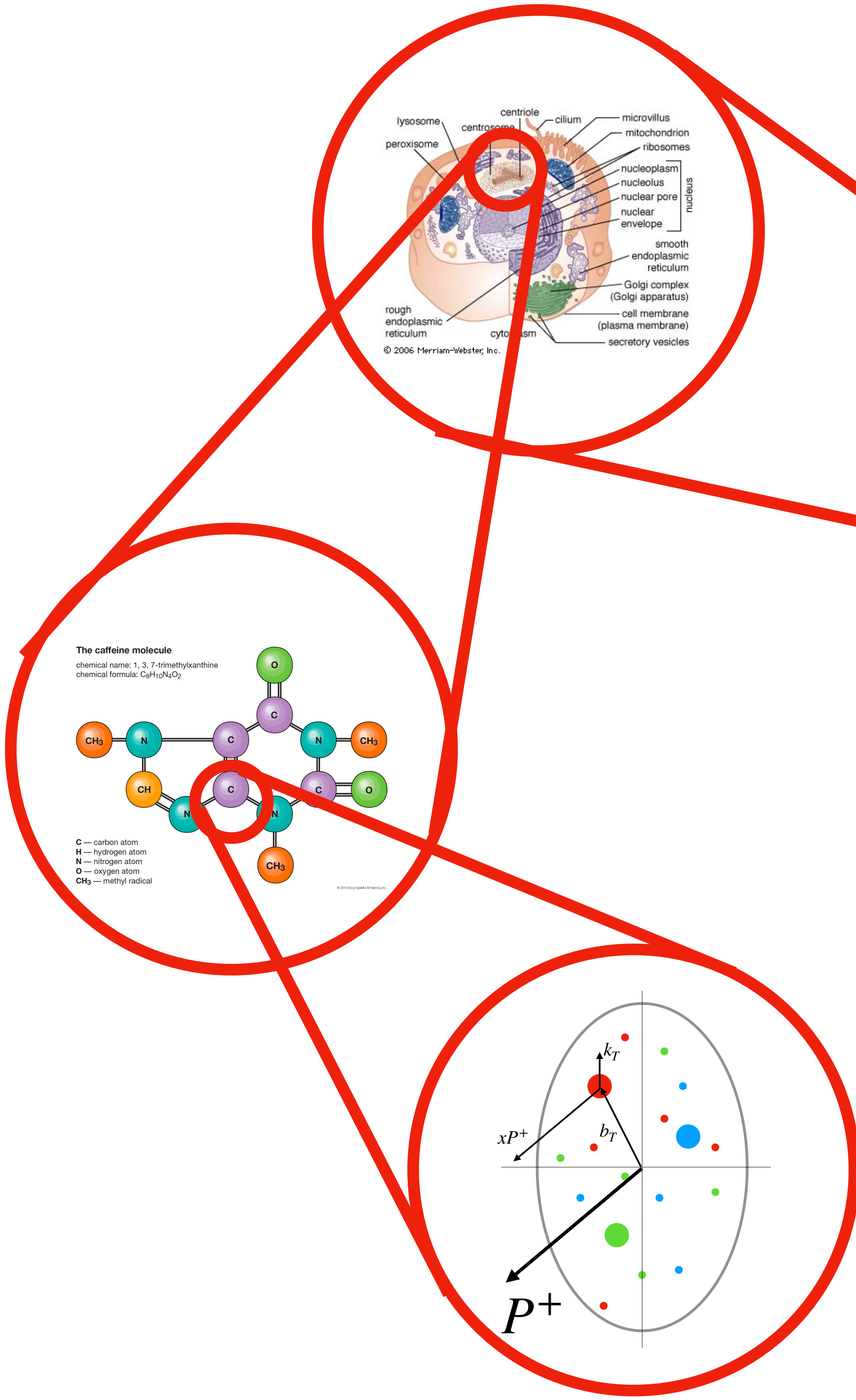
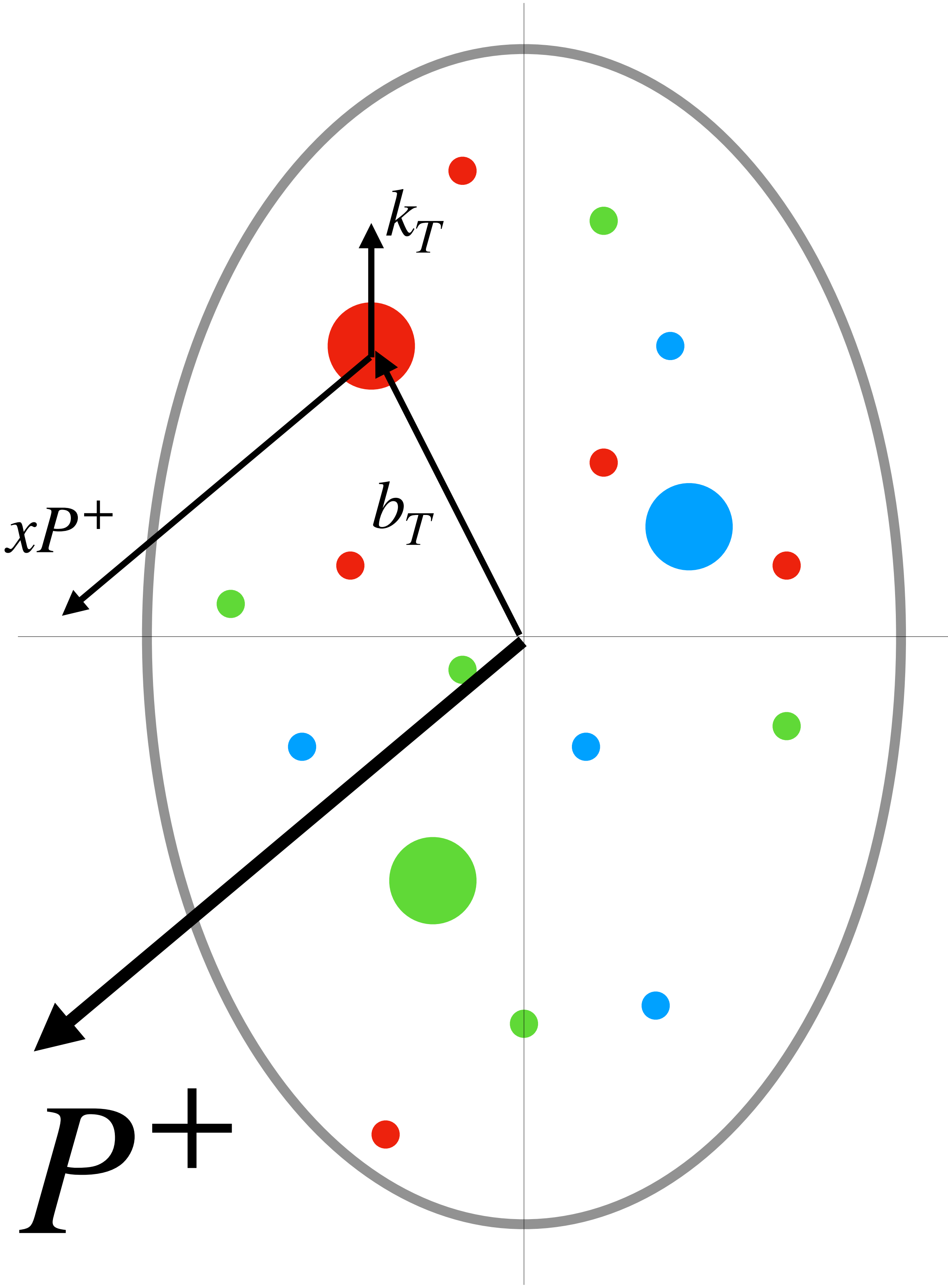


- Typical humans in Highline Park (NYC)

Parton Structure

- Internal structure of hadrons
- How are quarks and gluons distributed?
- How do they contribute to cross sections, hadron mass, spin, …..?
  - **Longitudinal** structure from momentum fraction x
  - **Transverse** structure from
    - Impact parameter b_T
    - Transverse momentum k_T



Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum Distribution (GTMD)

$F(x, b_T, k_T)$

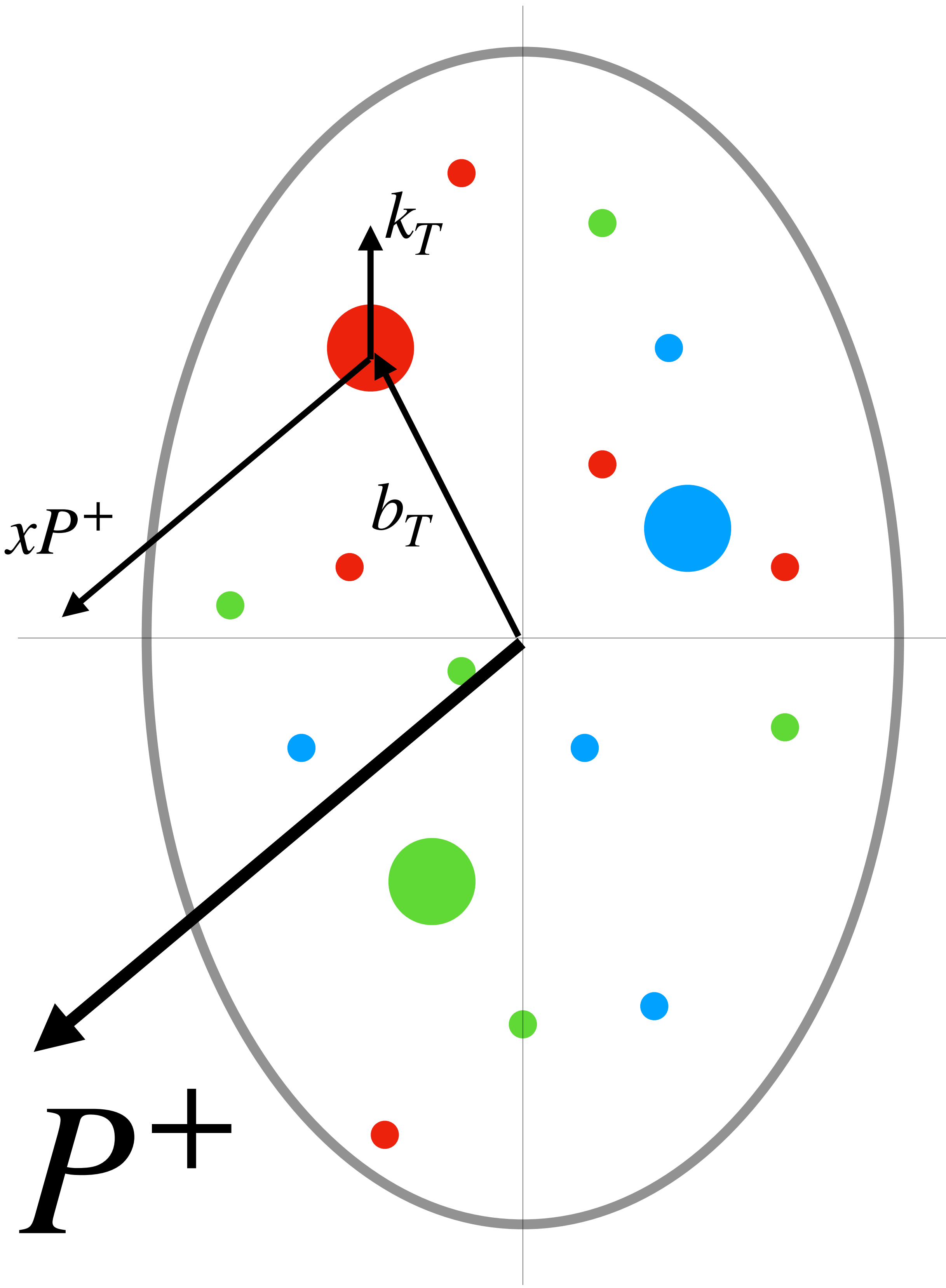
$\int d^2b_t$ $\int d^2k_t$

Transverse Momentum Distribution (TMD)
$f(x, k_T)$

Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2k_t$ $\int d^2b_t$

Parton Distribution Function (PDF)
$f(x)$



Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions

Ioffe time: $\nu = p \cdot z$

V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

- $$I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-)\gamma^+ W(z^-;0)\psi_q(0) \,|\, p \rangle_\mu$$

$$z^2 = 0$$

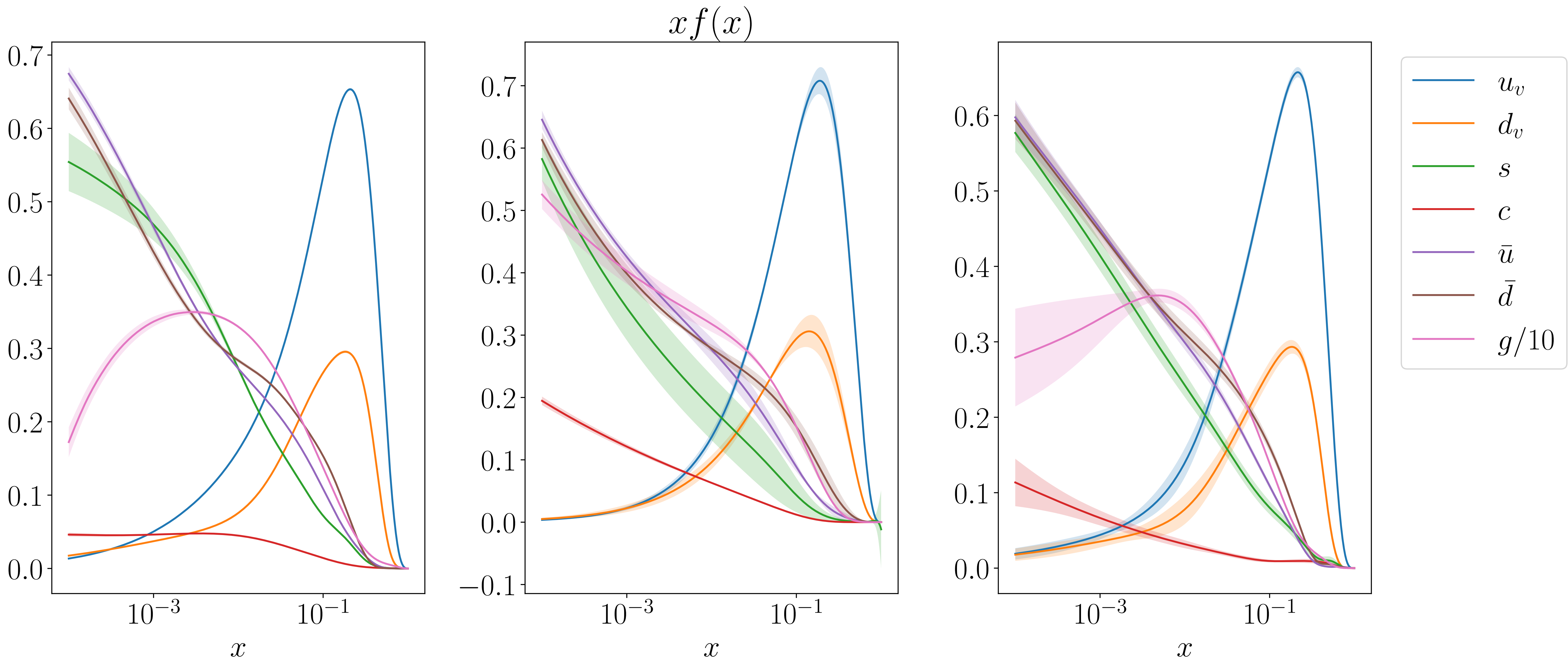
- $$I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-;0) F^i_+(0) \,|\, p \rangle_\mu$$

$$i = x, y$$

- Parton Distribution Functions
  - $$I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$$
  - $$I_g(\nu, \mu^2) = \int_0^1 dx\, \cos(x\nu)\, x f_g(x, \mu^2)$$

