

# **N<sup>3</sup>LO extraction of the Sivers functions from SIDIS, DY and W<sup>±</sup>/Z data**

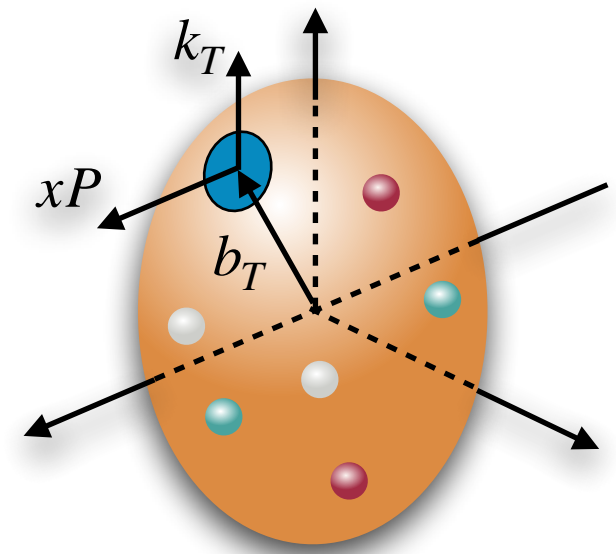
**Alexei Prokudin**



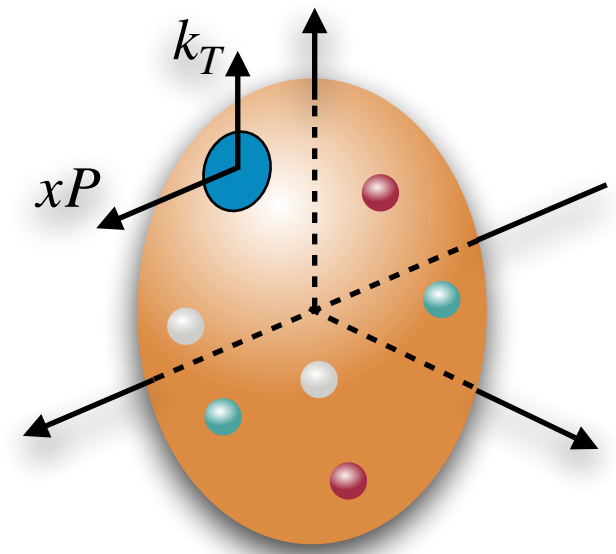
M. Bury, AP, A. Vladimirov, PRL 126, 112002 (2021)

M. Bury, AP, A. Vladimirov, JHEP 05 (2021) 151

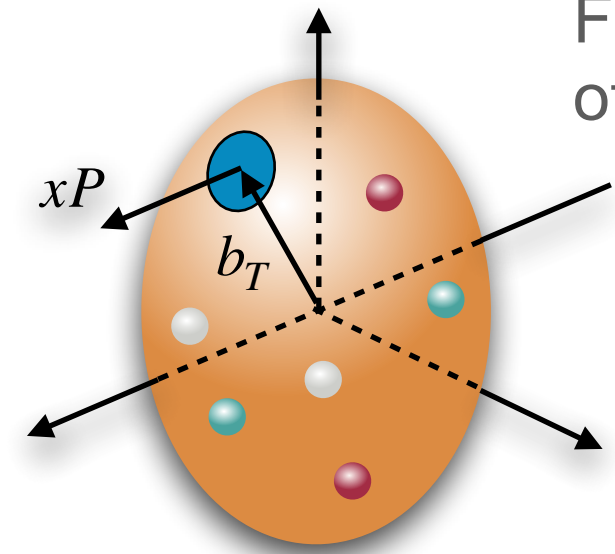
Wigner distributions  
 (Fourier transform of  
 GTMDs = Generalized  
 Transverse Momentum  
 Distributions)



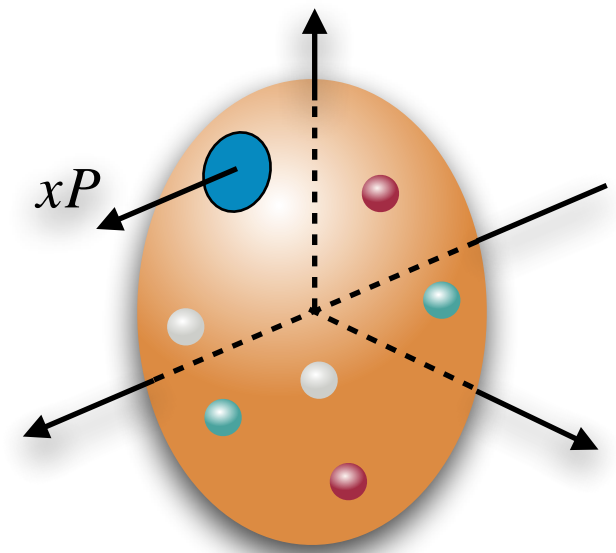
TMDs



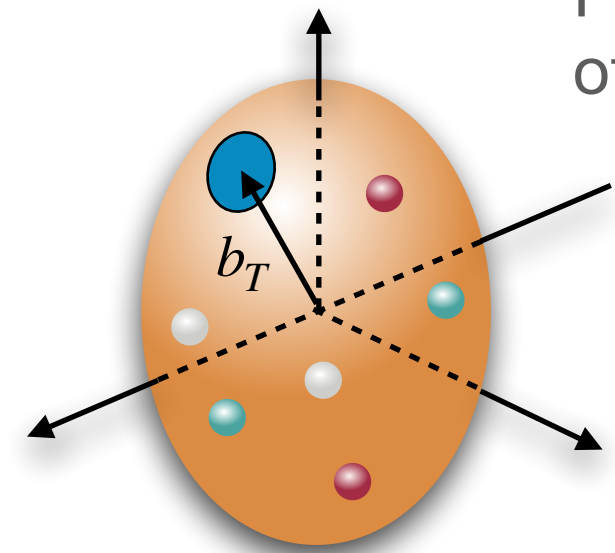
Fourier transform  
 of GPDs



PDFs



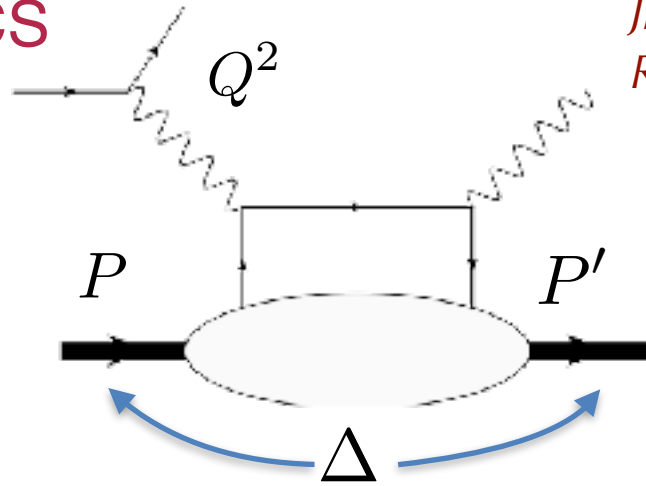
Fourier transform  
 of Form Factors





# GPD

## DVCS



Ji (1997)  
Radyushkin (1997)

$Q^2$  ensures hard scale, pointlike interaction

$\Delta = P' - P$  momentum transfer can be varied independently

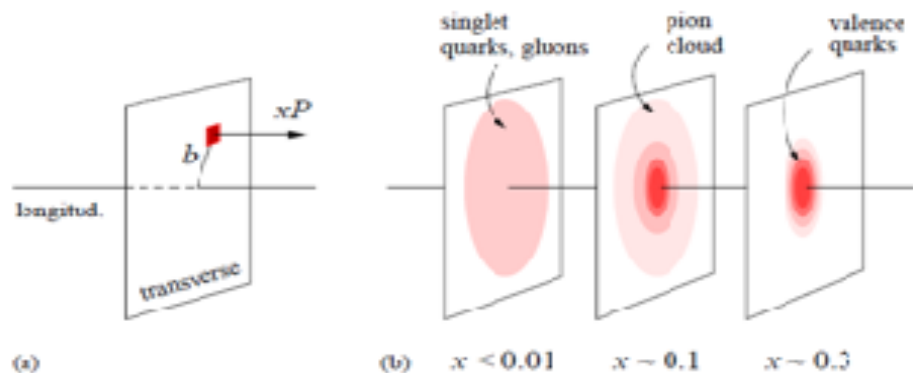
## Connection to 3D structure

Burkardt (2000)  
Burkardt (2003)

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

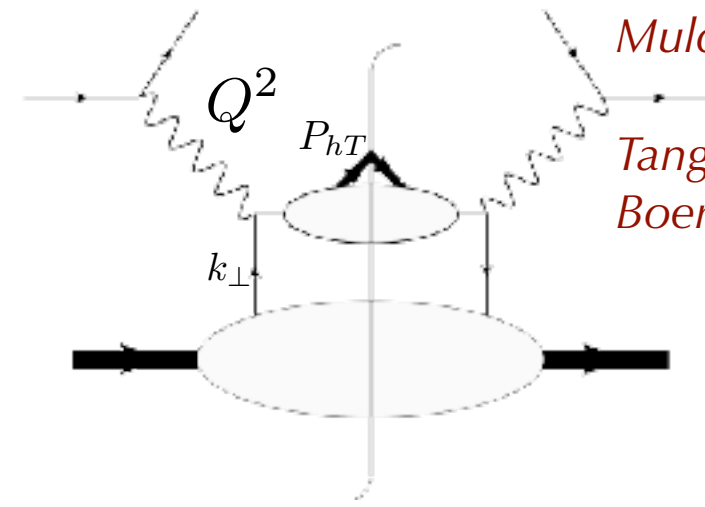
Drell-Yan frame  $\Delta^+ = 0$   $\xi = -\frac{\Delta^+}{2p^+}$

Weiss (2009)



# TMD

## SIDIS



Kotzinian (1995),  
Mulders,

Tangerman (1995),  
Boer, Mulders (1998)

$Q^2$  ensures hard scale, pointlike interaction

$P_{hT}$  final hadron transverse momentum can be varied independently

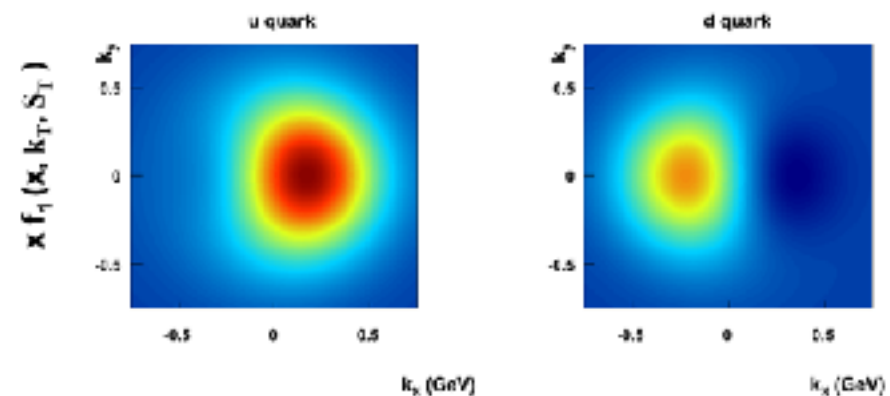
## Connection to 3D structure

Ji, Ma, Yuan (2004)  
Collins (2011)