Phenomenological Fits

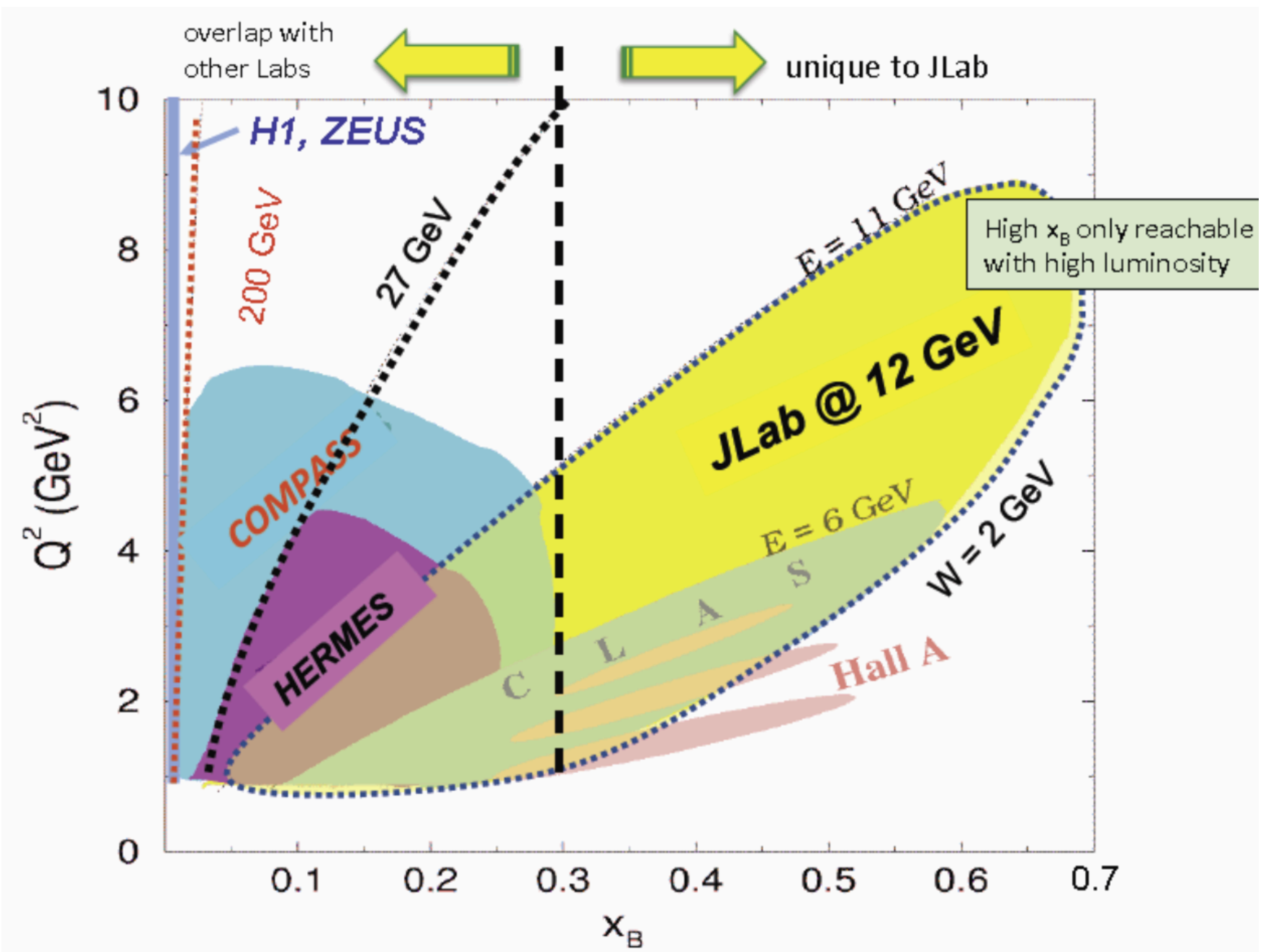
- **Nucleon unpolarized PDFs** from analysis of global experimental data
  - Best known distributions

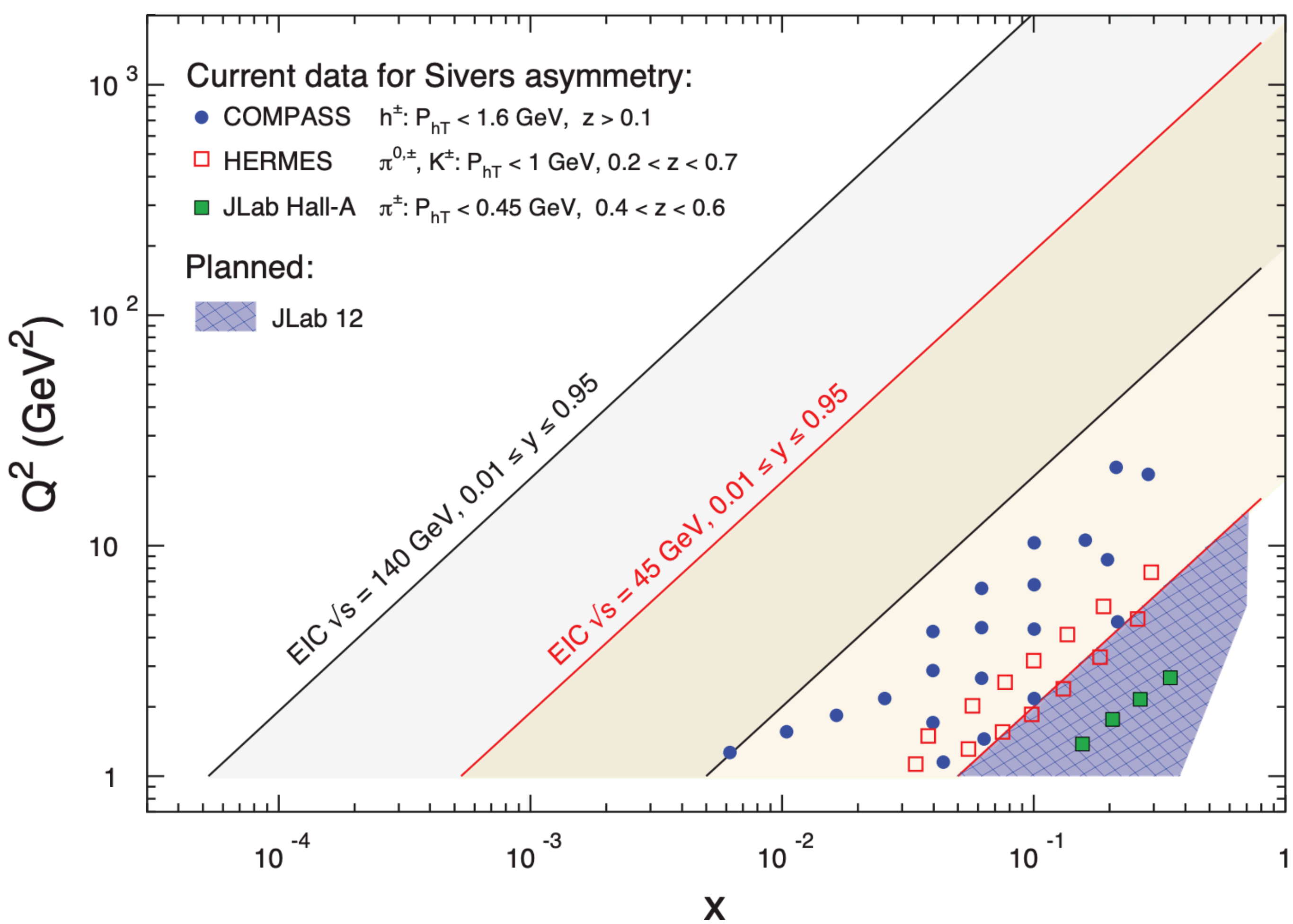


(Left) MSHT20 Eur. Phys. J. C 81 (2021) 4, 341. (Center) JAM20 Phys. Rev. D 104 (2021) 1, 016015. (Right) NNPDF4.0 Eur. Phys. J. C 82 (2022) 5, 428

JLab 12 GeV and the EIC

- Expand Kinematic Coverage
    - JLab extends to large x (also where lattice is most sensitive)
    - Unpolarized gluon PDF dominates low x and is goal of EIC
- Expand to polarized and 3D distributions (lattice goal as well)





JLab 12 GeV White Paper:
Eur. Phys. J. A 48 (2012) 187

EIC White Paper:
Eur. Phys. J. A 52 (2016) 9, 268

Positivity of the PDFs

- In parton model (LO without QCD interactions), $f_i^{\uparrow/\downarrow}(x) \geq 0$
  - Sometimes assumed for PDF analysis
  - For gluons implies,

$$2g(x) = g^{\uparrow}(x) + g^{\downarrow}(x) \qquad\qquad 2\Delta g(x) = g^{\uparrow}(x) - g^{\downarrow}(x)$$

$$|\Delta g(x)| \leq g(x)$$

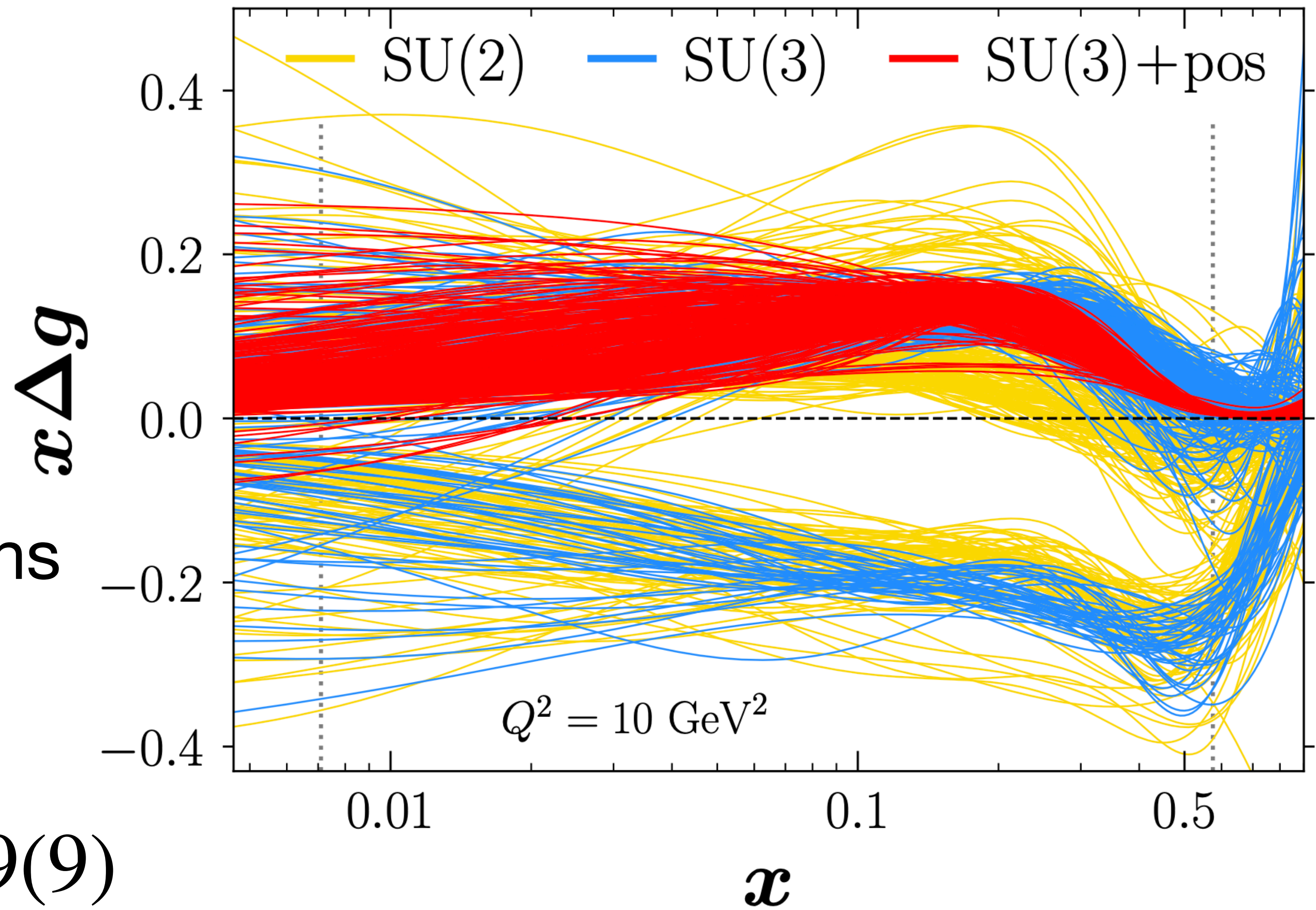
- With interactions, does not have to be true for $\overline{\mathrm{MS}}$ scheme

J. Collins, T. Rogers, N. Sato, Phys Rev D 105 (2022) 7,076010

- How will this effect analyses?

Spinning gluons

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

- Without constraint: $\Delta G = 0.3(5)$

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

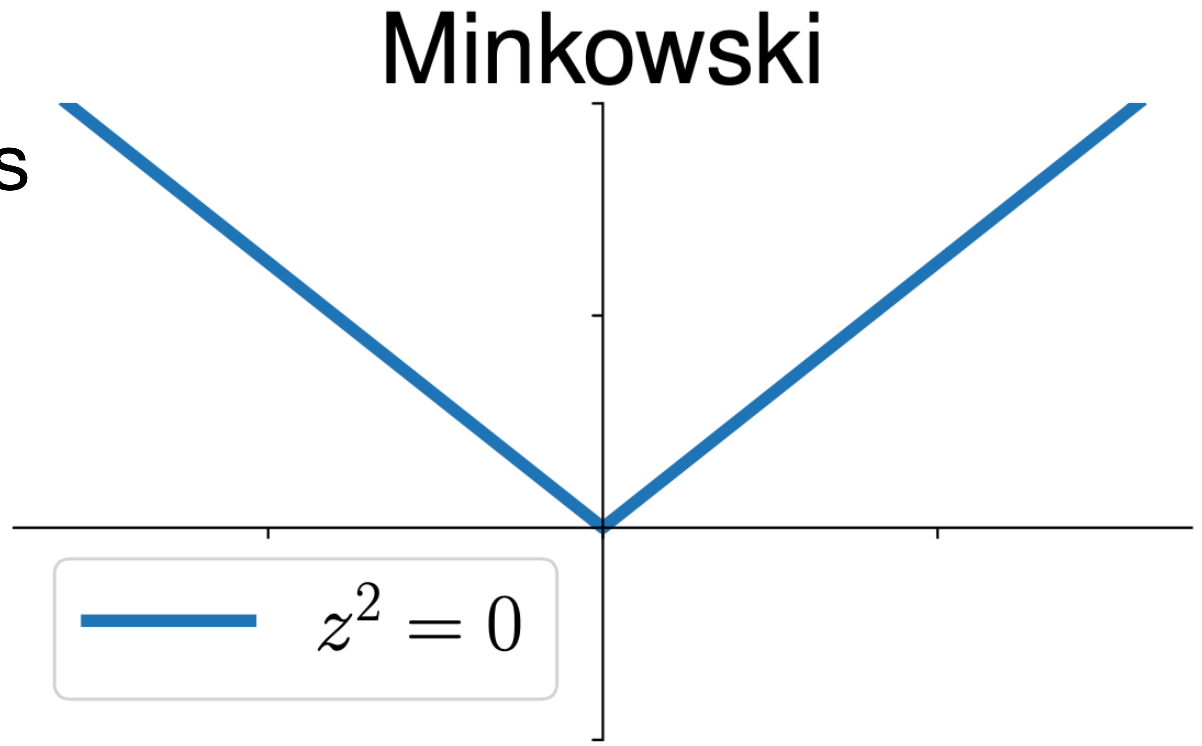
- Lattice: $\Delta G = 0.251(47)(16)$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

$$\Delta G = \int dx\, \Delta g(x)$$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations



- Use space-like separations

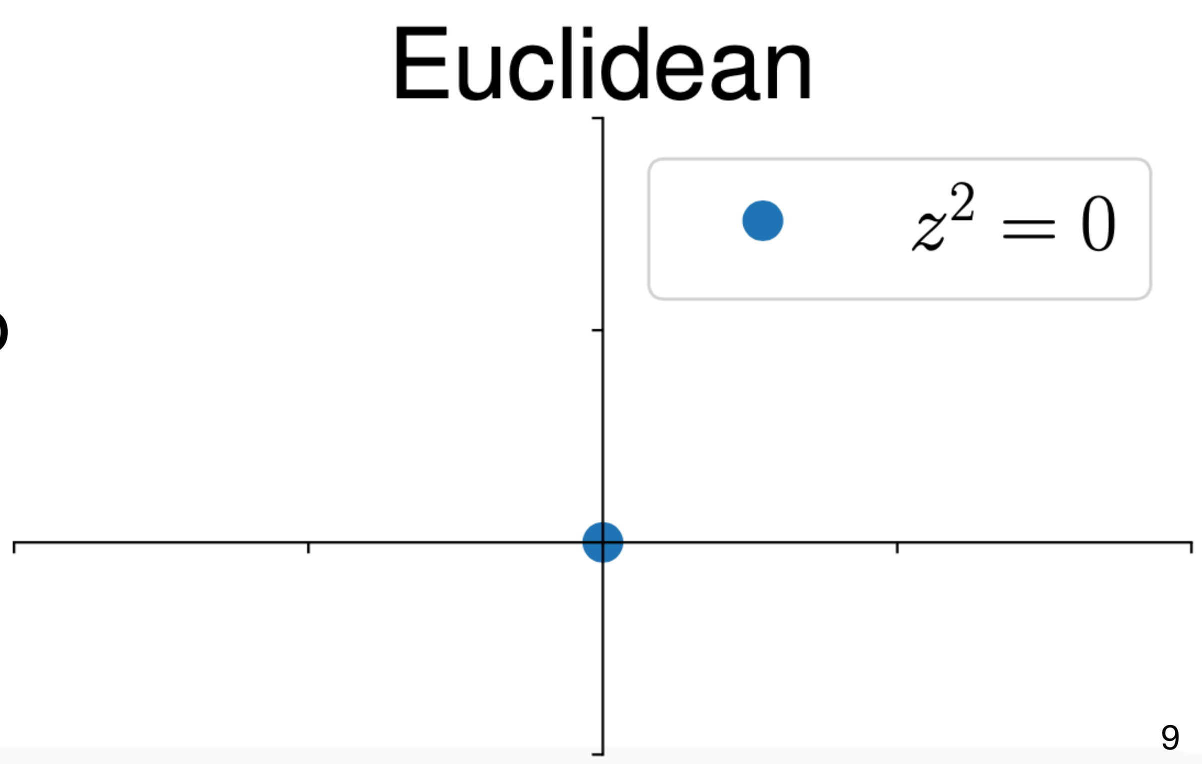
  X. Ji *Phys Rev Lett* 110 (2013) 262002

  - Wilson line operators

$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

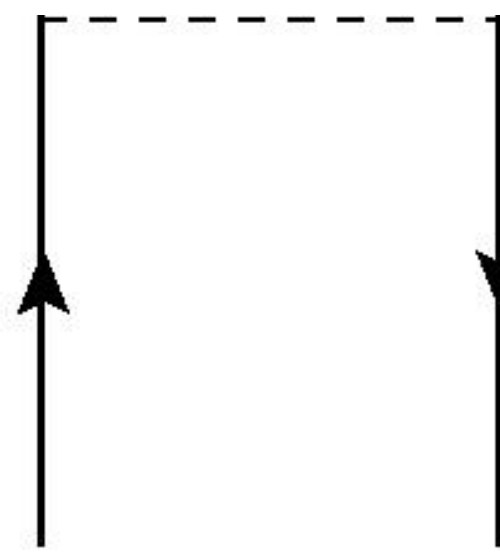
  - Factorizations exist analogous to cross sections



Many approaches

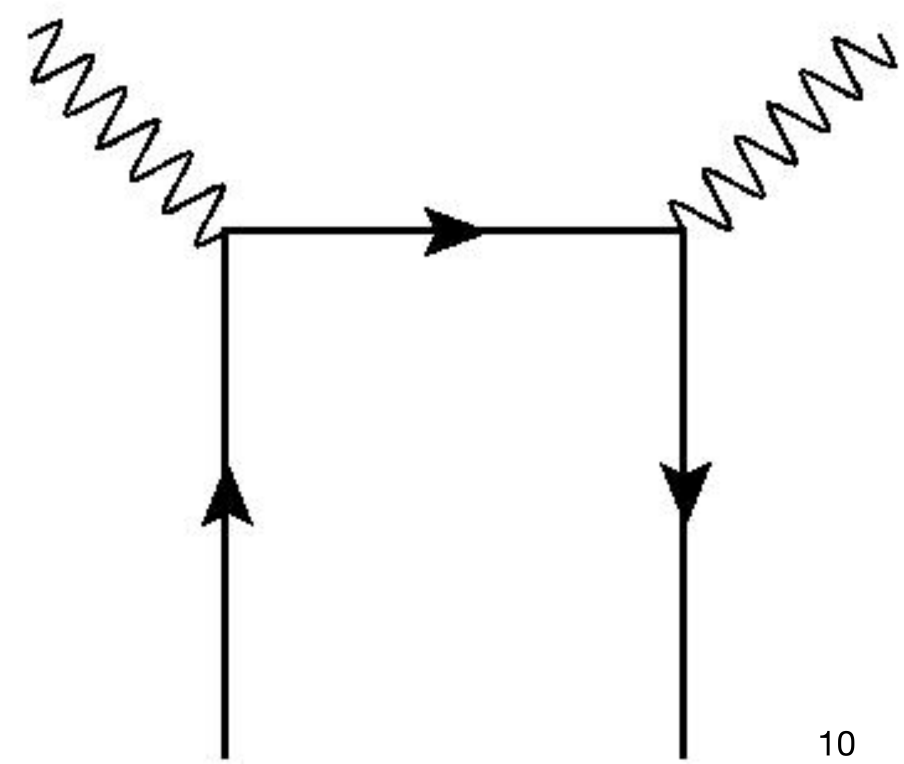
- **Wilson line operators**

$$O_{WL}(x;z) = \bar{\psi}(x+z)\Gamma W(x+z;x)\psi(x)$$



  - LaMET X. Ji *Phys. Rev. Lett.* 110 (2013) 262002
  - Pseudo-PDF A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025
- Two current correlators

$$O_{CC}(x,y) = J_{\Gamma}(x)J_{\Gamma'}(y)$$



  - Hadronic Tensor
    K.-F. Liu et al *Phys. Rev. Lett.* 72 1790 (1994)
    Phys. Rev. D 62 (2000) 074501
  - HOPE
    W. Detmold and C.-J. D. Lin, *Phys. Rev. D* 73 (2006) 014501
  - Short distance OPE
    V. Braun and D. Muller *Eur. Phys. J. C* 55 (2008) 349
  - OPE-without-OPE
    A. Chambers et al, *Phys. Rev. Lett.* 118 (2017) 242001
  - Good Lattice Cross Sections
    Y.-Q. Ma and J.-W. Qiu *Phys. Rev. Lett.* 120 (2018) 2, 022003

Wilson Line Matrix Elements

- Matrix element $M(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0)\,|\, p\rangle$ $\qquad z^2 \neq 0$

$$= 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

- Quasi-PDF: $\tilde{q}(y, p_z^2) = \dfrac{1}{2p_\alpha}\displaystyle\int dz e^{iyp_z z} M(z, p_z)$

- Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

- $$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(xp_z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)p_z)^2}\right)$$

- Pseudo-ITD: A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda_{\mathrm{QCD}}^2 z^2)$$

$$= \int du C'(u, \mu^2 z^2) I_q(u\nu, \mu^2) + O(\Lambda_{\mathrm{QCD}}^2 z^2)$$

The Role of Separation and Momentum

- In quasi-PDF and pseudo-PDF, separation and momentum swap roles

Scale:

$p_z^2 \,/\, z^2$

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ^2_{QCD}
- Technically only requires single value

Fourier variable:

$z \,/\, p_z$, or $\nu = p \cdot z$

- Variable for inverse Fourier Transform
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

Lattice QCD

Why Lattice QCD?

- QCD lacks controllable perturbative expansion
- Numerical Lattice Field Theory at all couplings, masses, ….
- Systematically improvable approximations

Summit (ORNL/OLCF)

Lattice QCD

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Need to watch out for

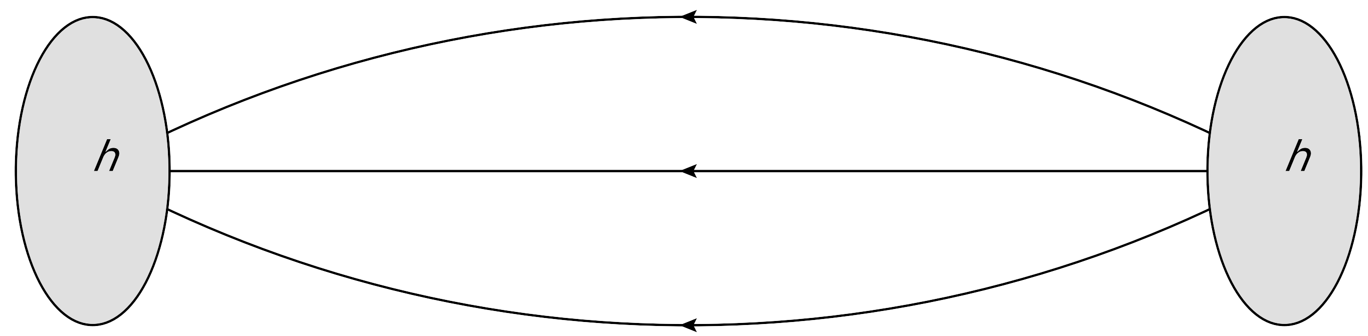
- Finite lattice spacing
- Finite physical volume
- Unphysical pion mass
- Euclidean vs Minkowski space
- $u - d$ iso-vector to avoid noisy diagrams

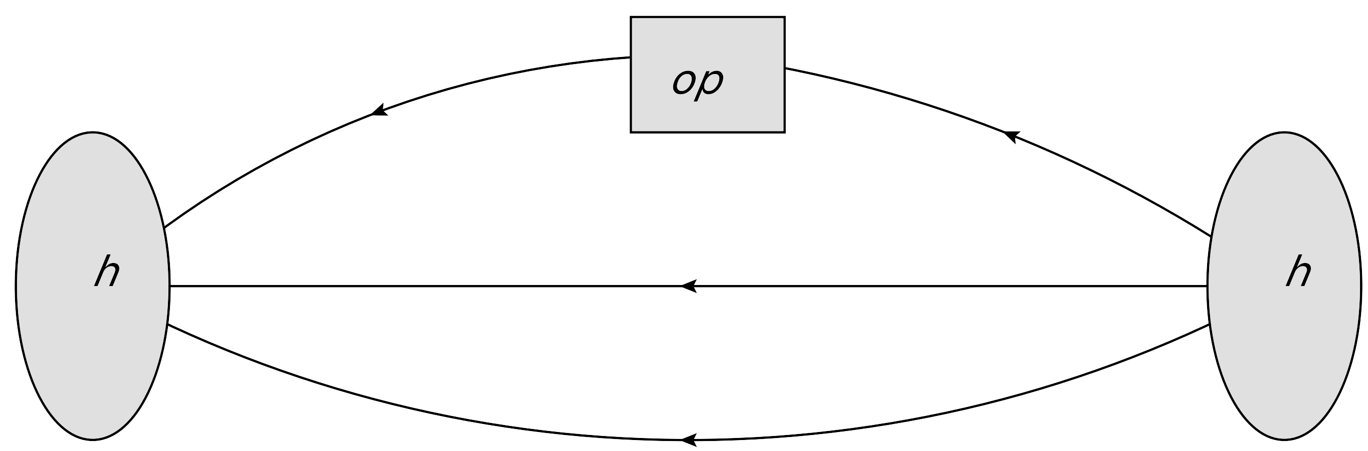
Correlation Functions

$$C_2(T) = \langle O_h(T)\bar{O}_h(0)\rangle = \sum_n |Z_n|^2 e^{-E_n T}$$

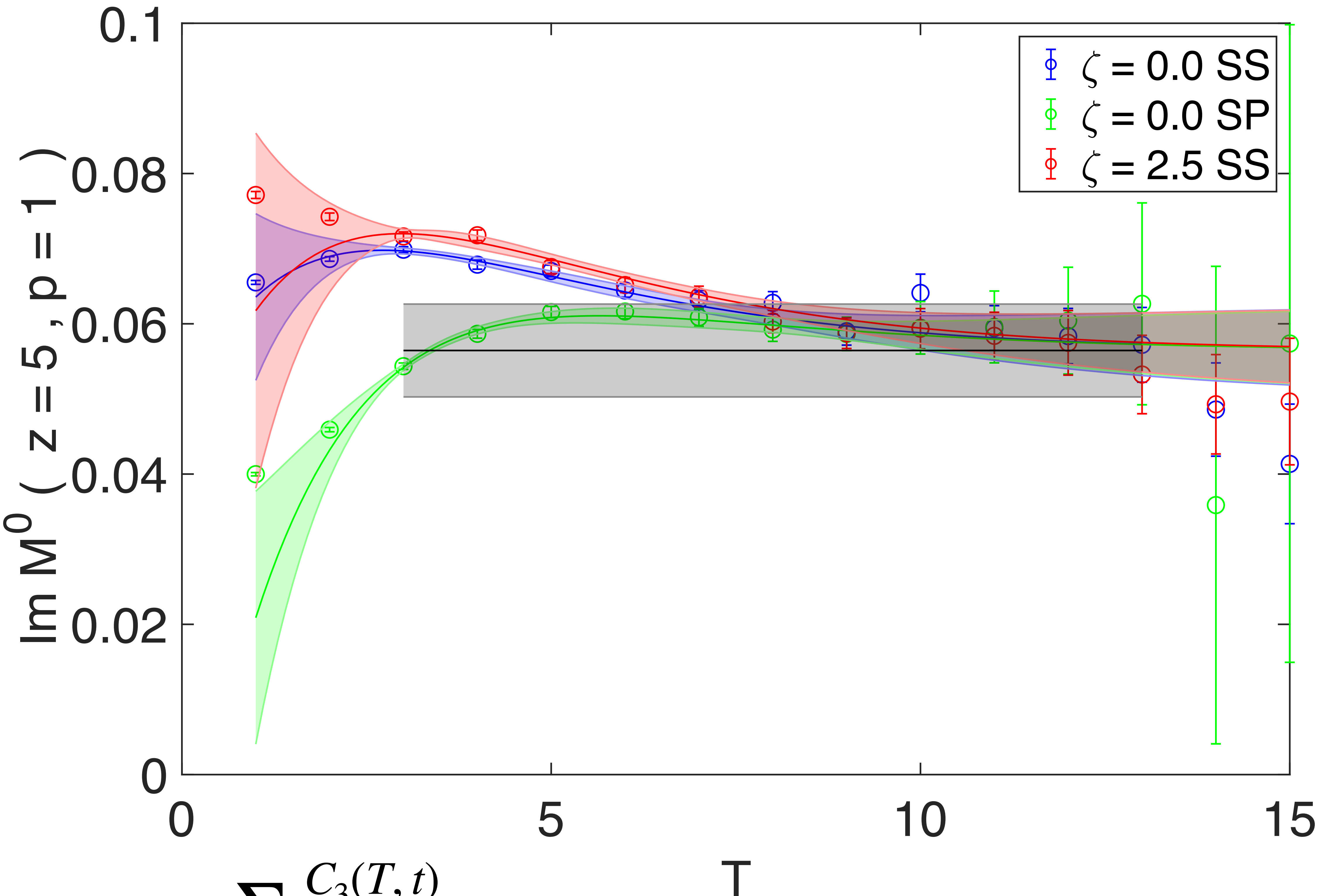
$$C_3(T,t) = \langle O_h(T)O_{op}(t)\bar{O}_h(0)\rangle$$