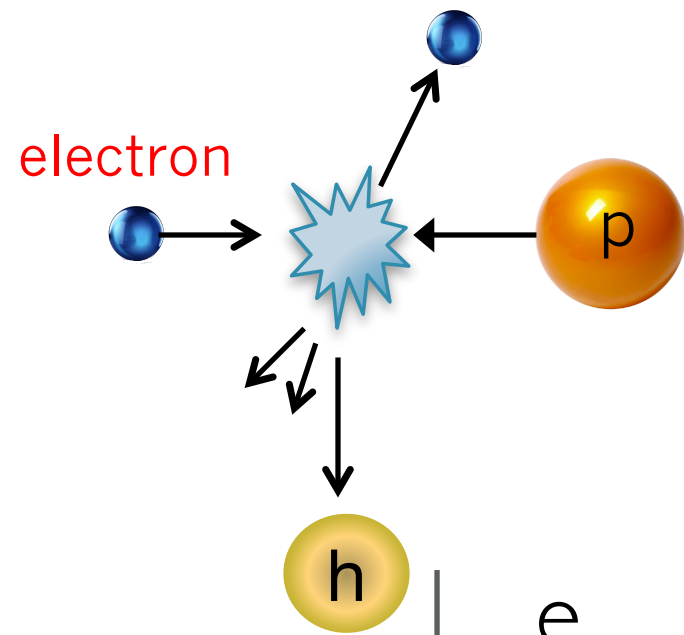
$$\tilde{f}(x, \vec{b}_T) = \int d^2 \vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{b}_T} f(x, \vec{k}_\perp)$$

$\vec{b}$  is the transverse separation of parton fields in configuration space

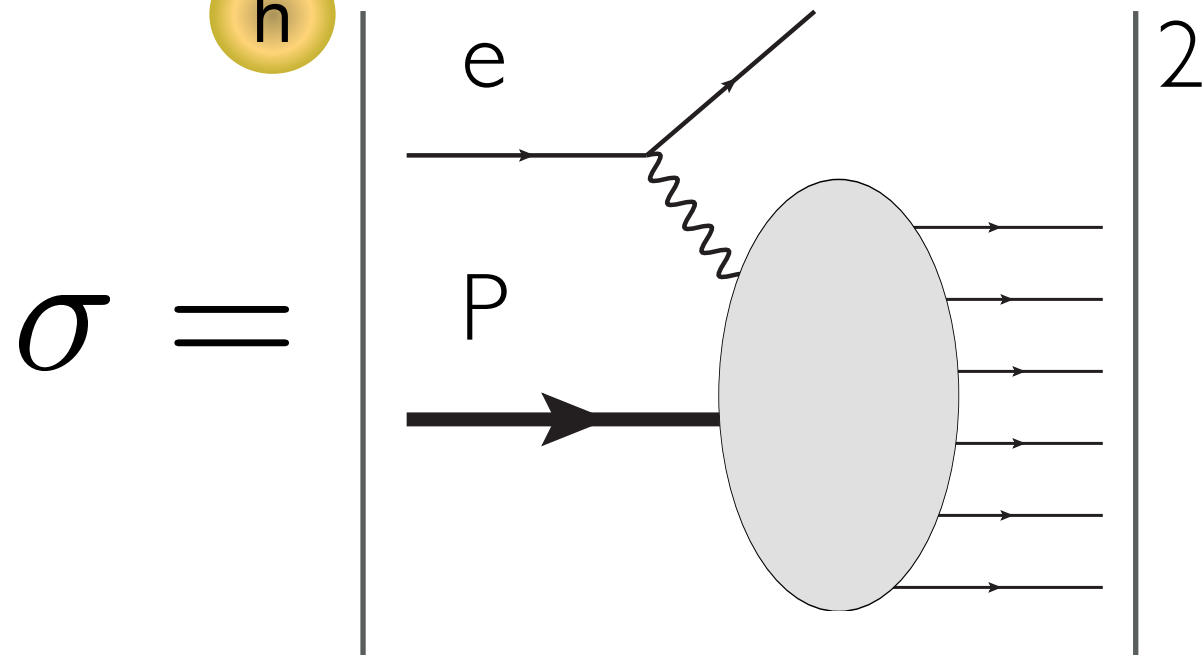
AP (2012)



# QCD FACTORIZATION IS THE KEY!



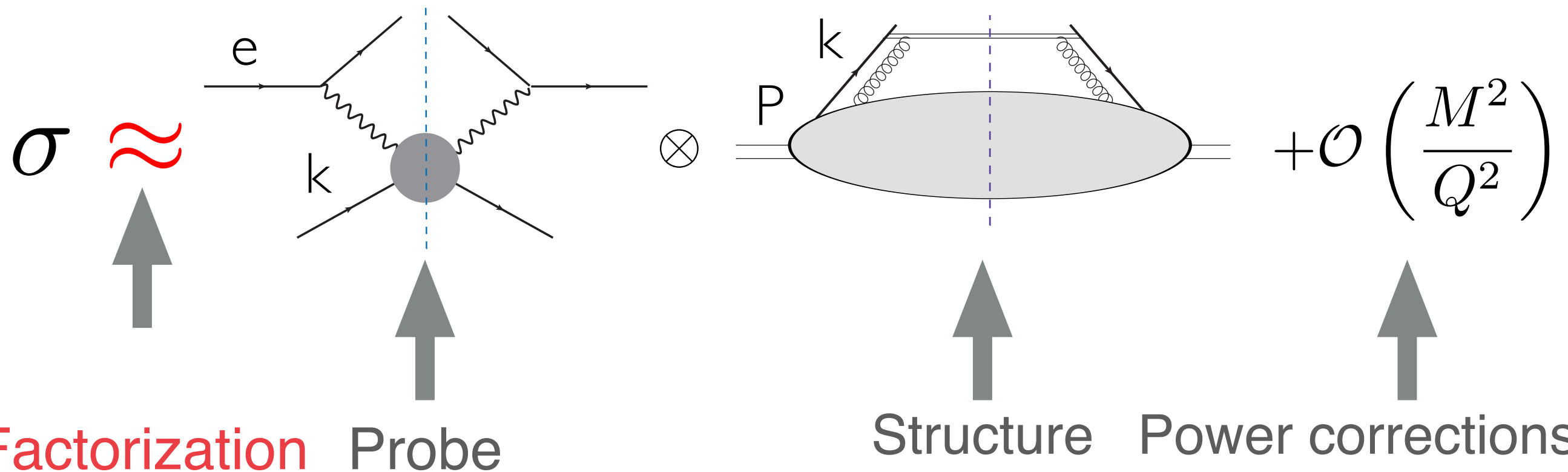
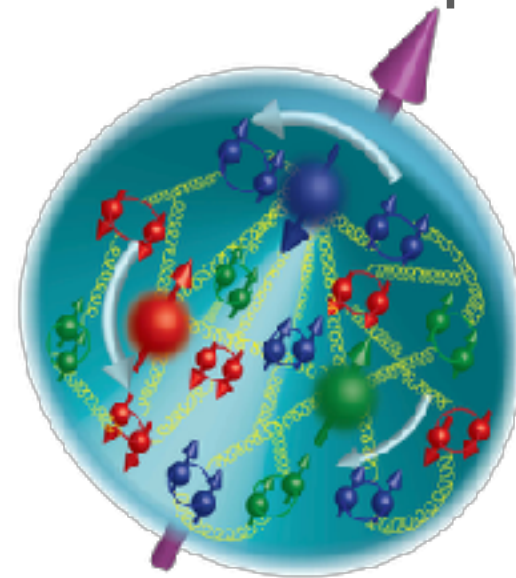
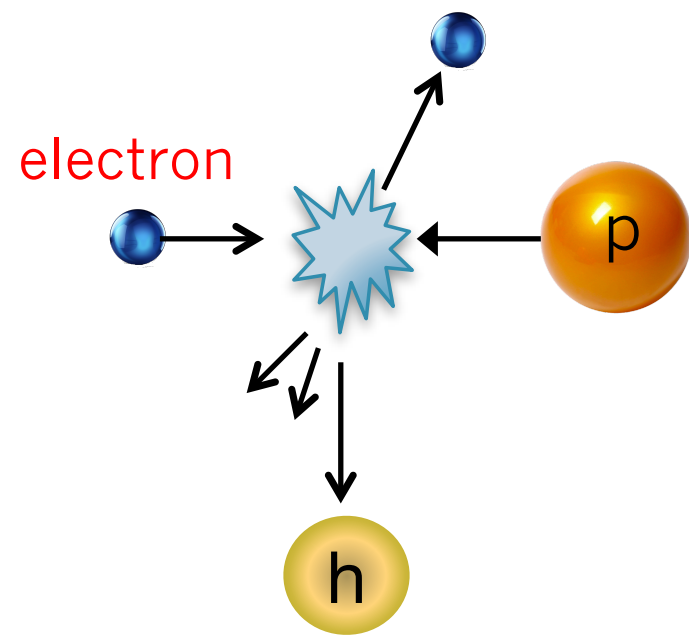
We need a probe to “see” quarks and gluons





# QCD FACTORIZATION IS THE KEY!

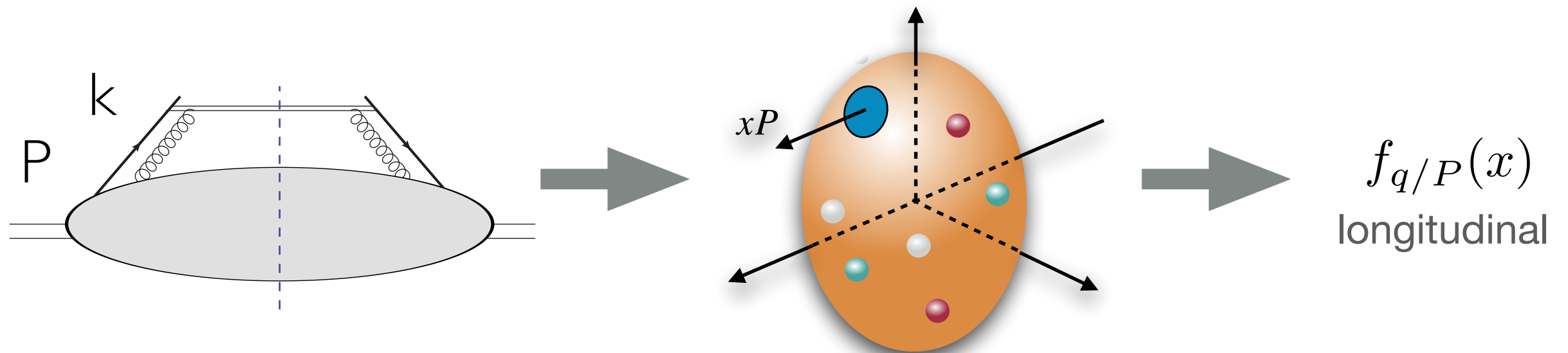
We need a probe to “see” quarks and gluons