- Ratios lead to hadronic matrix elements when $T \to \infty$
  - Short T: Excited state contamination
  - Large T: Exponentially growing signal-to-noise ratio





Correlation Functions



$$R(T) = \sum_t \frac{C_3(T,t)}{C_2(T)} \quad M^{\text{eff}}(T) = R(T+1) - R(T) = M + O(e^{-(E_1-E_0)T})$$

Difficulty Reaching High Momentum

- **Creating operator with high overlap and signal**
  - Momentum smearing improves overlap with moving states
    G. Bali et al Phys. Rev. D 93 (2016) 9, 094515
  - Distillation from all time slices improves signal
  - GEVP optimizes the overlap with ground state

M. Peardon, et al,
Phys. Rev. D 80 (2009) 054506
C. Egerer et al
Phys. Rev. D 103 (2021) 3, 034502

- **Excited state energy gap shrinks**
  - Larger times needed for ground state
  - Summed GEVP techniques can remove lowest states and suppress remaining

    J. Bulava, M. Donnellan, R. Sommer JHEP 01 (2012) 140

- **Exponentially suppressed signal-to-noise ratio**
  - No current solutions (technically a sign problem)

What do we do with this data?

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$q(x) = \int_0^\infty d\nu\, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$q(x) = \int_0^\infty d\nu\, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

- Forward integral to an ill posed matrix equation

$$\mathfrak{M}(\nu) = \int_0^1 dx\, C(x\nu)\, q(x) \to [\mathbf{C}][\mathbf{q}]$$

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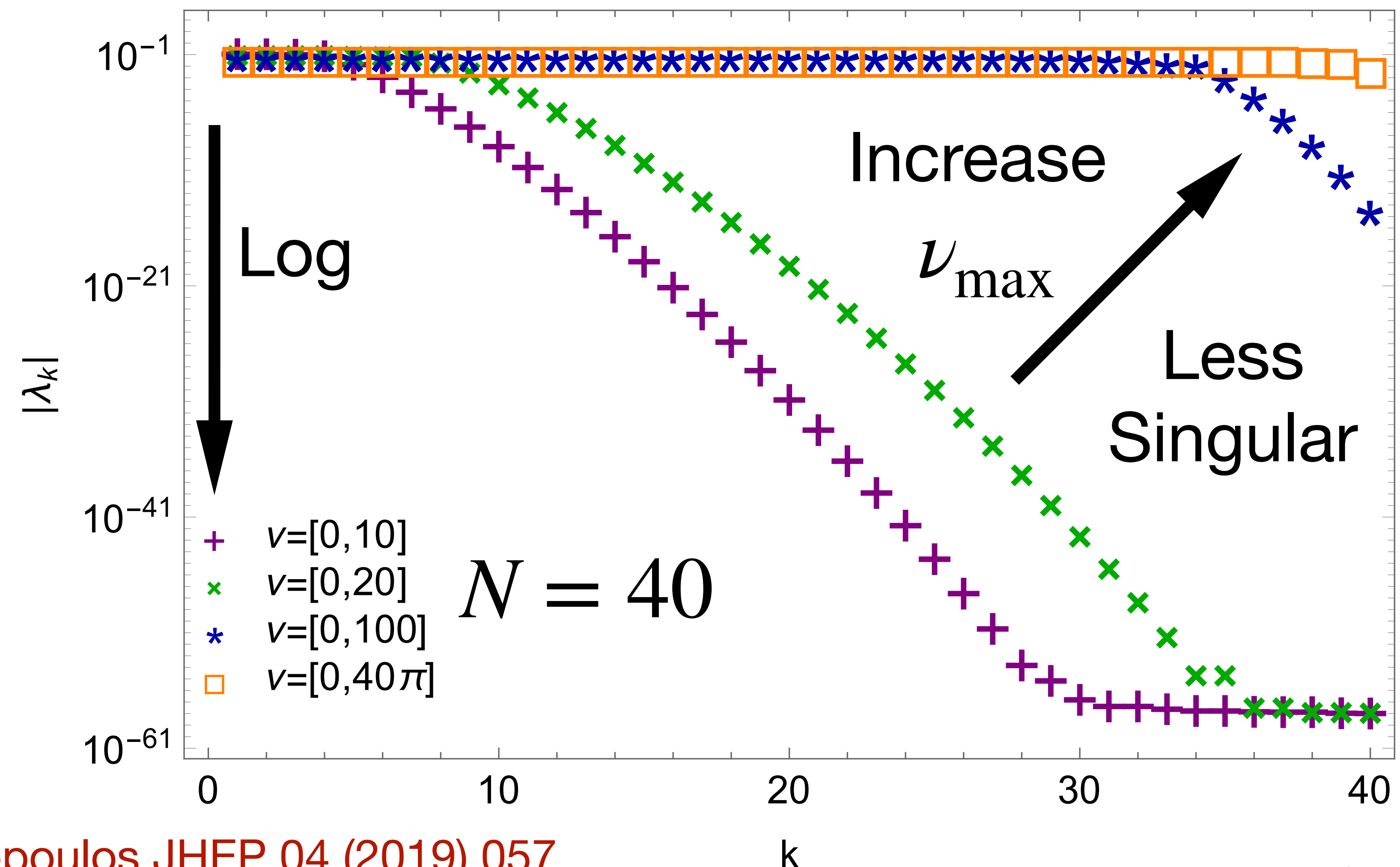

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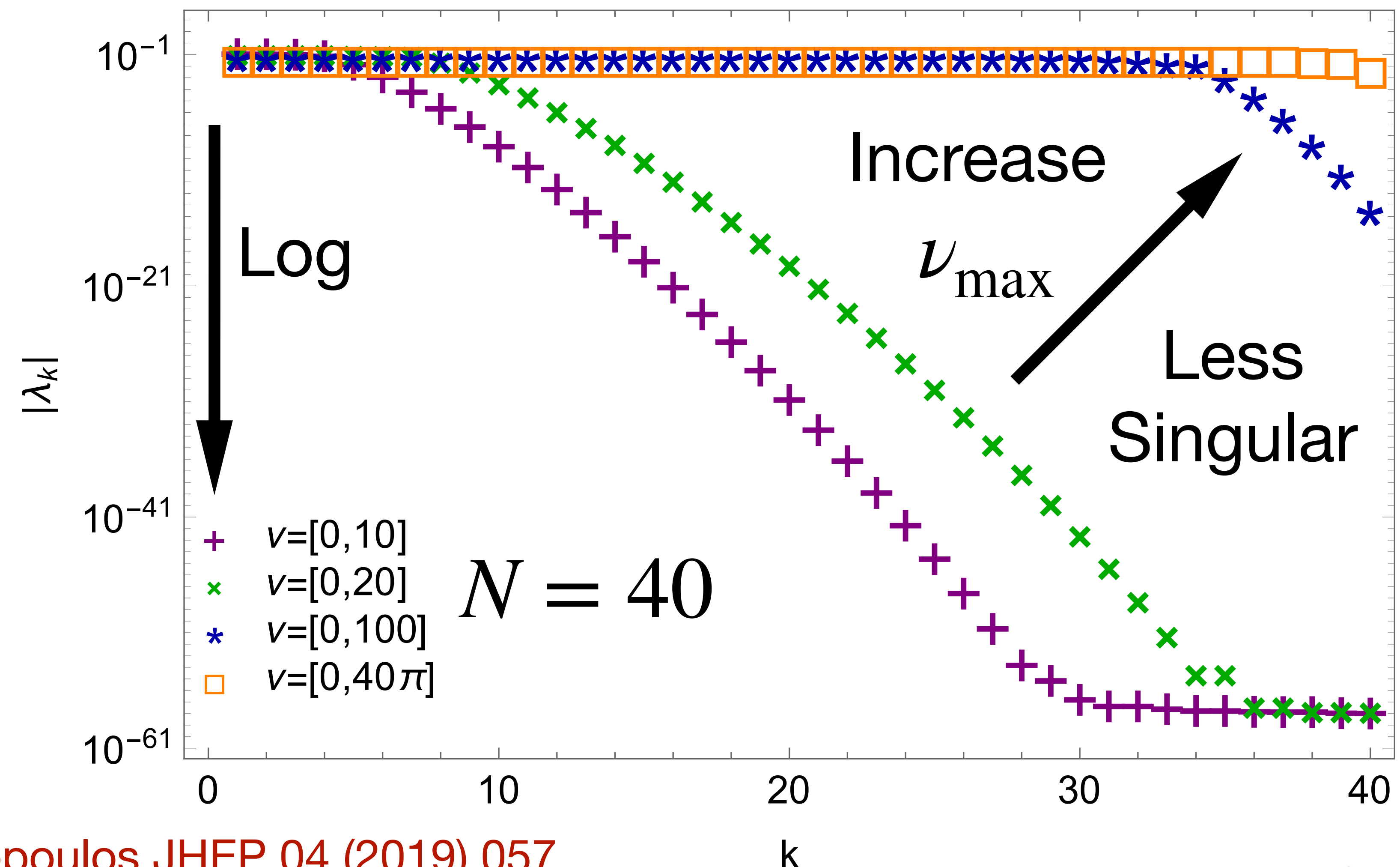
$$\mathfrak{M}(\nu) = \int_0^1 dx\, C(x\nu)\, q(x) \to [\mathbf{C}][\mathbf{q}]$$

- Must be regulated by additional information
  - Restricted functional form
  - Constraints on the PDF or parameters
  - Assumptions of smoothness, continuity, ….



JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for Parton Physics

- **Structure Functions**

$$F_2(x, Q^2) = \sum_i \int_x^1 d\xi\, C(\xi, \frac{\mu^2}{Q^2})\, q(\frac{x}{\xi}, \mu^2)$$

- **LaMET (on the lattice)**

$$M(p_z, z) = \int_{-\infty}^{\infty} dy\, e^{iyp_z z}\, \tilde{q}(y, p_z^2)$$

- **pseudo-Distributions / Good Lattice Cross Sections**

$$\mathfrak{M}(\nu, z^2) = \int_{-1}^{1} dx\, C(x\nu, \mu^2 z^2)\, q(x, \mu^2)$$

- **Local Matrix elements / HOPE / OPE-without-OPE**

$$a_n(\mu^2) = \int_{-1}^{1} dx\, x^{n-1}\, q(x, \mu^2)$$

- **Hadronic Tensor**

$$\tilde{W}_{\mu\nu}(\tau) = \int d\nu\, e^{-\nu\tau}\, W_{\mu\nu}(\nu)$$

Approaches to the Inverse Problem

- Parametric Solutions:
  - Phenomenological model
  - **Neural Networks**

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos
JHEP 04 (2019) 057

- Non-Parametric Solutions:
  - **Backus Gilbert**
  - **Bayesian Reconstruction / Maximum Entropy**
  - Bayes-Gauss-Fourier Transform / Gaussian Processes

Unknown Functions

- **Lattice systematic errors**

$$\mathfrak{M}^{\mathrm{latt}}(p, z, a) = \mathfrak{M}^{\mathrm{cont}}(\nu, z^2) + \sum_n \left(\frac{a}{|z|}\right)^n P_n(\nu) + \left(a\Lambda_{\mathrm{QCD}}\right)^n R_n(\nu) + \ldots$$

- **Power Corrections**

$$\mathfrak{M}^{\mathrm{cont}}(p, z, a) = \mathfrak{M}^{\mathrm{lt}}(\nu, z^2) + \sum_n \left(z^2\Lambda^2_{\mathrm{QCD}}\right)^n B_n(\nu)$$

- **Factorization**

$$[\mathrm{Re/Im}]\mathfrak{M}_{\mathrm{lt}}(\nu, z^2) = \int_0^1 dx\, K_{\mathrm{R/I}}(x\nu, \mu^2 z^2) q_{\mp}(x, \mu^2)$$

$$K_R(x\nu, \mu^2 z^2) = \cos(x\nu) + O(\alpha) \qquad K_I(x\nu, \mu^2 z^2) = \sin(x\nu) + O(\alpha)$$

Jacobi Polynomials

- Change of variables from true Jacobi polynomials

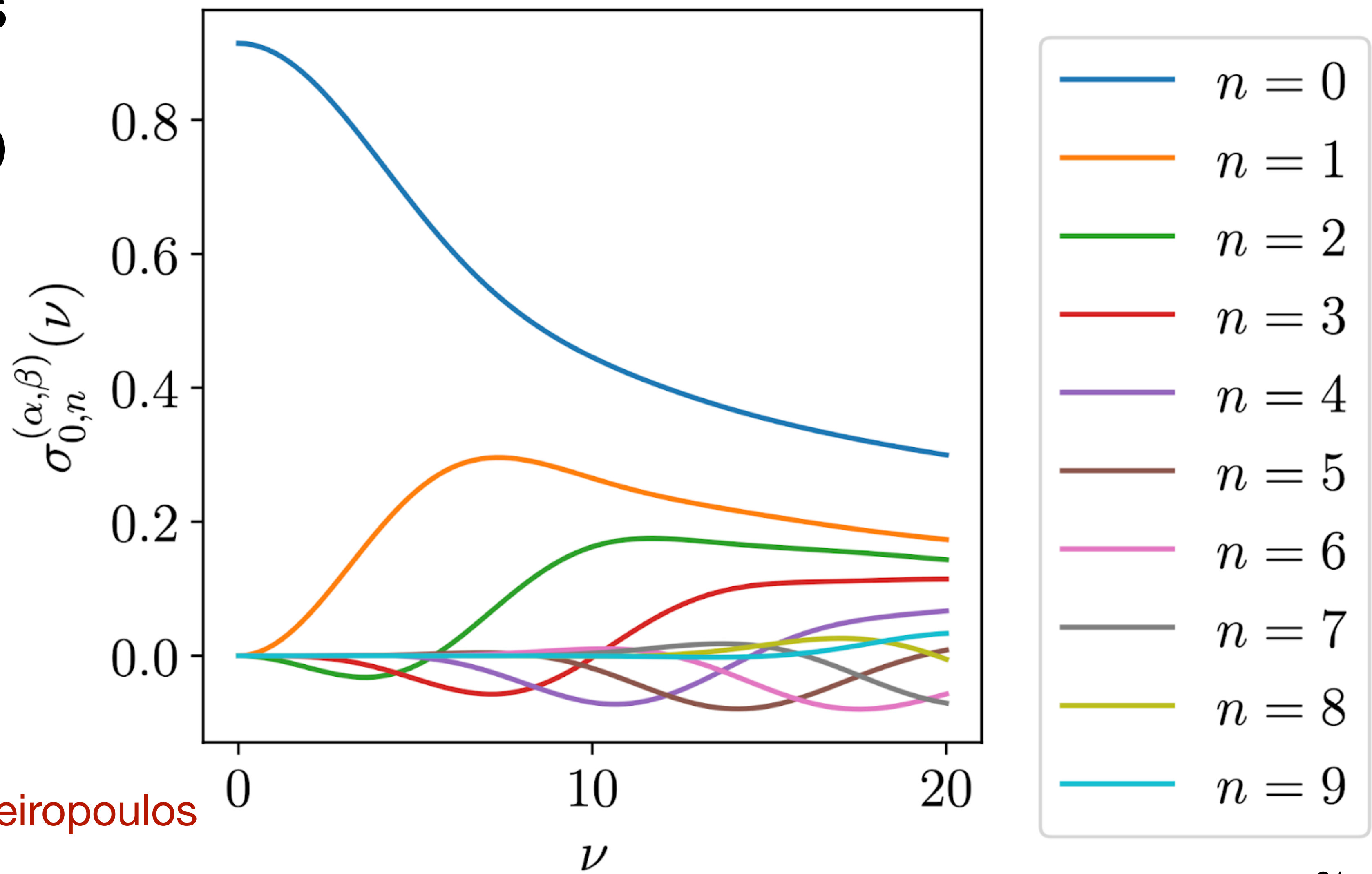
$$\int_{-1}^{1} dz(1+z)^{\alpha}(1-z)^{\beta}P_n(z)P_m(z) = \delta_{nm}\tilde{N}_n \rightarrow \int_0^1 dx x^{\alpha}(1-x)^{\beta}J_n(x)J_m(x) = \delta_{nm}N_n$$

- PDF Parameterized as

$$q_- = x^{\alpha}(1-x)^{\beta}\sum_n {}_-d_n P_n(x)$$

- ITD becomes

$$\mathrm{Re}\, I_q(\nu) = \sum_n {}_-d_n \sigma_n(\nu)$$

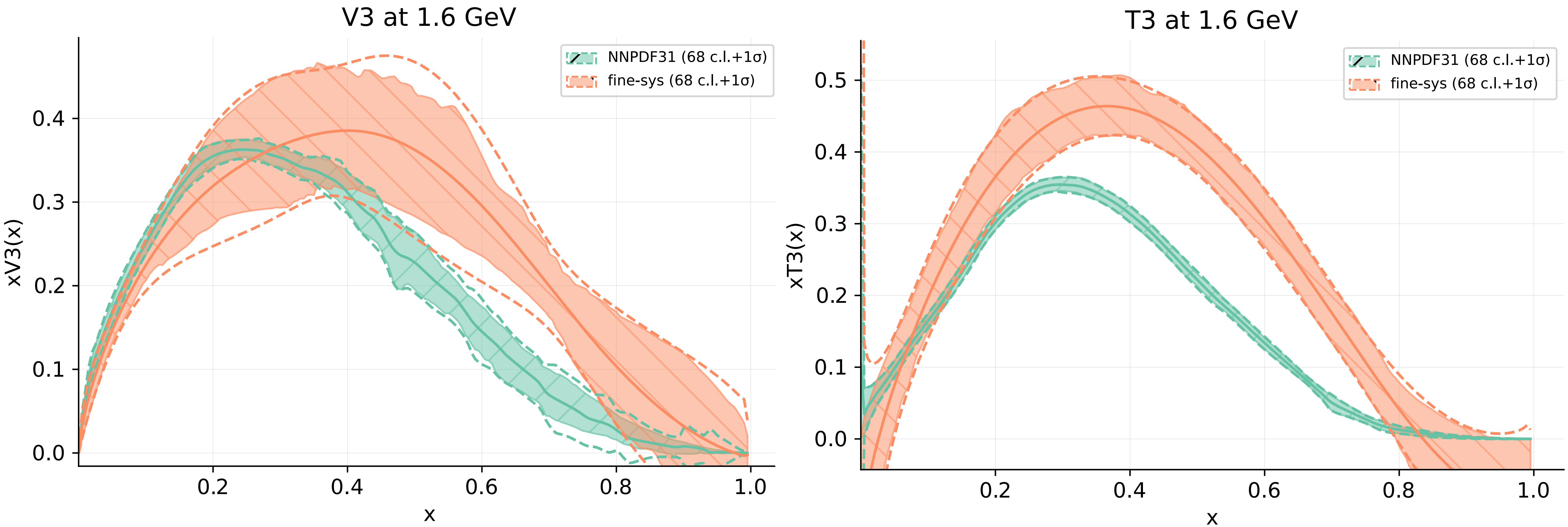


JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos
JHEP 11 (2021) 024

Neural Networks for Inverse Problems

- NNPDF Analysis with NN geom 2-5-3-1

L. Del Debbio, T. Giani, JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos JHEP 02 (2021) 138



$V_3 = q_- = q_v \quad T_3 = q_+$

- Higher dimensional distributions will need larger networks
  - Sivers asymmetry and TMDs with NNs I. Fernando et al SciPost Phys.Proc. 8 (2022) 035

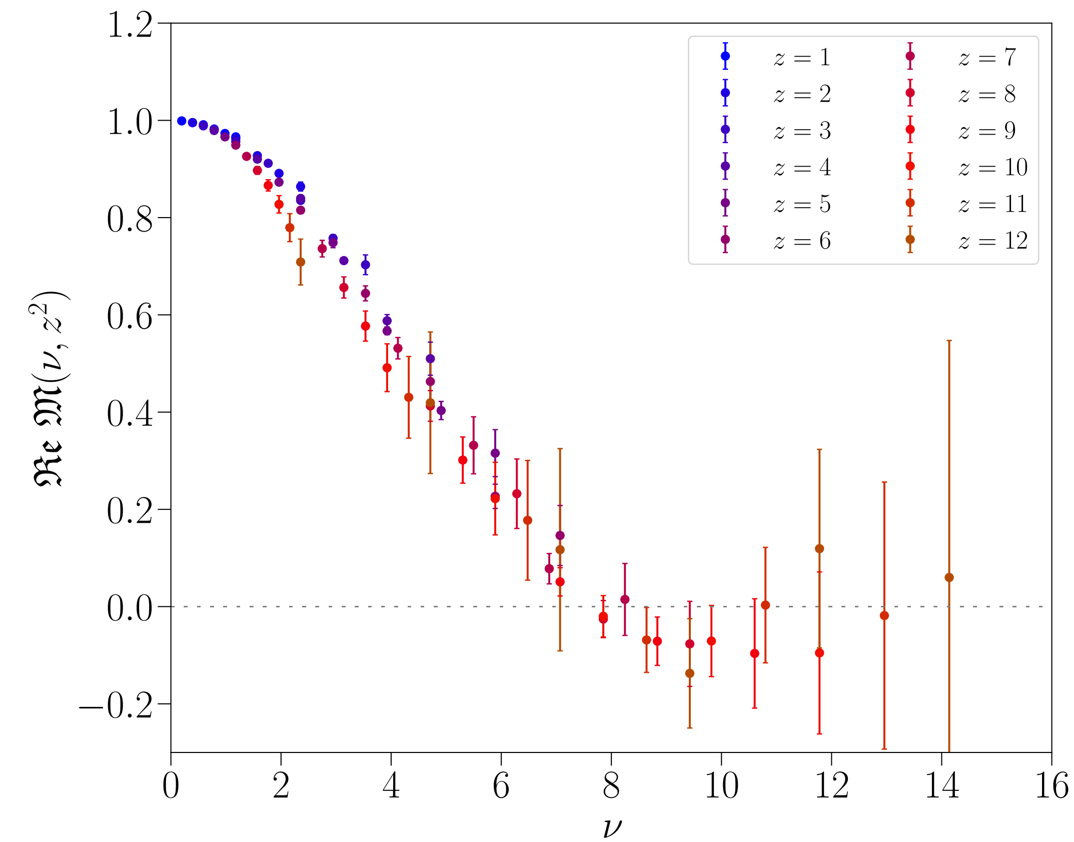
Quark pseudo-PDFs

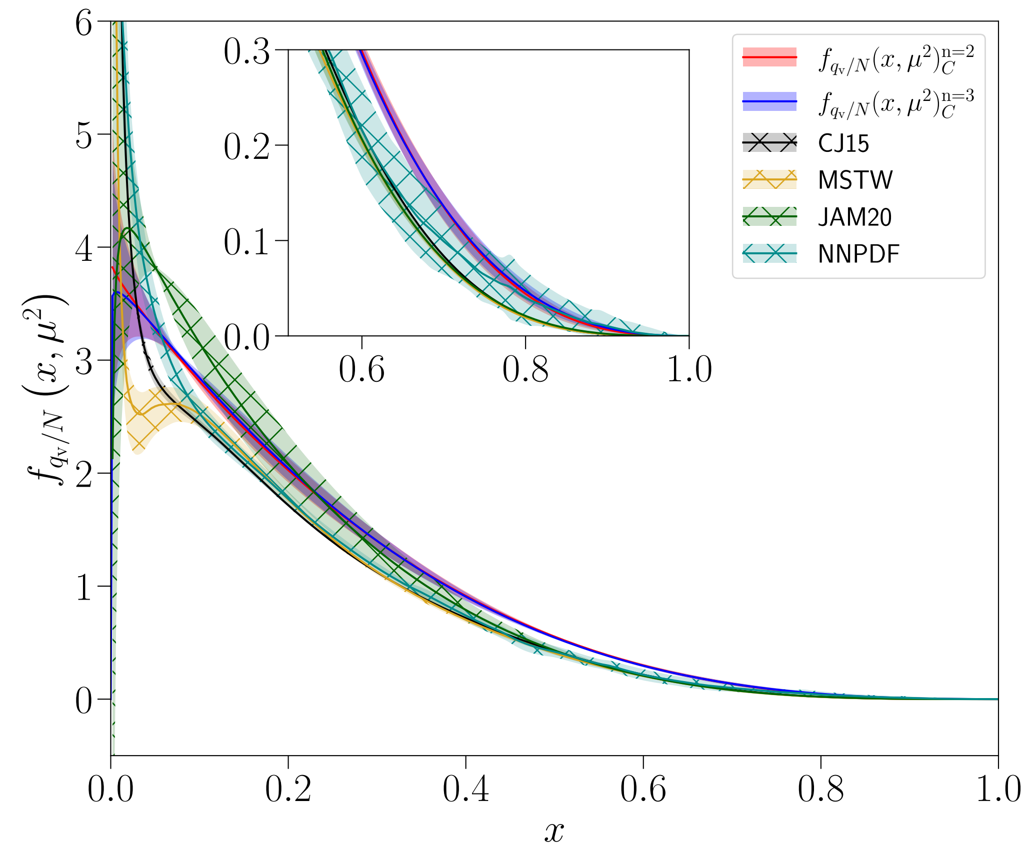
- Bare Matrix element: $M = \langle p \,|\, \bar{\psi}(z)\Gamma W(z;0)\psi(0)p\rangle$
- Choose multiplicatively renormalizable operators
  - Unpolarized PDF : $\Gamma = \gamma^t$
  - Helicity PDF : $\Gamma = \gamma^z\gamma^5$
  - Transversity PDF : $\Gamma = \gamma^5\gamma^t\gamma^i \quad ; \quad i = x, y$
- First study with distillation with unphysically heavy quarks and coarse ensemble