# HADRON'S PARTONIC STRUCTURE

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## Collinear Parton Distribution Functions



Probability density to find a quark with a momentum fraction  $x$

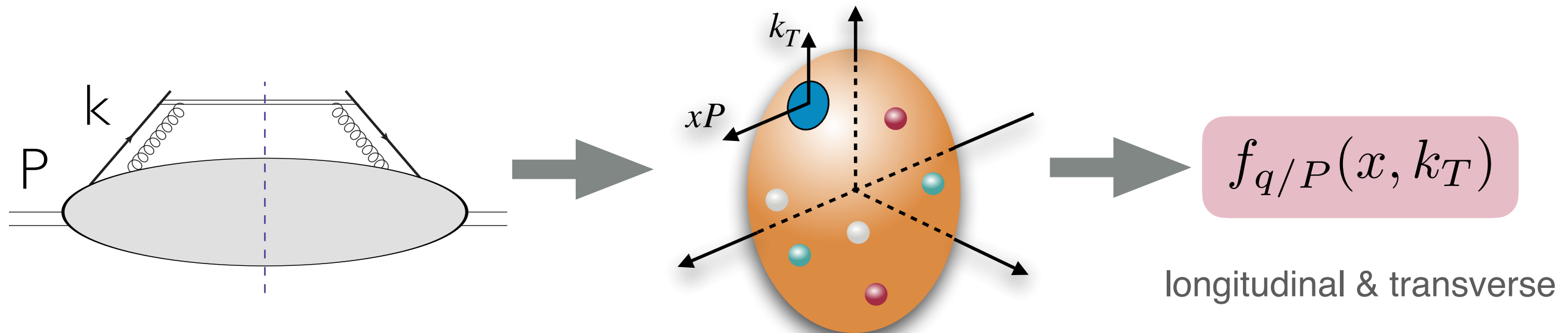
Hard probe resolves the particle nature of partons, but is not sensitive to hadron's structure at  $\sim$ fm distances.



# HADRON'S PARTONIC STRUCTURE

To study the physics of *confined motion of quarks and gluons* inside of the proton one needs a new type “hard probe” with two scales.

Transverse Momentum Dependent functions



One large scale ( $Q$ ) sensitive to particle nature of quark and gluons

One small scale ( $k_T$ ) *sensitive to how QCD bounds partons* and to the detailed structure at  $\sim$ fm distances.

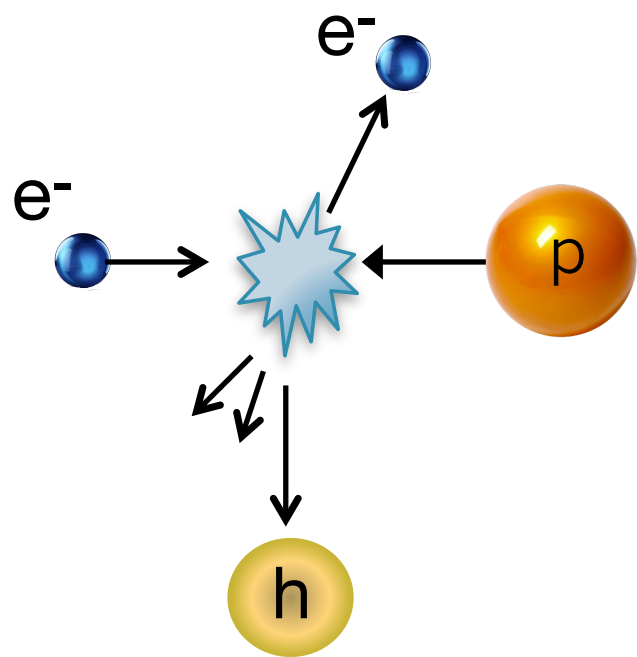
# TRANSVERSE MOMENTUM DEPENDENT FACTORIZATION

Small scale  $\longrightarrow q_T \ll Q \longleftarrow$  Large scale

The confined motion ( $k_T$  dependence) is encoded in TMDs

## Semi-Inclusive DIS

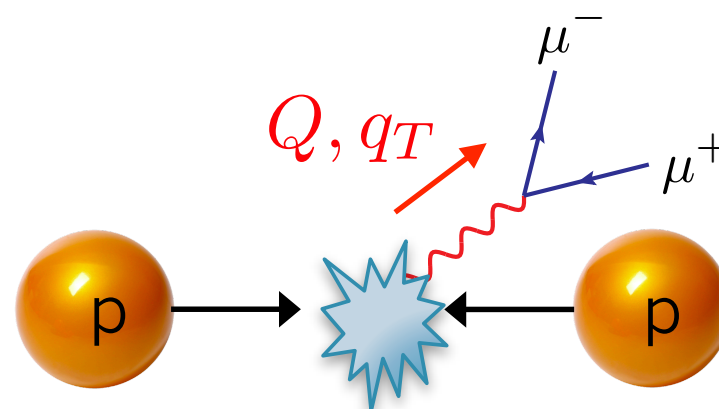
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$



Meng, Olness, Soper (1992)  
 Ji, Ma, Yuan (2005)  
 Idilbi, Ji, Ma, Yuan (2004)  
 Collins (2011)

## Drell-Yan

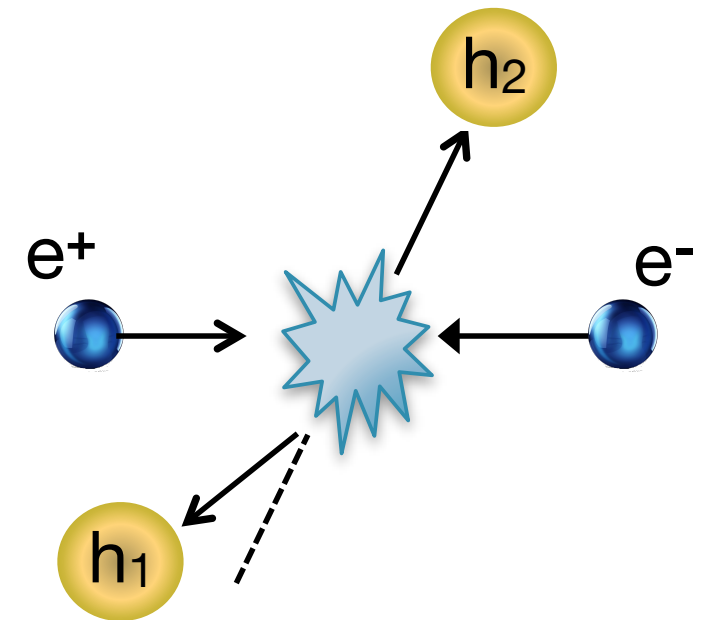
$$\sigma \sim f_{q/P}(x_1, k_T) f_{\bar{q}/P}(x_2, k_T)$$



Collins, Soper, Serman (1985)  
 Ji, Ma, Yuan (2004)  
 Collins (2011)

## Dihadron in $e^+e^-$

$$\sigma \sim D_{h_1/q}(z_1, k_T) D_{h_2/\bar{q}}(z_2, k_T)$$

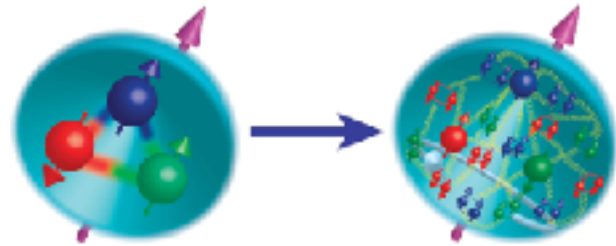


Collins, Soper (1983)  
 Collins (2011)

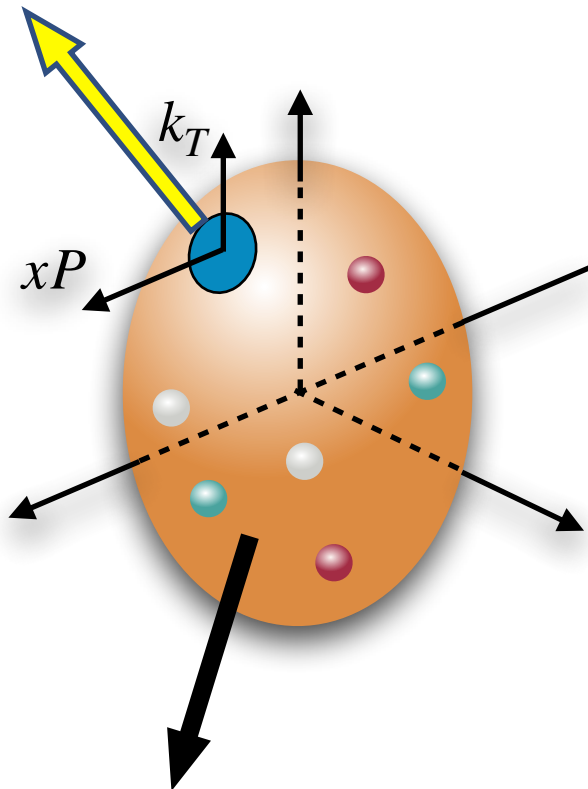


# Our understanding of hadron evolves: TMDs with Polarization

## Quark Polarization



Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons



## Nucleon Polarization

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x, k_T^2)$ <i>Unpolarized</i>		$h_1^\perp(x, k_T^2)$ <i>Boer-Mulders</i>
	L		$g_1(x, k_T^2)$ <i>Helicity</i>	$h_{1L}^\perp(x, k_T^2)$ <i>Kozinian-Mulders, "worm" gear</i>
	T	$f_{1T}^\perp(x, k_T^2)$ <i>Sivers</i>	$g_{1T}(x, k_T^2)$ <i>Kozinian-Mulders, "worm" gear</i>	$h_1(x, k_T^2)$ <i>Transversity</i>  $h_{1T}^\perp(x, k_T^2)$ <i>Pretzelosity</i>

Analogous tables for:

Gluons  $f_1 \rightarrow f_1^g$  etc

Fragmentation functions

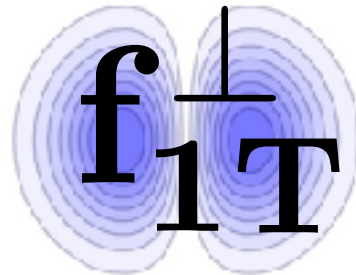
Nuclear targets  $S \neq \frac{1}{2}$

# THE SIVERS FUNCTION



# THE SIVERS FUNCTION

## Sivers function



Sivers 1989

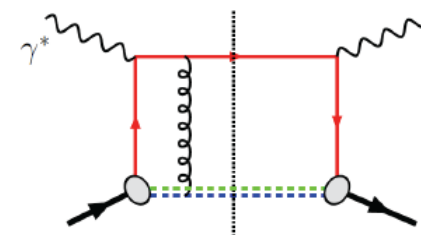
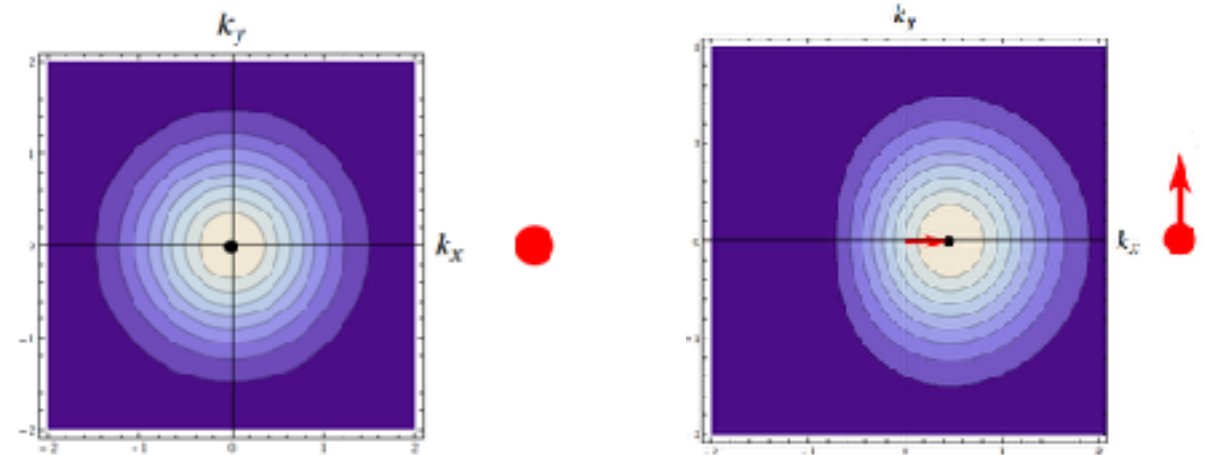
- Describes unpolarized quarks inside of transversely polarized nucleon

- Generates asymmetries in SIDIS and DY

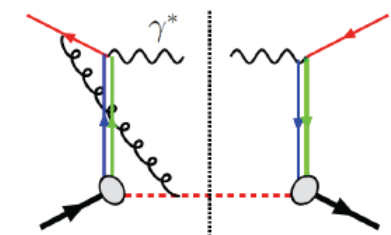
Kotzinian (1995)  
 Mulders, Tangerman (1995)  
 Boer, Mulders (1998)

- Changes sign in DY w.r.t. SIDIS

Brodsky, Hwang, Schmidt (2002)  
 Collins (2002)



$r$  wavy line  $(gb)$   
 attractive



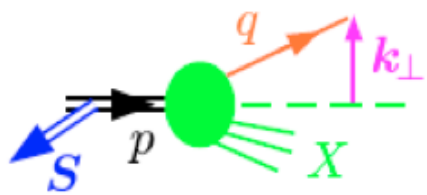
$r$  wavy line  $r$   
 repulsive

$$f_{1T}^\perp \text{SIDIS} = -f_{1T}^\perp \text{DY}$$

# THE SIVERS FUNCTION

The Sivers function: unpolarized quark distribution inside a transversely polarized nucleon

Sivers 1989



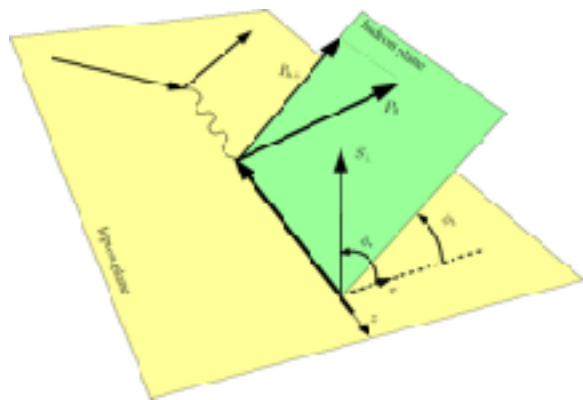
Spin independent

Spin dependent

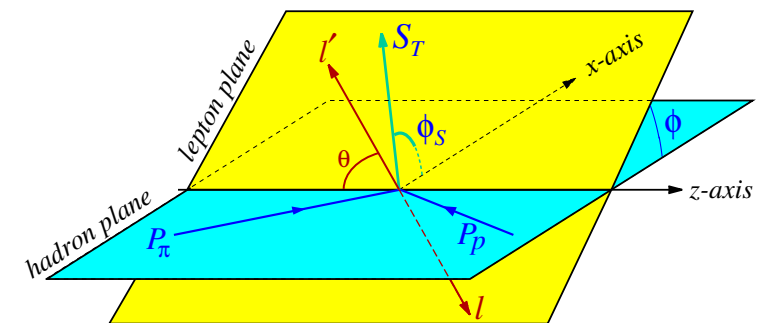
$$f_{q/h^\uparrow}(x, \vec{k}_\perp, \vec{S}) = f_{q/h}(x, k_\perp^2) - \frac{1}{M} f_{1T}^\perp(x, k_\perp^2) \vec{S} \cdot (\hat{P} \times \vec{k}_\perp)$$

$$\ell P \rightarrow \ell' \pi X$$

$$\pi^-(P_\pi) + N(P_p, S) \rightarrow \ell^+ \ell^- + X$$



Kotzinian (1995),  
Mulders,  
Tangerman (1995),  
Boer, Mulders (1998)



Collins-Soper frame

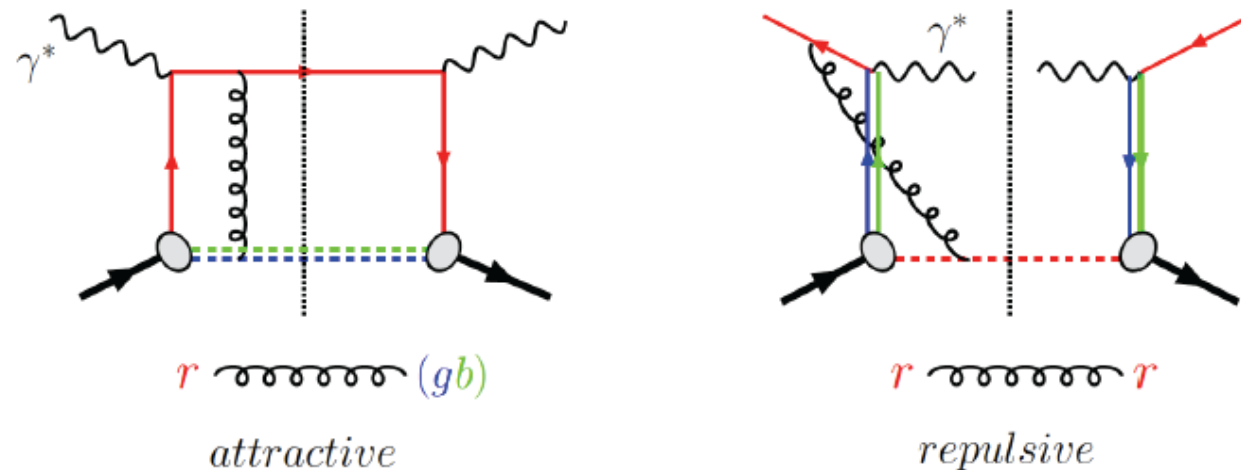
$$\sigma(S_T) \sim \sin(\phi_h - \phi_S) f_{1T}^\perp \otimes D_1$$

$$\sigma(S_T) \sim \sin(\phi_S) f_{1T}^\perp \otimes f_1$$

# SIGN CHANGE OF THE SIVERS FUNCTION

.....  
 Colored objects are surrounded by gluons, profound consequence of gauge invariance:

The Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky, Hwang, Schmidt `02  
 Belitsky, Ji, Yuan `04  
 Collins `02  
 Boer, Mulders, Pijlman `04  
 Kang, Qiu `08  
 Kovchegov, Sievert `18  
 etc

$$f_{1T}^{\perp \text{SIDIS}} = -f_{1T}^{\perp \text{DY}}$$

Crucial test of TMD factorization and collinear twist-3 factorization  
 Several labs worldwide measure Sivers effect in SIDIS and Drell-Yan  
 BNL, CERN, FERMILAB etc

The verification of the sign change is an NSAC (DOE and NSF) milestone

# THE SIVERS FUNCTION

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Large –  $N_c$  result  $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$

Pobylitsa 2003

- Confirmed by phenomenological extractions
- Confirmed by experimental measurements

Relation to GPDs ( $E$ ) and anomalous magnetic moment Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

- Predicted correct sign of Sivers asymmetry in SIDIS
- Shown to be model-dependent
- Used in phenomenological extractions

Meissner, Metz, Goeke 2007

Bacchetta, Radici 2011

# THE SIVERS FUNCTION

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Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

$$f_{1T}^{\perp(1)q}(x) = \int d^2 k_{\perp} \frac{k_{\perp}^2}{2M^2} f_{1T}^{\perp q}(x, k_{\perp}^2)$$

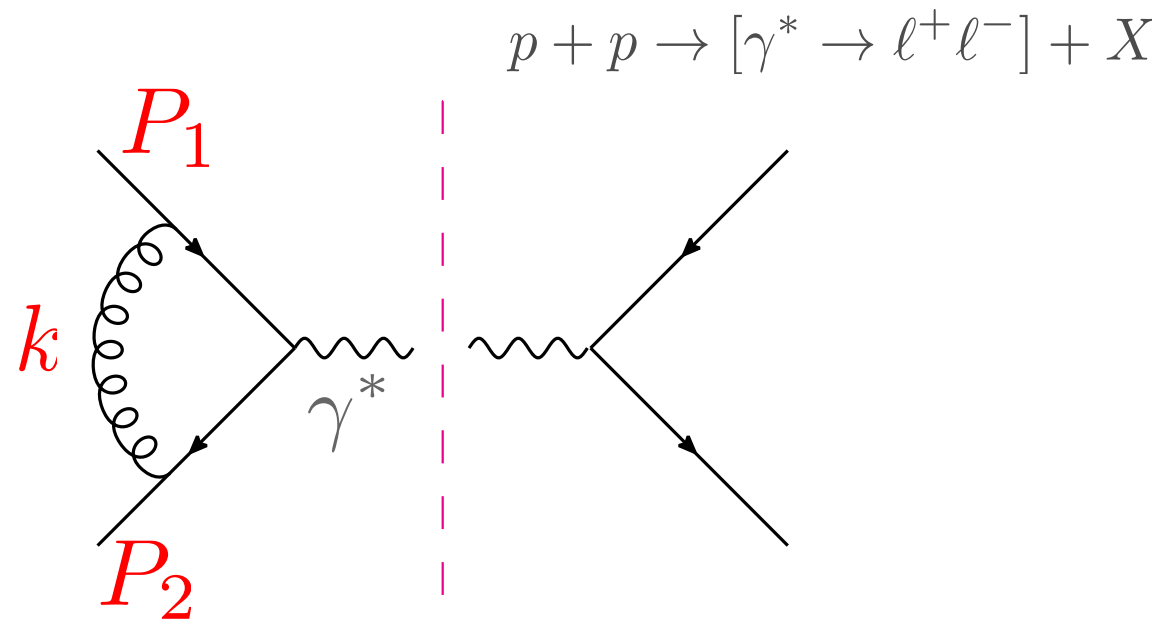
→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \quad \sum_{a=q,g} \int_0^1 dx f_{1T}^{\perp(1)a}(x) = 0$$