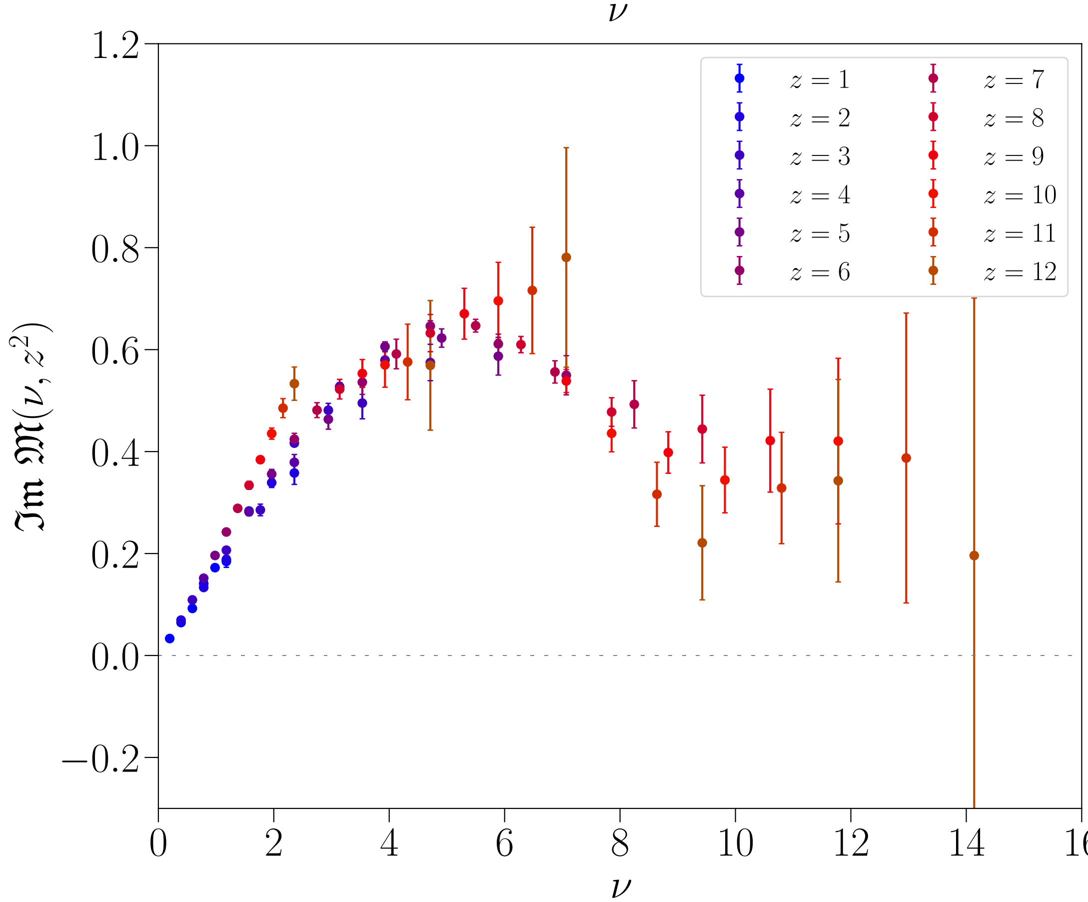
$a = 0.094$ fm
$m_\pi = 358$ MeV

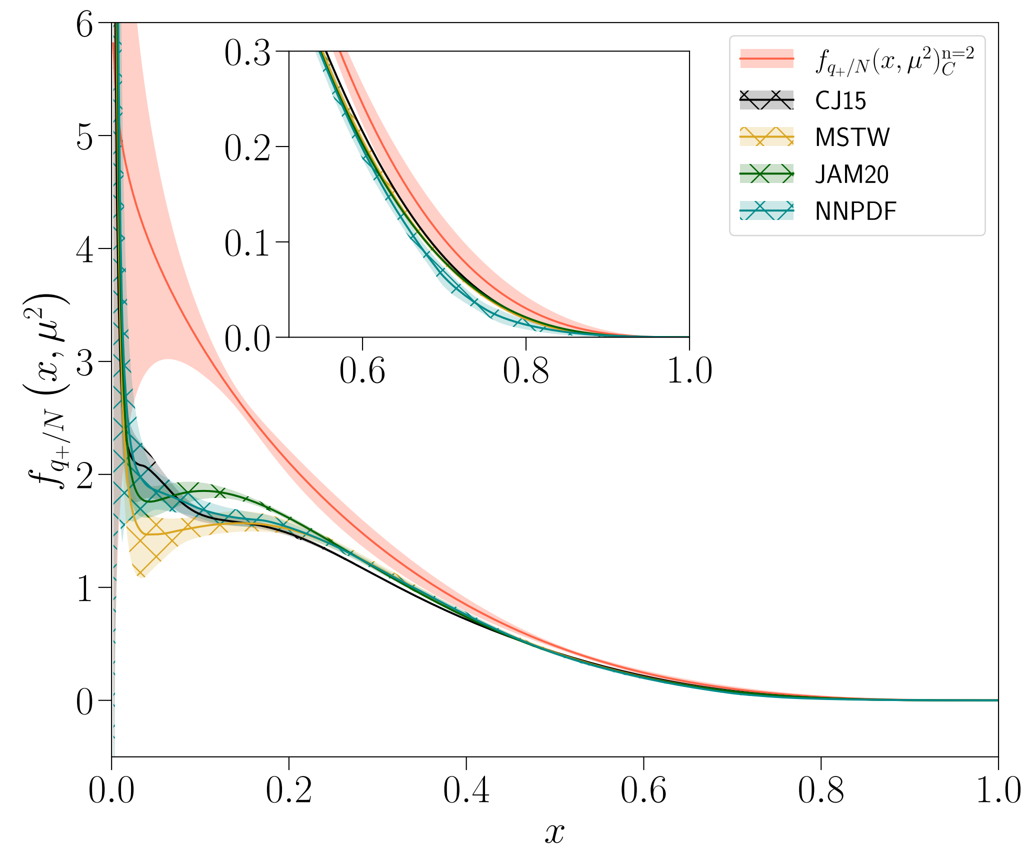
Unpolarized Quark PDF

C. Egerer et al (HadStruc
JHEP 11 (2021) 148



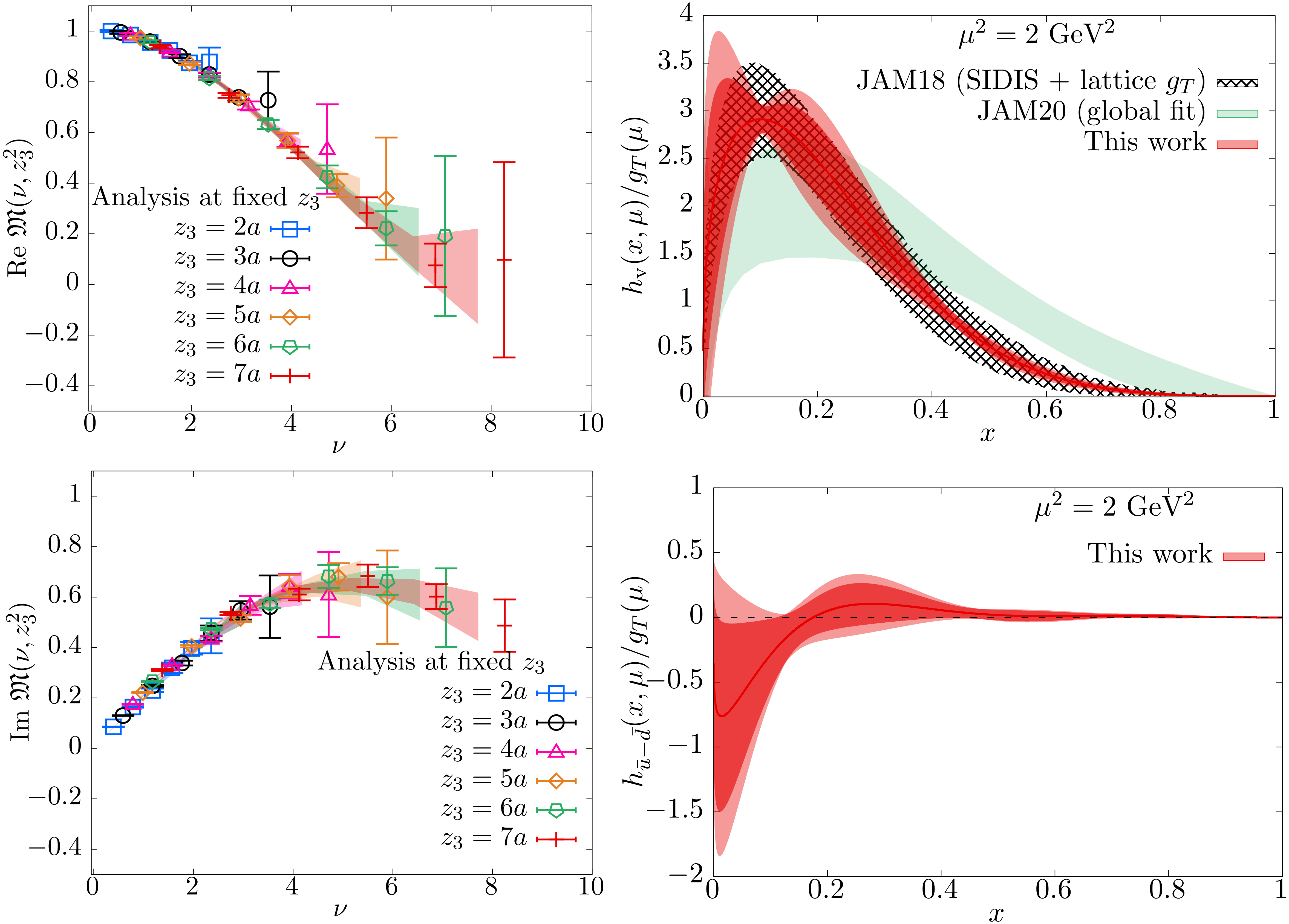






Transversity Quark PDF

C. Egerer et al (HadStruc)
Phys Rev D 105 (2022) 3, 034507



Gluon Matrix Elements

- **General Matrix Element**

$$M^{\mu\alpha;\nu\beta}(z,p,s) = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)F^{\nu\beta}(0)\Big]\,|\,p,s\rangle$$

  - Assume z is along cardinal direction (eventually lattice axis)

- **Renormalization**

  Z-Y. Li, Y-Q. Ma, J-W. Qiu.
  Phys. Rev. Lett. 122 (2019) 6, 062002

  - Multiplicatively renormalizable
  - Depends on how many of μ, ν, ρ, σ are in z direction.

- Matrix element has complicated Lorentz decomposition in terms of p^μ, z^μ, s^μ
  - Need to isolate amplitudes with leading twist contributions

Spin Averaged matrix element

- Spin averaged combination $\mathscr{M}(\nu, z^3) = \frac{1}{2p_0^2}\left[M_{ti;it} + M_{ij;ij}\right]$
  - Gives **one** amplitude with leading twist contribution

$i, j = x, y$

I. Balitsky, W. Morris, A. Radyushkin Phys Rev D 105 (2022) 1, 014008
T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

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- Use **ratio** with finite continuum limit

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)\ \mathcal{M}(0,0)\,|_{p=0,z=0}}{\mathcal{M}(\nu,0)\,|_{z=0}\,\mathcal{M}(0,z^2)\,|_{p=0}}$$

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T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

- Use **ratio** with finite continuum limit

$$\mathfrak{M}(\nu, z^2) = \frac{\mathscr{M}(\nu, z^2)\ \mathscr{M}(0,0)\,|_{p=0,z=0}}{\mathscr{M}(\nu,0)\,|_{z=0}\,\mathscr{M}(0,z^2)\,|_{p=0}}$$

- Relation to **gluon** and **quark singlet** ITD

$$\langle x\rangle_g \mathfrak{M}(\nu, z^2) = \int_0^1 C^{gg}(u, \mu^2 z^2) I_g(u\nu, \mu^2) + C^{qg}(u, \mu^2 z^2) I_s(u\nu, \mu^2)$$

Spin Averaged matrix element

- Spin averaged combination $\mathcal{M}(\nu, z^3) = \dfrac{1}{2p_0^2}\left[M_{ti;it} + M_{ij;ij}\right]$
  - Gives **one** amplitude with leading twist contribution

$i, j = x, y$

I. Balitsky, W. Morris, A. Radyushkin Phys Rev D 105 (2022) 1, 014008
T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

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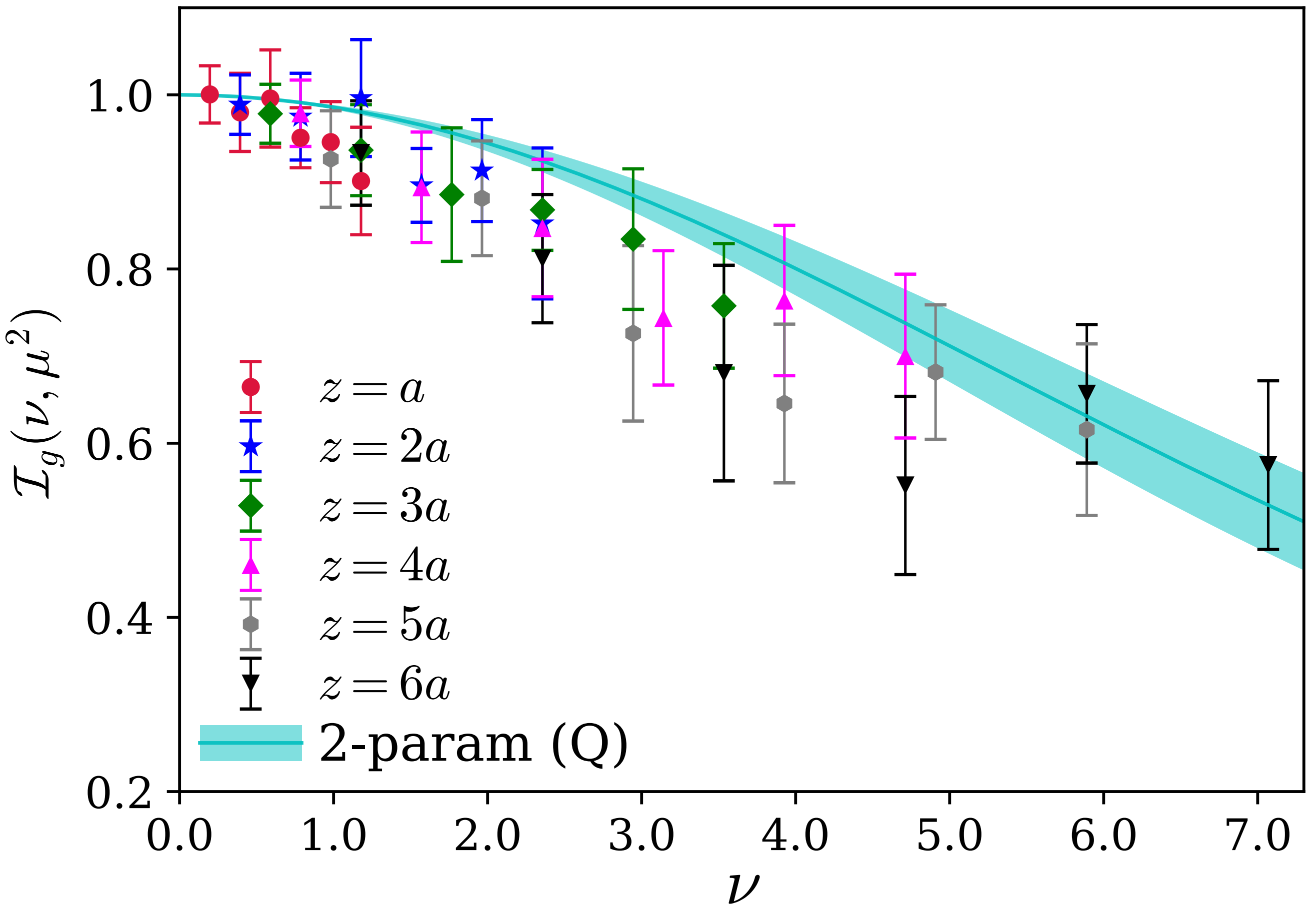
- Relation to **gluon** and **quark singlet** ITD

Neglected for now

$$\langle x\rangle_g \mathfrak{M}(\nu, z^2) = \int_0^1 C^{gg}(u, \mu^2 z^2) I_g(u\nu, \mu^2) + \cancel{C^{qg}(u, \mu^2 z^2) I_s(u\nu, \mu^2)}$$

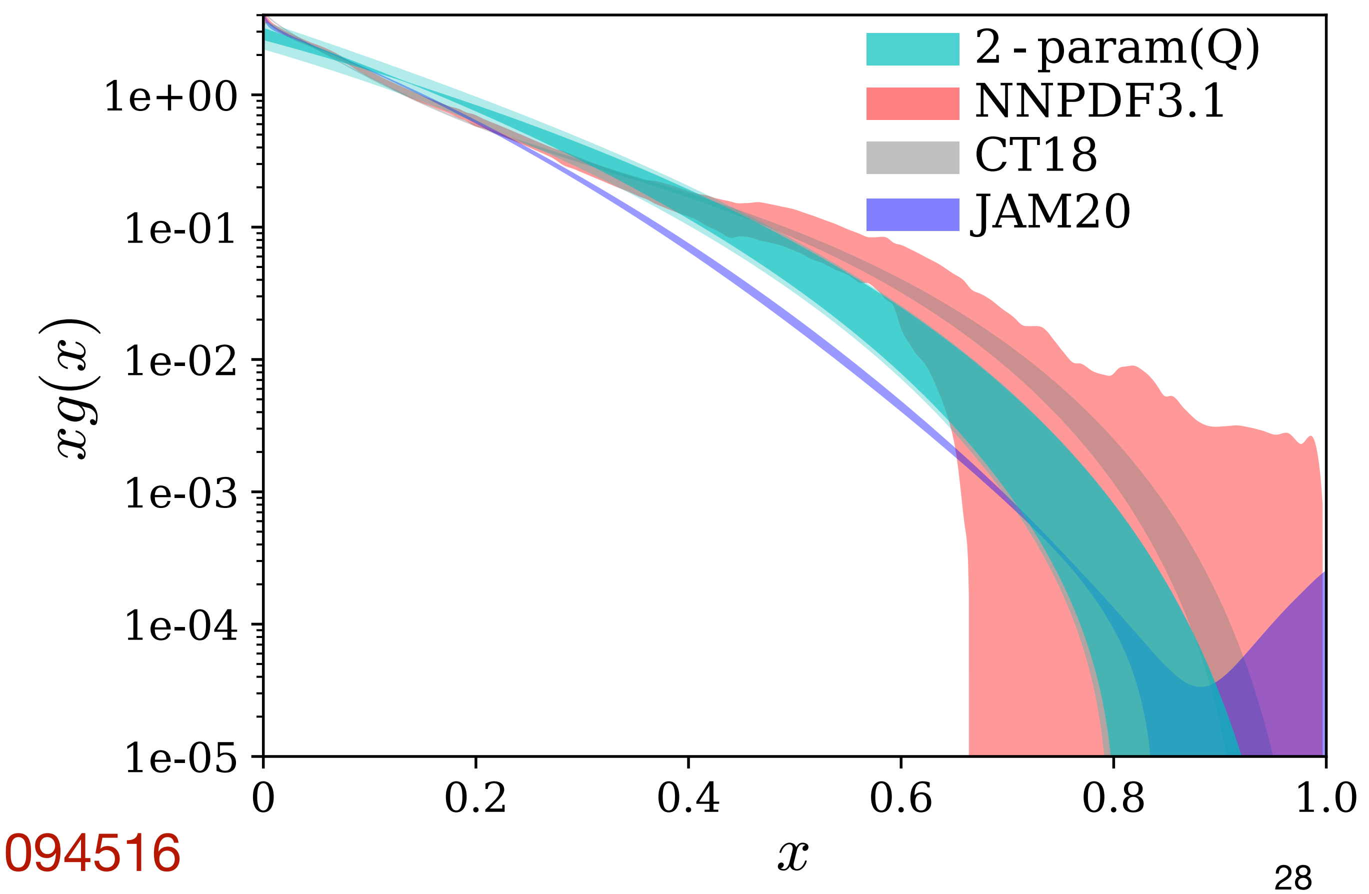
Unpolarized Gluon PDF



- Extrapolated to $\tau \to 0$ flow time
- Modified by NLO formula

$a = 0.094$ fm
$m_\pi = 358$ MeV

- ITD fit to cosine transform of
  $xg(x) = x^a(1-x)^b/B(a+1, b+1)$
- Qualitative agreement with global analysis



T. Khan et al (HadStruc) Phys. Rev. D 104 (2021) 9, 094516

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s \,|\, \mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\widetilde{F}^{\nu\beta}(0)\right] |p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
    - Gives **two** amplitudes, one has no leading twist contribution

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I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
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- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\,\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

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I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
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- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

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I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
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- Useful Combination $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
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$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathcal{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathcal{M}(0,z^2)/m^2}$$

- Relation to gluon and quark singlet ITD

Neglected for now

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2 z^2)\widetilde{I}_g(u\nu,\mu^2) + \cancel{\widetilde{C}^{qg}(u,\mu^2 z^2)\widetilde{I}_s(u\nu,\mu^2)}$$

Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{zp}$$
$$+(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}$$

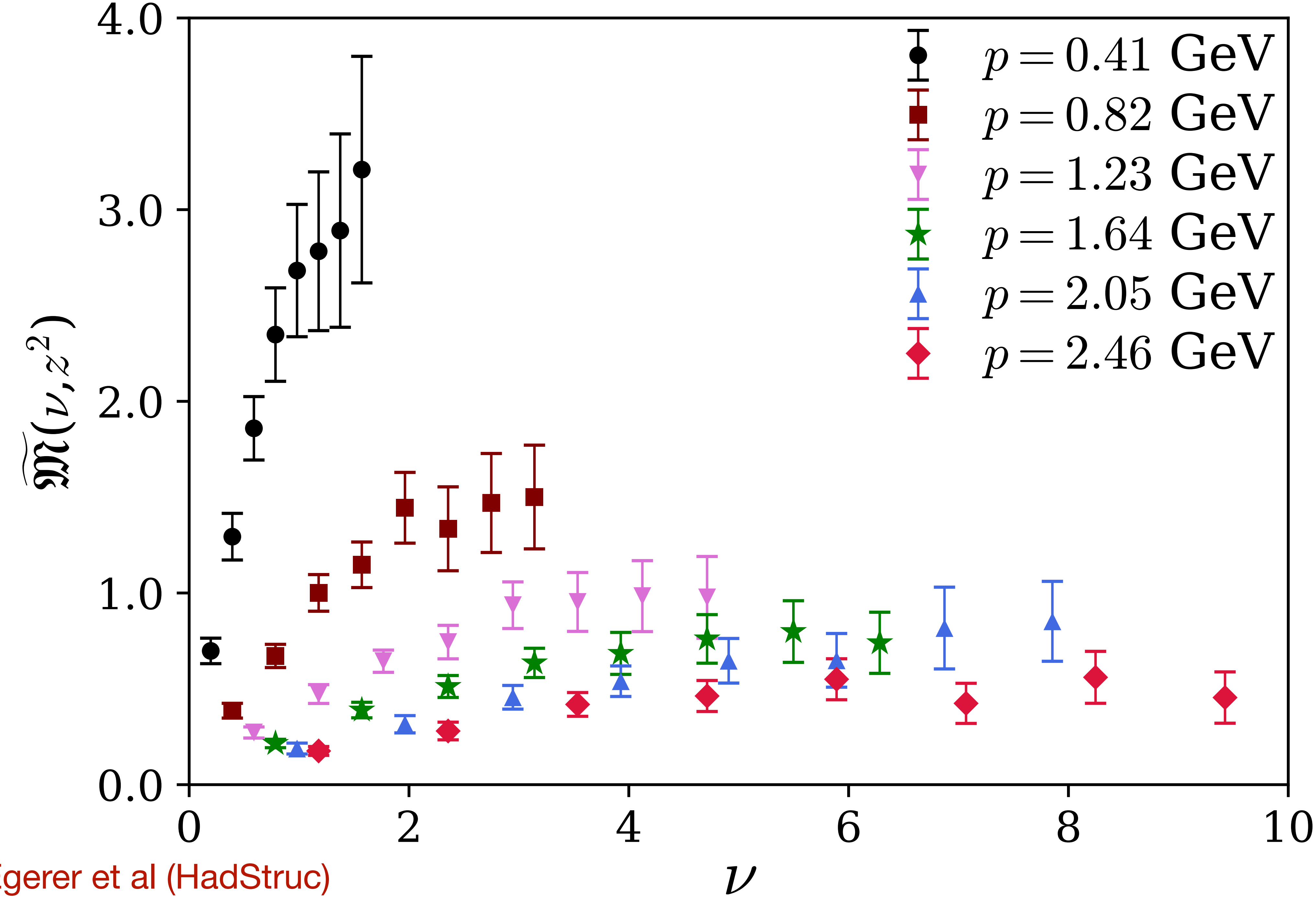
$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp}$$
$$+\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps}$$
$$+\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz}$$
$$+\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

Want: $M_{\Delta g}(\nu, z^2) = \big[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\widetilde{\mathcal{M}}_{pp}\big]$

Can get: $$\widetilde{\mathcal{M}}(z,p) = \big[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\big]$$
$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$
$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon Matrix Element

- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$ will need to be removed



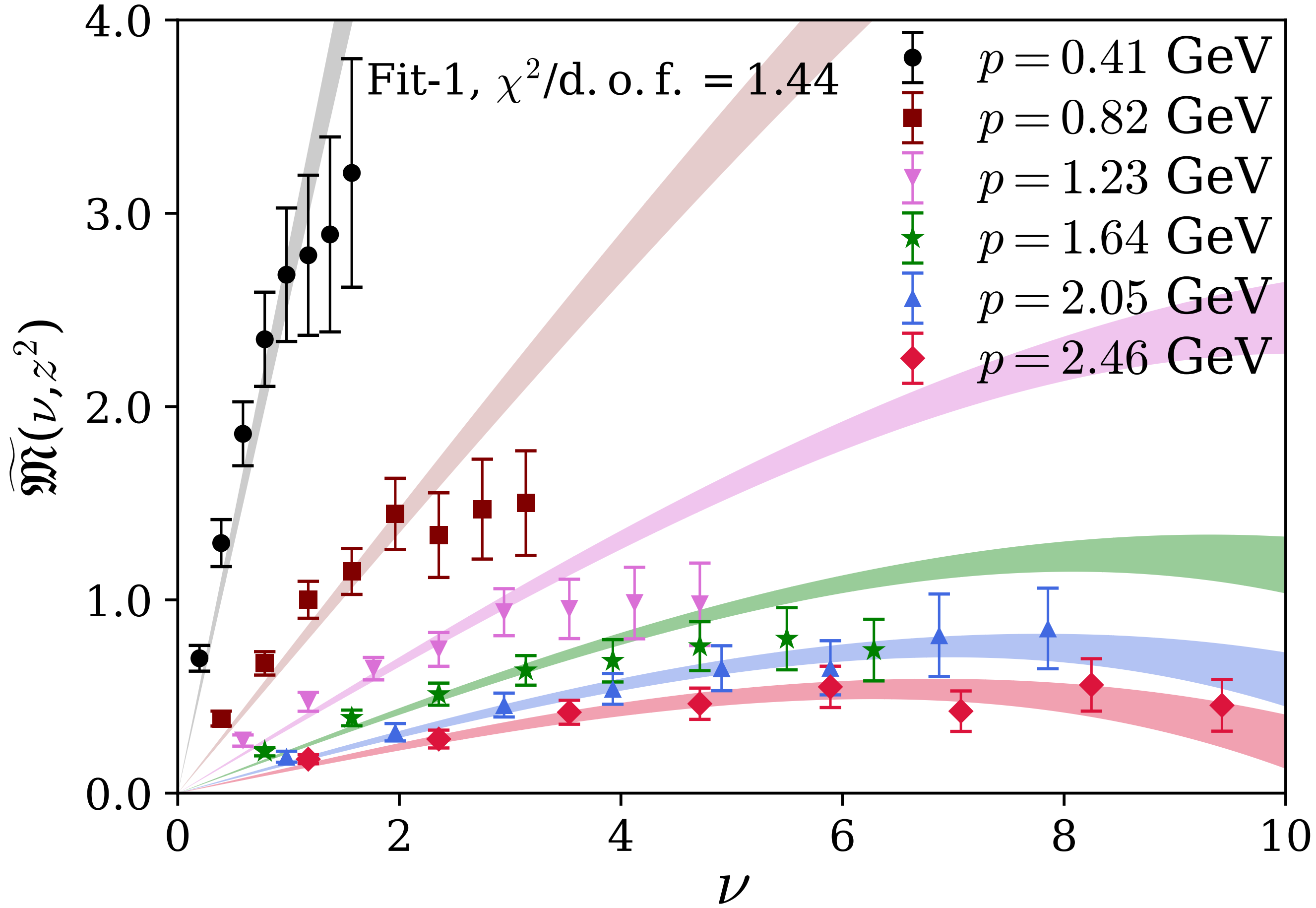
$a = 0.094$ fm

$m_\pi = 358$ MeV

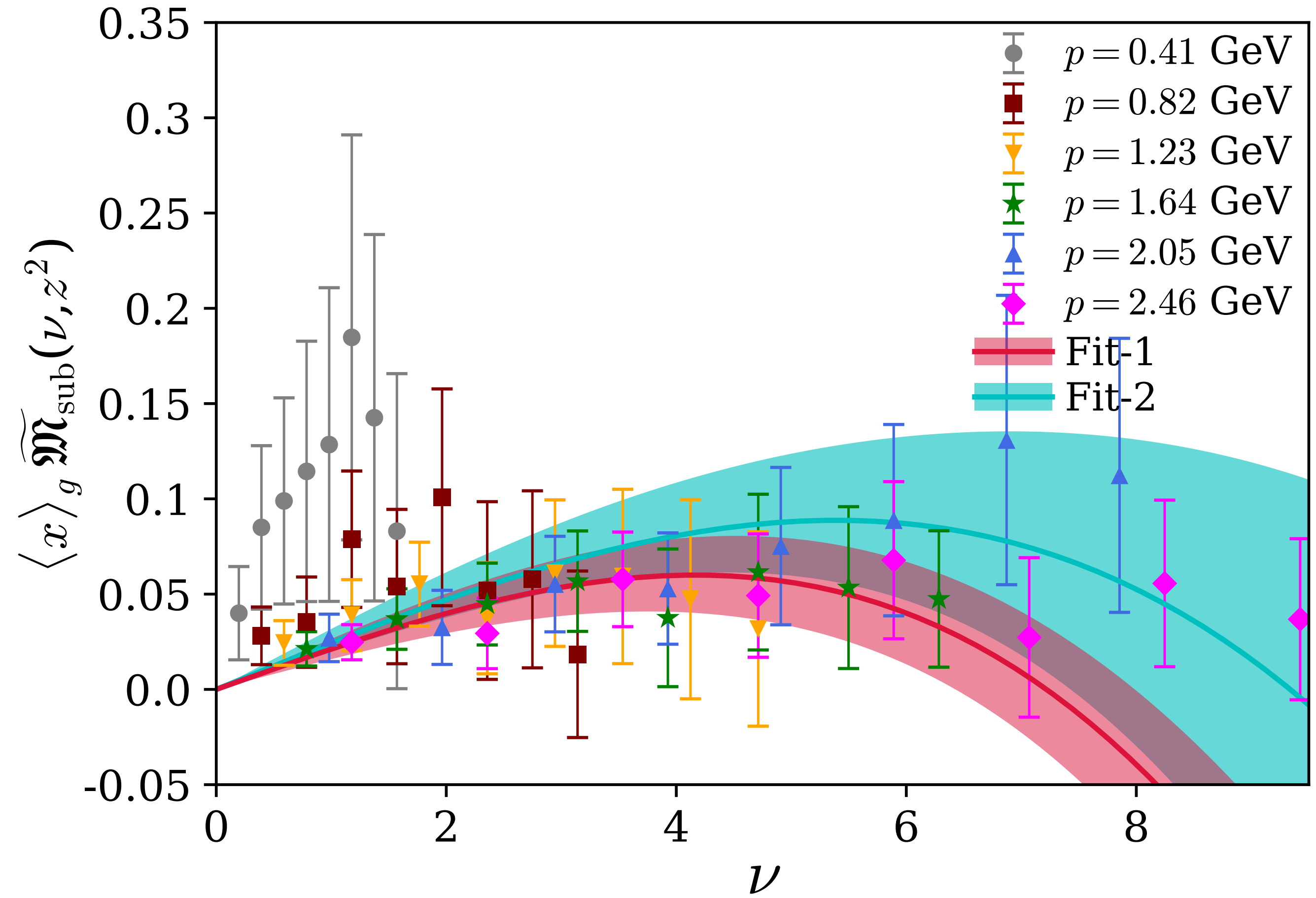
C. Egerer et al (HadStruc)
arXiv: 2207.08733