# THE SCALE DEPENDENCE

# TMD FACTORIZATION IN A NUT-SHELL



Factorization of regions:

(1)  $k \ll P_1$ , (2)  $k \ll P_2$ , (3)  $k$  soft, (4)  $k$  hard

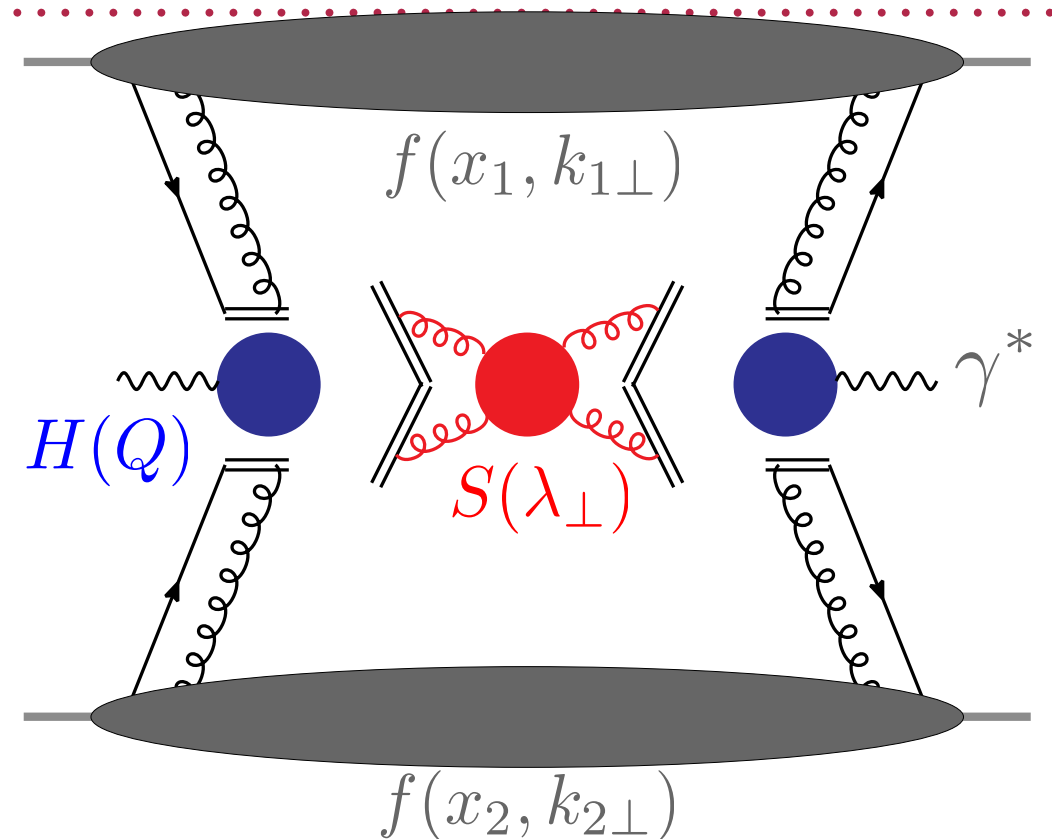
$$\frac{d\sigma}{dQ^2 dy d^2q_\perp} = \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)$$

$$\mu^2 \frac{dF(x, b; \mu, \zeta)}{d\mu^2} = \frac{\gamma_F(\mu, \zeta)}{2} F(x, b; \mu, \zeta),$$

$\mu$  = renormalization scale

$$\zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -\mathcal{D}(b, \mu) F(x, b; \mu, \zeta),$$

$\zeta$  = Collins-Soper parameter



$$F(x, b) = f(x, b) \sqrt{S(b)}$$

**Collins-Soper Equations**

# TMD FACTORIZATION

Collins, Soper, Sterman (85), Collins (11), Rogers, Collins (15)

$$F(x, k_{\perp}; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{i k_{\perp} \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^{\infty} db b J_0(k_{\perp} b) F(x, b; Q)$$

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp \left\{ - \int_{c/b^*}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp \left( -S_{\text{non-pert}}(b, Q) \right)$$

OPE/collinear part

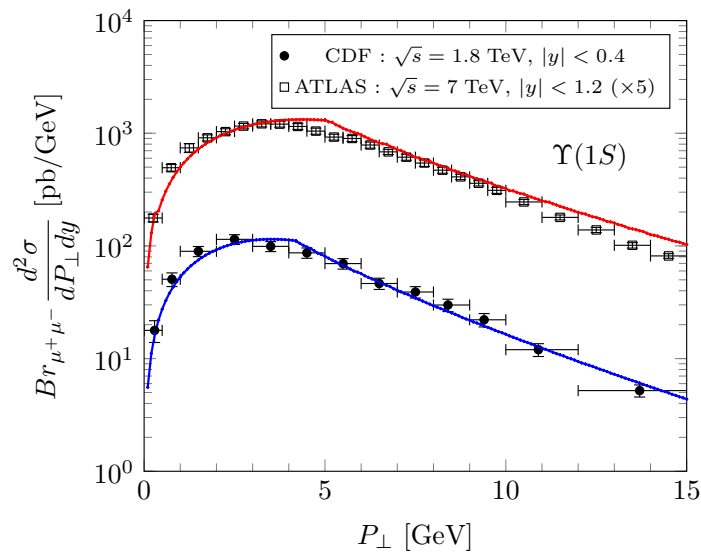
transverse part, Sudakov FF

✓ **Non-perturbative: fitted from data**

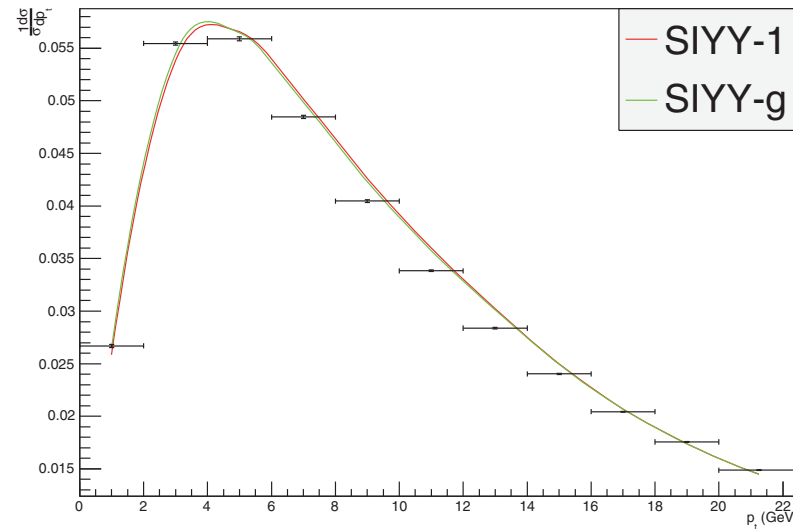
- The evolution is complicated as one evolves in 2 dimensions
- The presence of a non-perturbative evolution kernel makes calculations more involved
- Theoretical constraints exist on both non-perturbative shape of TMD and the non-perturbative kernel of evolution

- ✓ The key ingredient –  $\ln(Q)$  piece is spin-independent
- ✓ Non-perturbative shape of TMDs is to be extracted from data
- ✓ One can use information from models or ab-initio calculations, such as lattice QCD: shape of TMDs, non-perturbative kernel.

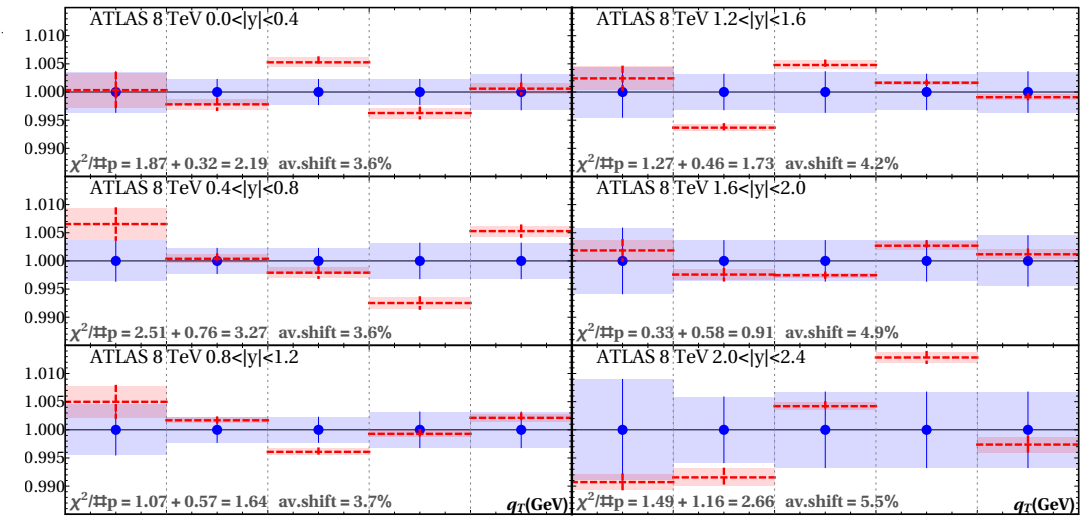
# SUCCESS OF TMD FACTORIZATION PREDICTIVE POWER



Qiu, Watanabe arXiv:1710.06928



Sun, Isaacson, Yuan, Yuan arXiv:1406.3073



Bertone, Scimemi, Vladimirov arXiv:1902.08474

## Upsilon production

## Z boson production at the LHC

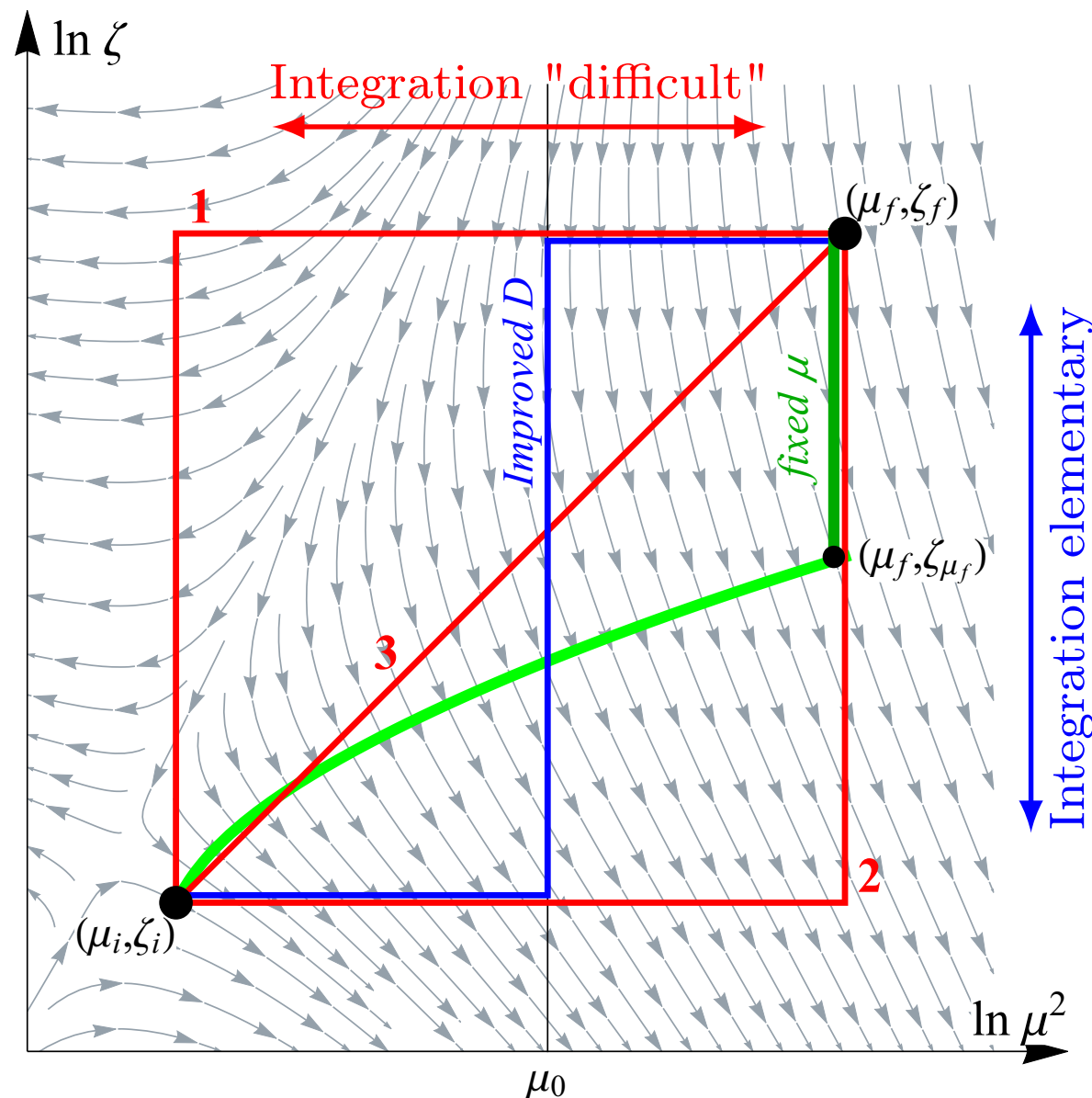
- TMD factorization (with an appropriate matching to collinear results) aims at an accurate description (and prediction) of a differential in  $q_T$  cross section in a wide range of  $q_T$
- LHC results at 7 and 13 TeV are accurately predicted from fits of lower energies

# TMD EVOLUTION CONTAINS NON-PERTURBATIVE COMPONENT

- ▶ TMD evolution is a two scale evolution
- ▶ Remarkably simple in the zeta-prescription

Scimemi, Vladimirov (18), (20)

Vladimirov (20)



$$F(x, b; \mu, \zeta) = \left( \frac{\zeta}{\zeta_\mu(b)} \right)^{-\mathcal{D}(b, \mu)} F(x, b)$$

- $F(x, b)$  is the “optimal” TMD
- $\zeta_\mu(b)$  calculable function
- $\mathcal{D}(b, \mu)$  Collins-Soper kernel or rapidity anomalous dimension. Fundamental universal function related to the properties of QCD vacuum

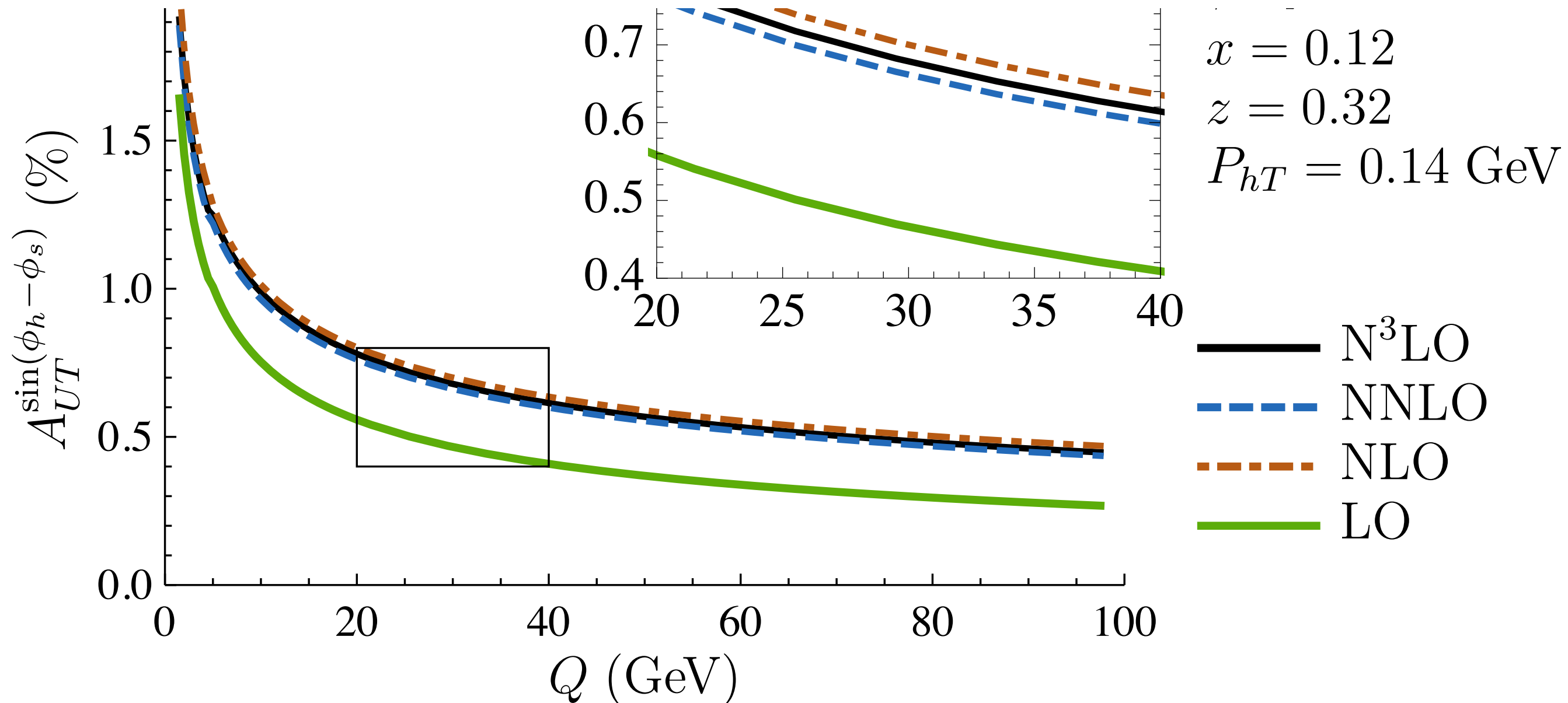


# THE ANALYSIS

# THE SIVERS ASYMMETRY

$$f_{1T, \perp, \alpha \perp b}^\perp(x, b) = N_\alpha \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{\alpha_s^3} \exp\left(-\frac{r_0 + xr_1}{\alpha_s^3} b^2\right)$$

$$\mathbf{N^3LO} = \frac{C_V \text{ (cancel)}}{\alpha_s^3} \mid \frac{\gamma_V}{\alpha_s^3} \mid \frac{\Gamma_{cusp}}{\alpha_s^4} \mid \frac{\text{CS-kernel at small-b}}{\alpha_s^3} \mid \frac{\text{unpol.TMD at small-b}}{\alpha_s^2}$$



# THE PARAMETRIZATION

Large-x not constrained

$$\sim (1-x)$$

Possibility of a node

$$\epsilon_u, \epsilon_d$$

$$f_{1T;q \leftarrow h}^\perp(x, b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q, \epsilon_q)} \exp\left(-\frac{r_0 + xr_1}{\sqrt{1+r_2x^2b^2}}b^2\right)$$

u, d, s, sea = ubar, dbar, bar

Sea is not constrained

$$\beta_s = \beta_{sea} \quad \epsilon_s = \epsilon_{sea} = 0$$

Common for all flavors  
Similar to unpolarized SV19

- 12 free parameters, flavor independent  $r_0, r_1, r_2$
- Valence quarks:  $N_{u,d}, \beta_{u,d}, \epsilon_{u,d}$
- Sea quarks:  $N_{s,sea}, \beta_s = \beta_{sea}$
- Data driven fit, no constraints
- Positivity is satisfied in the region covered by the data

# DATA SELECTION

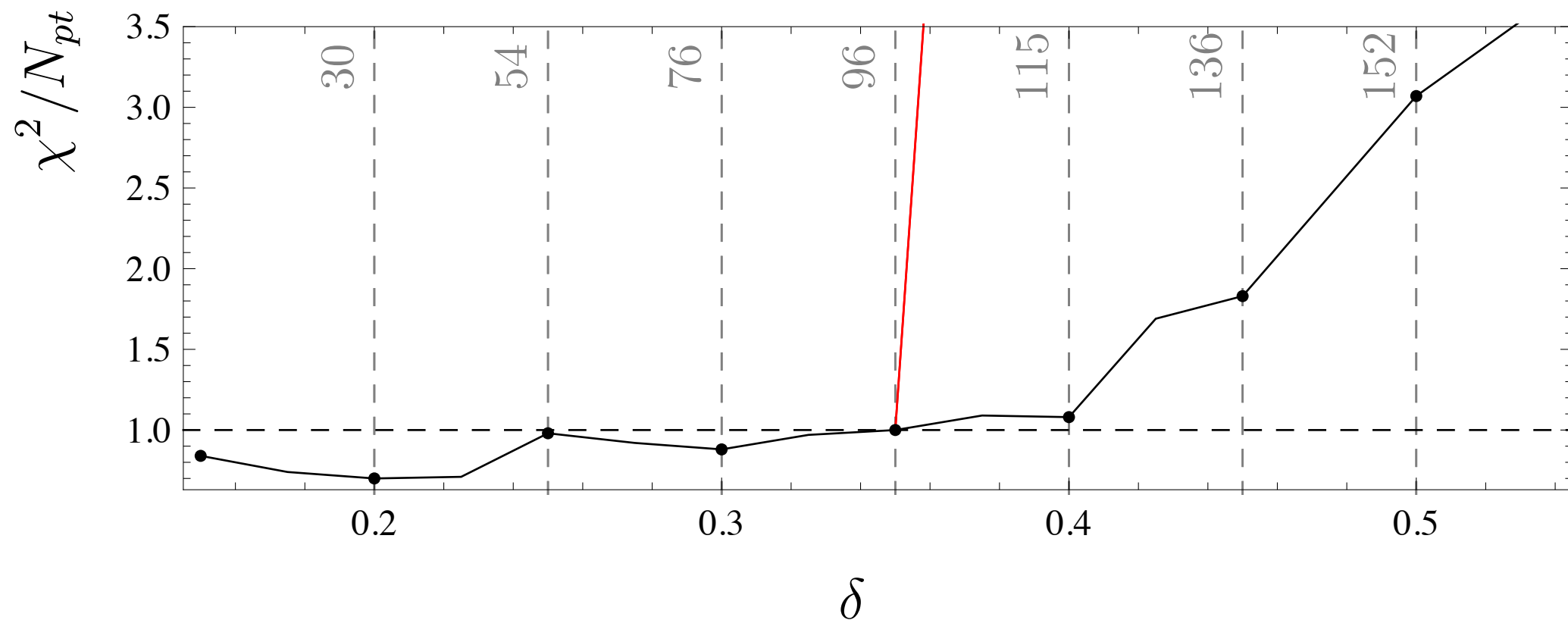
Bury, Prokudin, Vladimirov (2021)