Helicity Gluon PDF

- Model both terms



- Subtract rest frame



$a = 0.094$ fm

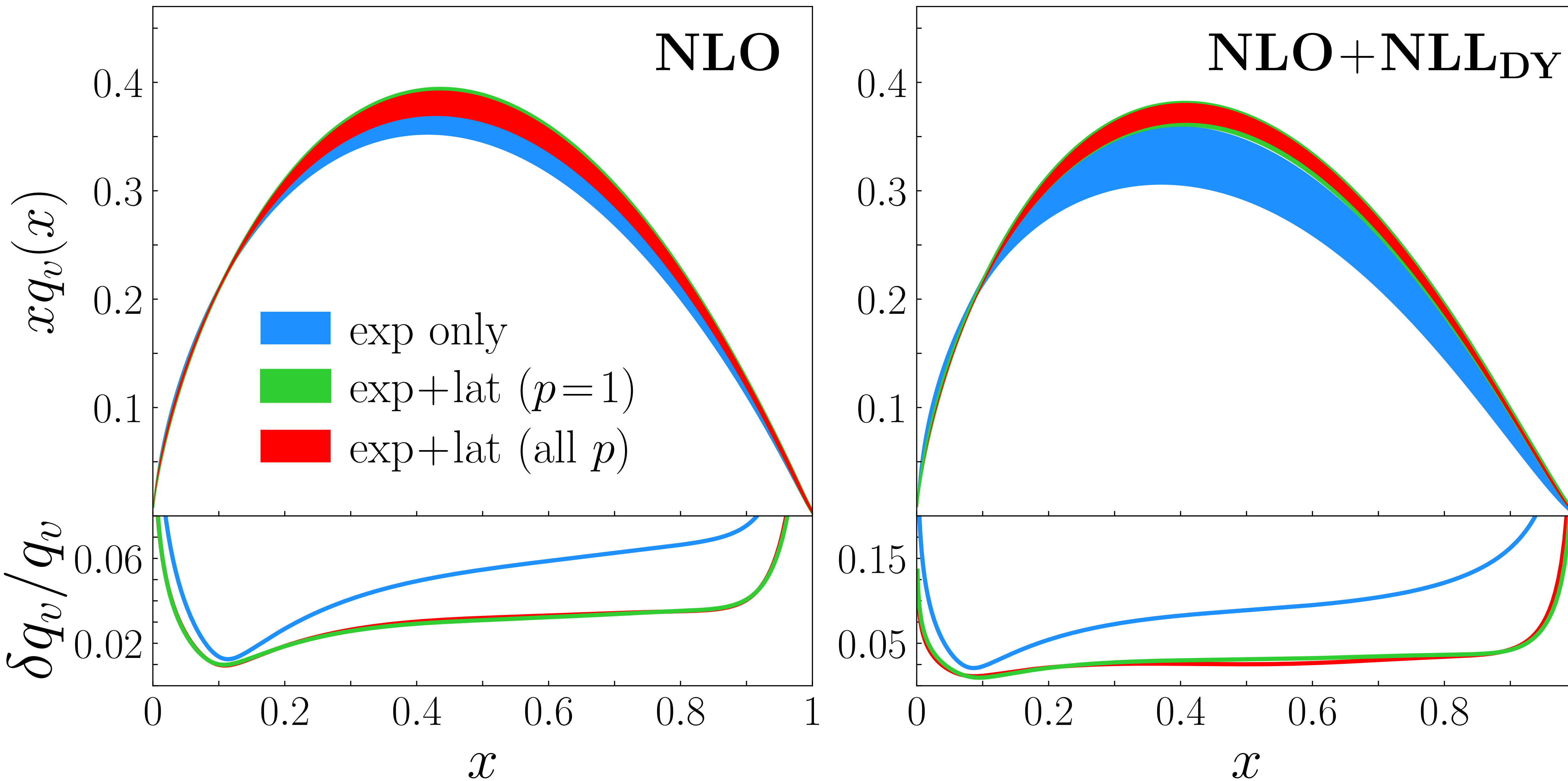
$m_\pi = 358$ MeV

C. Egerer et al (HadStruc) arXiv: 2207.08733

Lattice data aren’t that different from experimental data

Combining Lattice and Experiment

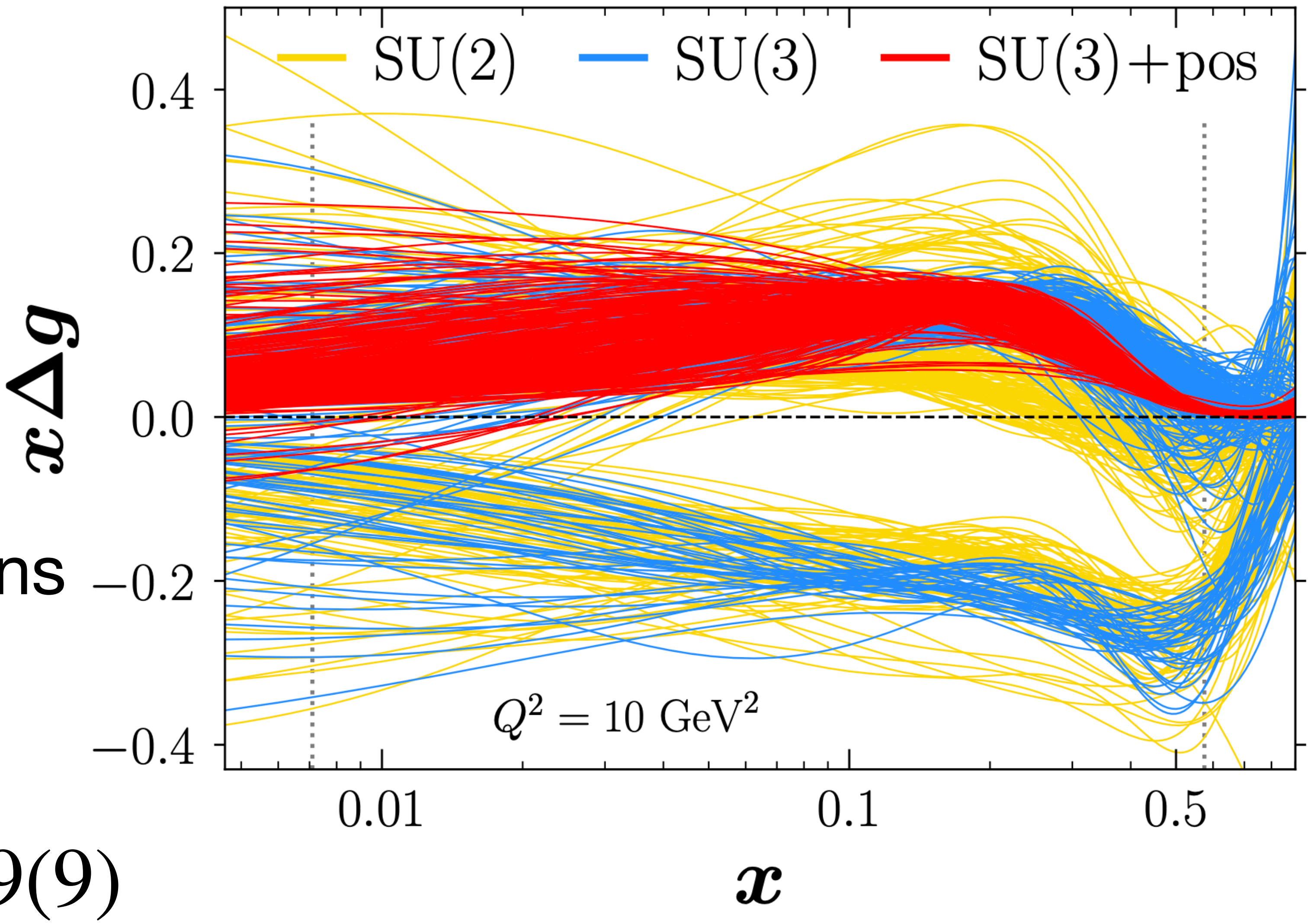
- Simultaneously fit Lattice and Experimental pion PDF data
- Each gives unique information complementing each other



P. Barry et al (HadStruc and JAM), *Phys. Rev. D* 105 (2022) 11, 114051

Spinning gluons (revisited)

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g(x)$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$
- Without constraint: $\Delta G = 0.3(5)$
- Lattice: $\Delta G = 0.251(47)(16)$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

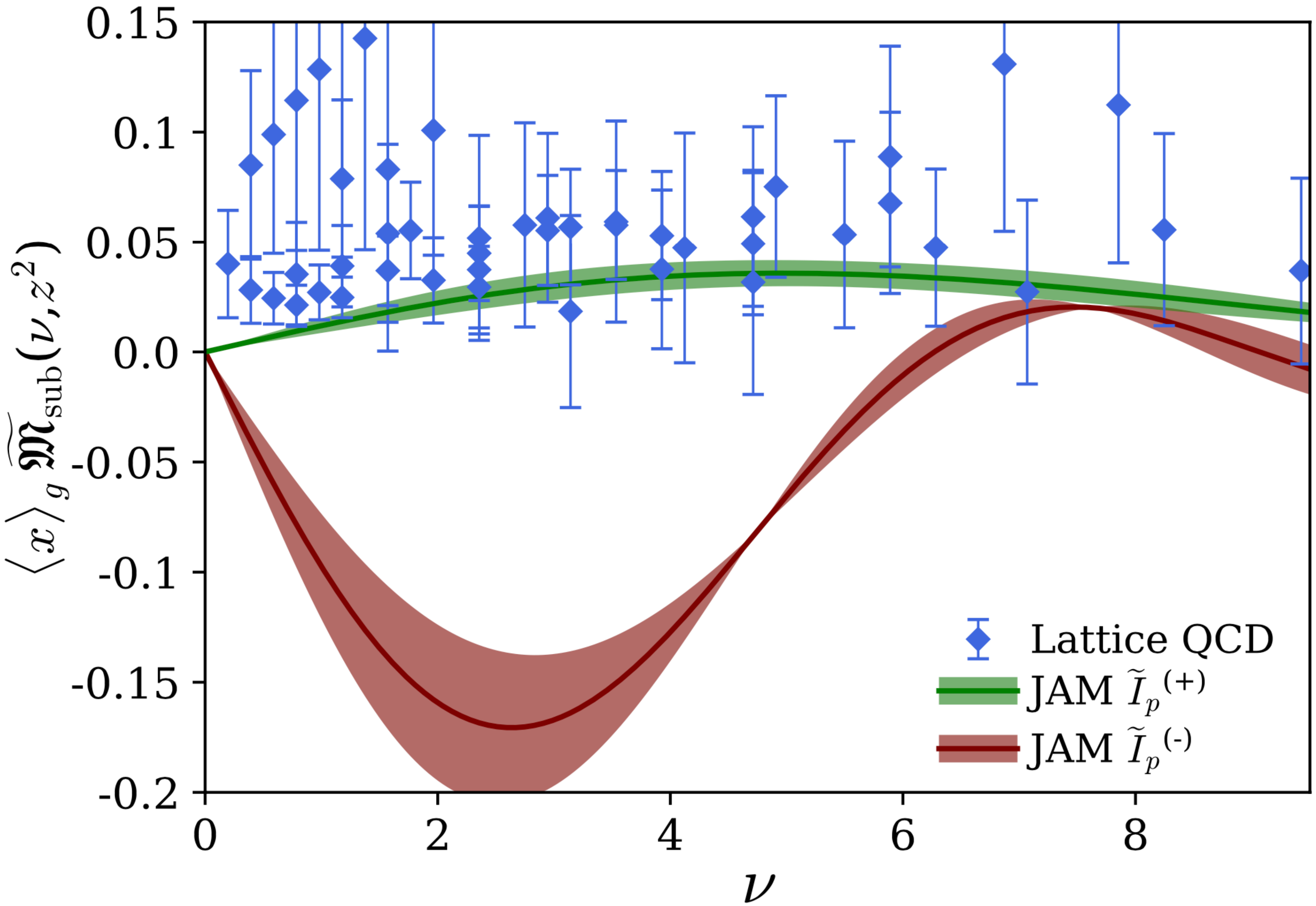
R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

Spinning gluons (revisited)

Can lattice data affect phenomenological polarized gluon analysis?



$$\Delta G = \int d\nu \, I_g(\nu)$$

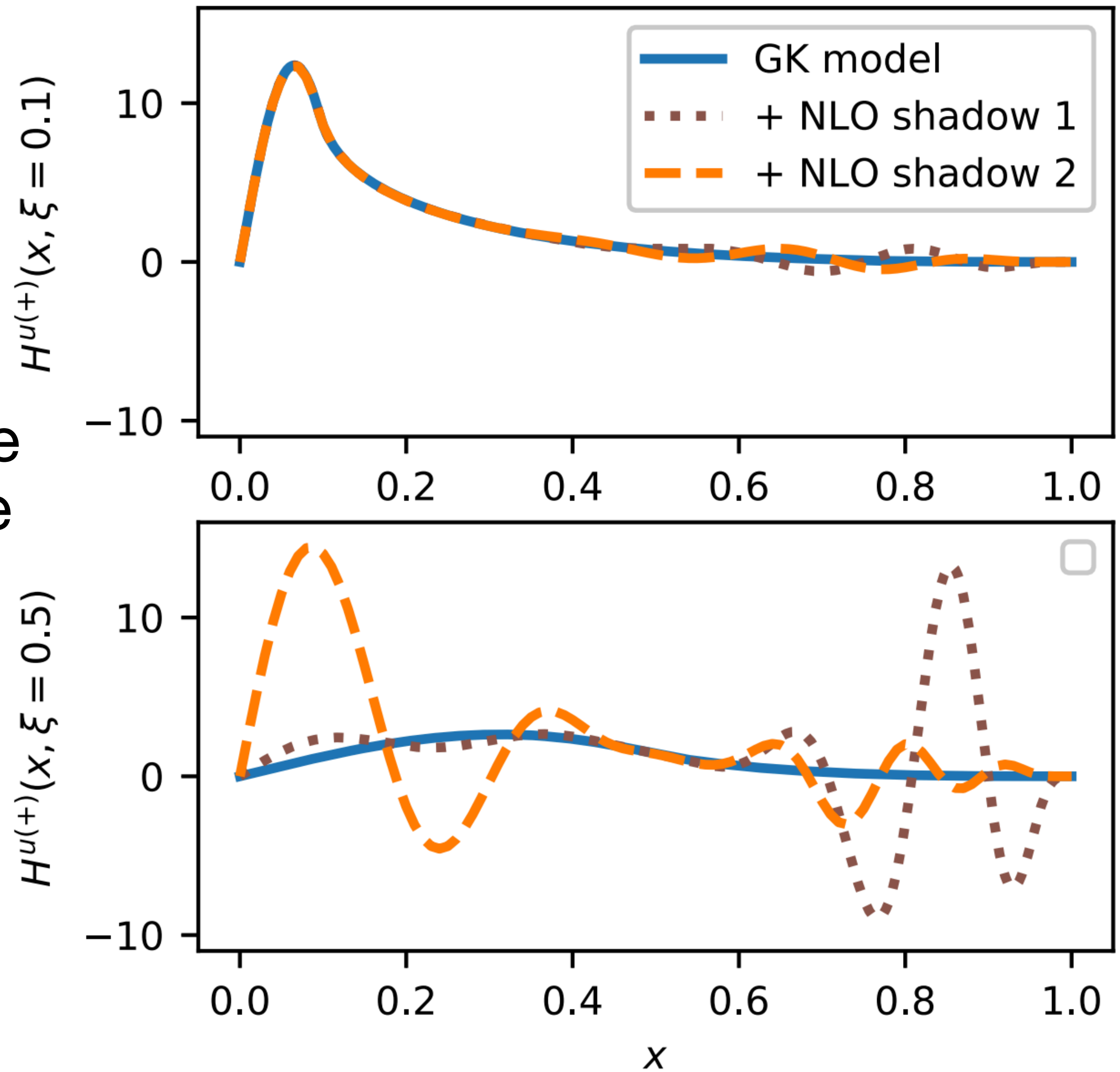
- The positive and negative solutions without positivity constraints plotted in ν space

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733

- Only positive band consistent with lattice data

GPDs and their shadows

- First lattice calculations of GPDs have begun
- Model dependence in GDPs from DVCS alone
- “Shadow GPDs” added to the “true” GPD would not change DVCS cross sections
- Lattice will potentially lack GPDs
    - Improved control of kinematics



Shadow GPDs: Phys Rev D 103 (2021) 11, 114019

Conclusions

- Gluon x or ν dependent structure requires state-of-the-art calculations
  - Use of **distillation** with large number of configurations
  - **Summed GeVP** to control excited states
  - **Wilson flow** to improve signal
- Future work towards gluon 3D distributions, higher twist PDFs,…. is possible given enough resources to find signal
- Possibly impact phenomenological PDF analyses
  - **Today:** with polarized gluon PDF
  - **Future:** JLab 12GeV and EIC data on PDF and on new distributions

Thank you and the organizers!