Dataset name	Ref.	Reaction	# Points
Compass08	[36]	$d^\uparrow + \gamma^* \rightarrow \pi^+$	1 / 9
		$d^\uparrow + \gamma^* \rightarrow \pi^-$	1 / 9
		$d^\uparrow + \gamma^* \rightarrow K^+$	1 / 9

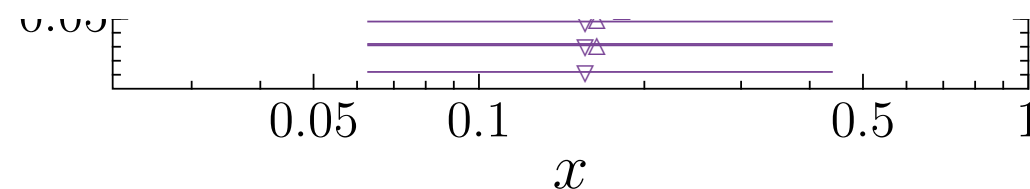
► Only  $P_T$  dependence used to avoid double counting

► Data selection compatible with

Com			
Herr			
JLak			
SIDJ			
Com			
Star			
Star			
Star			
DY			
<b>Total</b>			<b>76</b>



$\delta$



• Compass08

# FIT RESULTS

Bury, Prokudin, Vladimirov (2021)

- ▶ Replica method using Artemide framework
- ▶ Errors both from the data and the uncertainty due to unpolarized TMD

Name	$\chi^2 / N_{pt}$ [SIDIS]	$\chi^2 / N_{pt}$ [DY]	$\chi^2 / N_{pt}$ [total]
SIDIS at N <sup>3</sup> LO	$0.87^{+0.13}_{+0.03}$	$1.23^{+0.50}_{-0.24}$ no fit	$0.93^{+0.16}_{+0.01}$
SIDIS+DY at N <sup>3</sup> LO	$0.88^{+0.15}_{+0.04}$	$0.90^{+0.31}_{+0.00}$	$0.88^{+0.15}_{+0.05}$

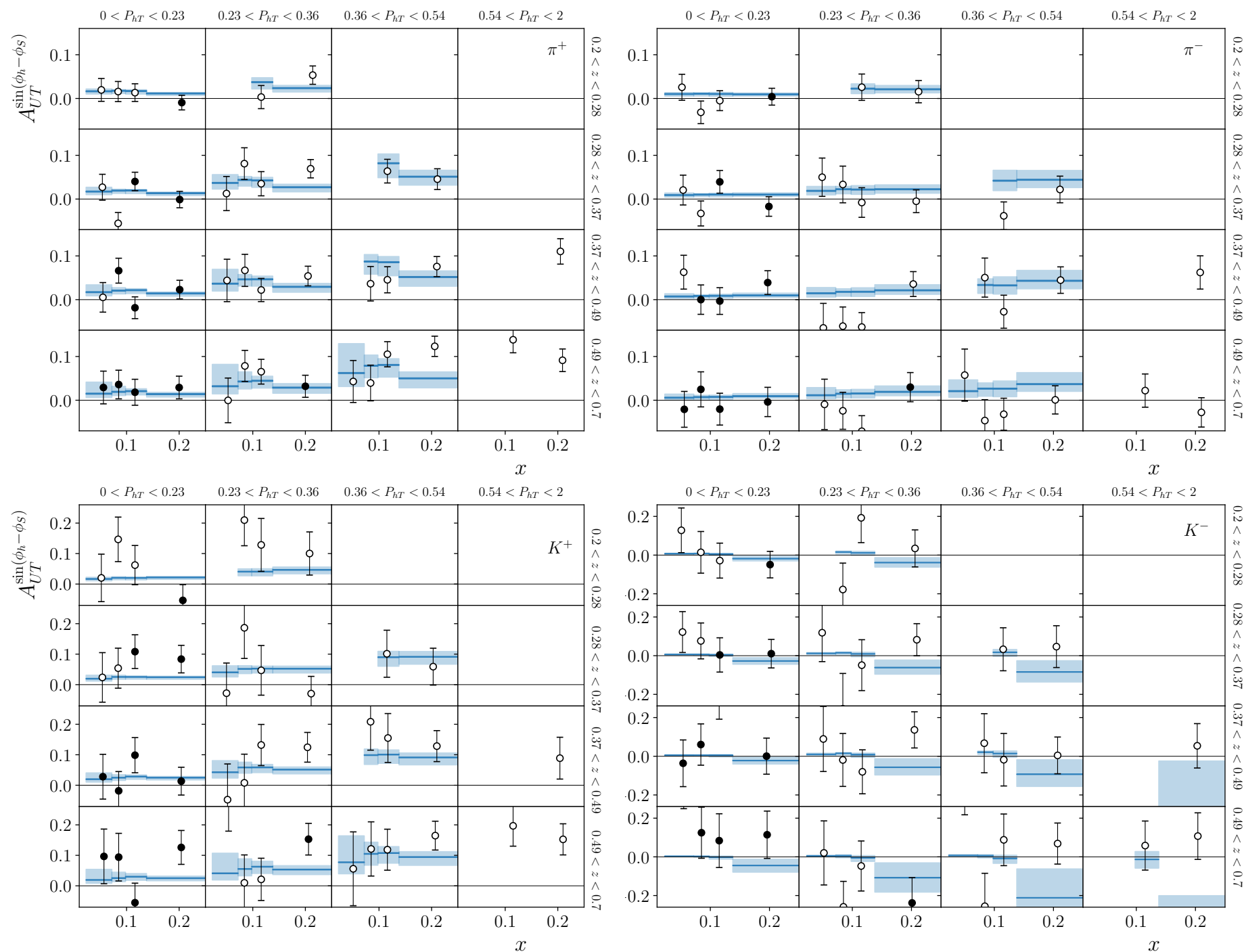
- ▶ Unbiased parametrization
- ▶ No tension between SIDIS and DY data — universality
- ▶ Good convergence of the fit for all data sets



# N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

## HERMES 2020 3D binning description



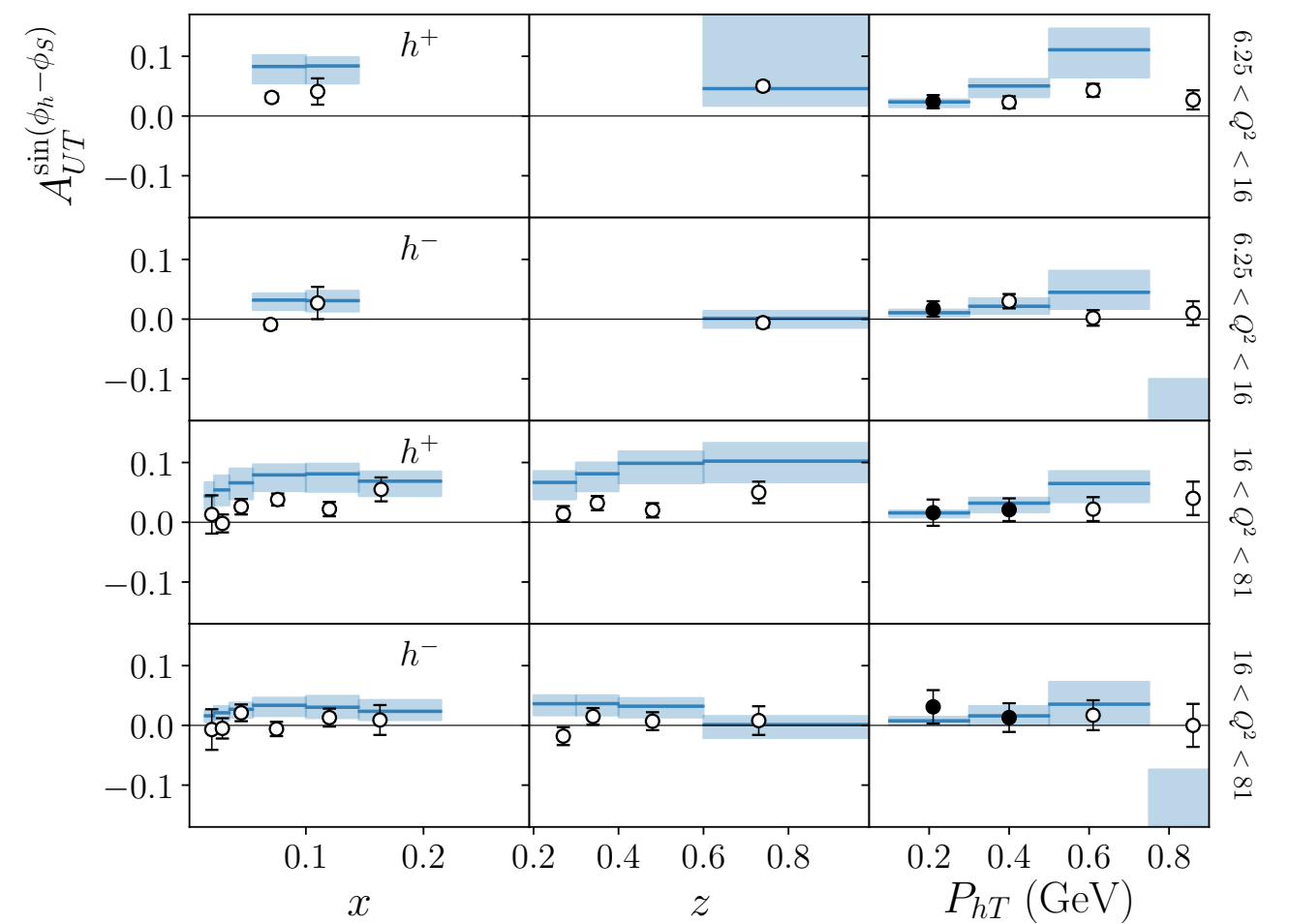
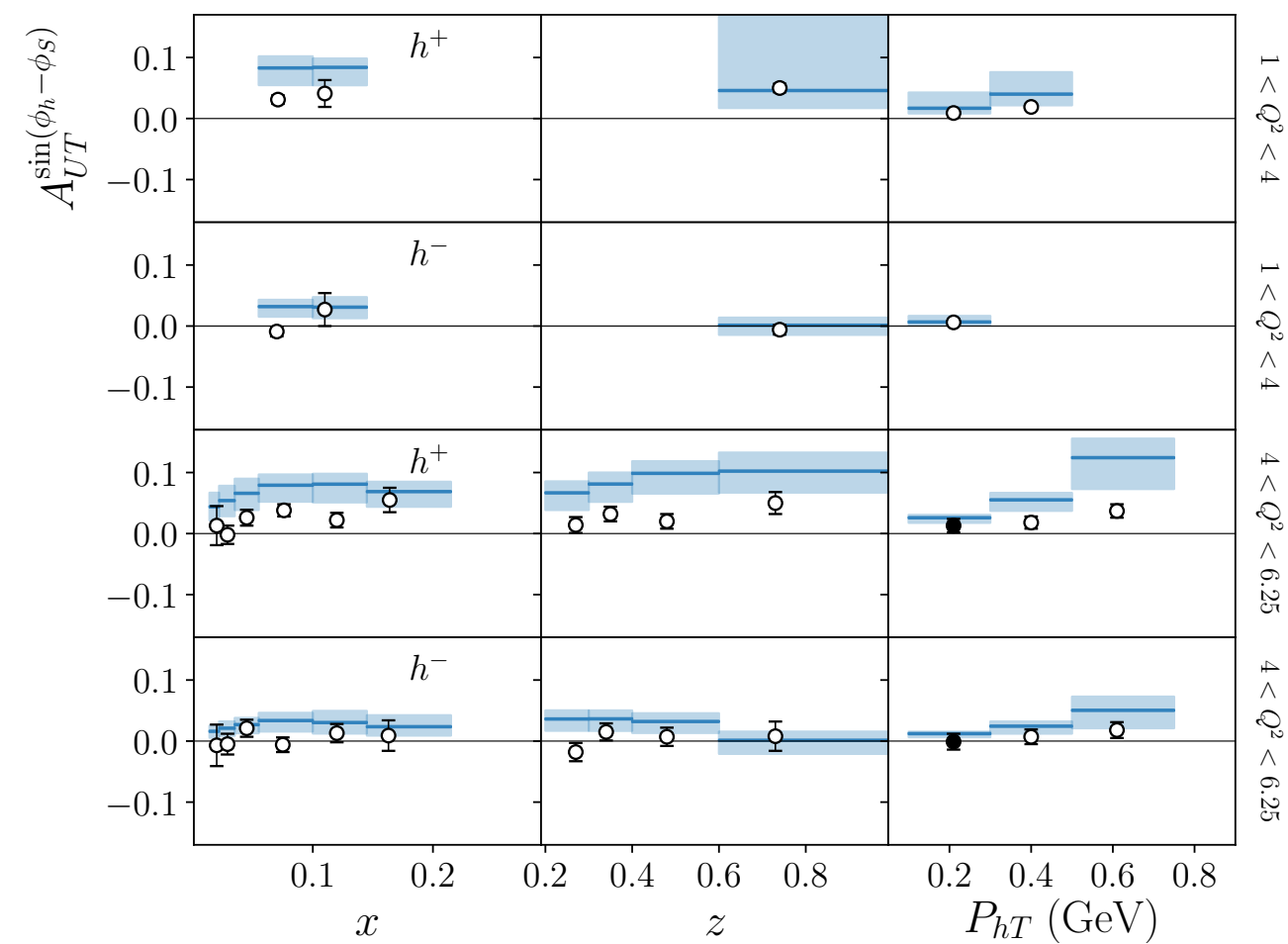
Filled in points  
used in the fit

Open points are  
predictions

# N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

## COMPASS SIDIS data

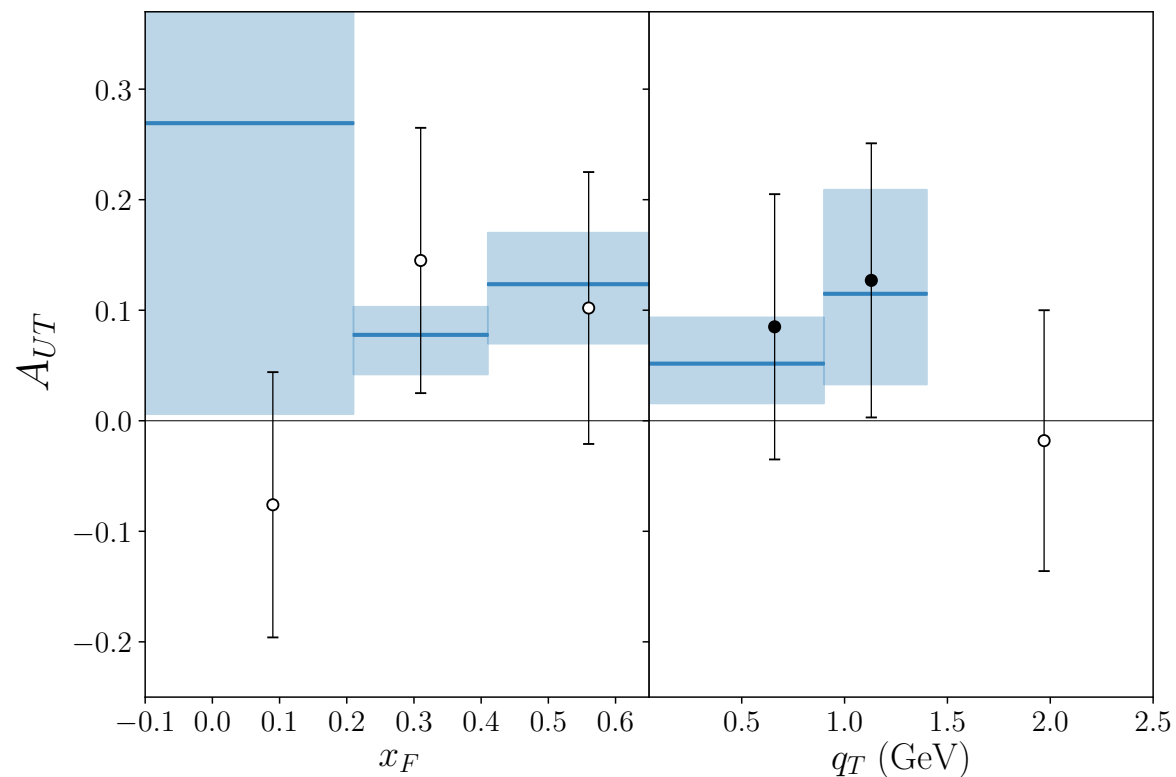


# N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

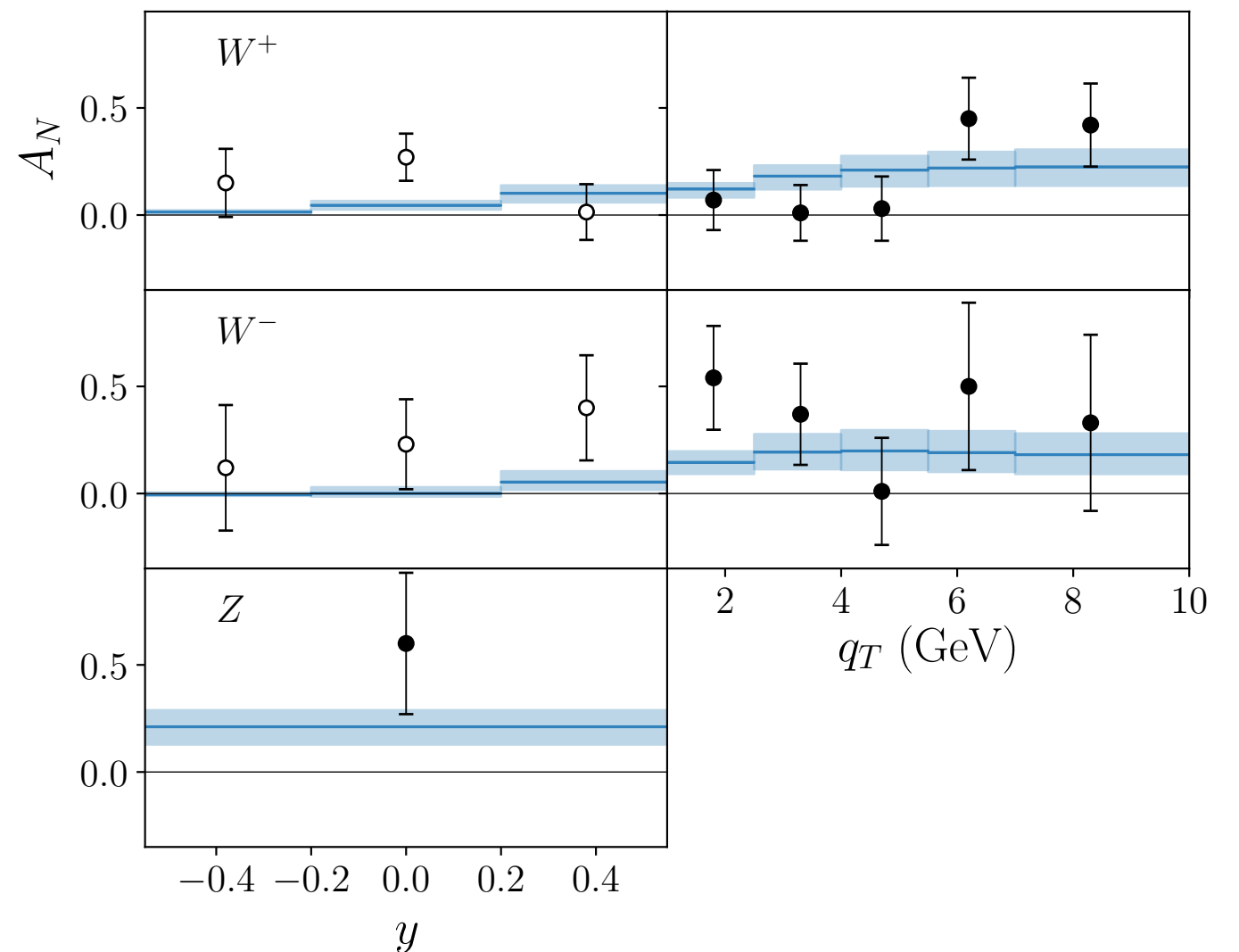
## Pion induced Drell-Yan, COMPASS

COMPASS Collab. Phys. Rev. Lett. 119, 112002 (2017)



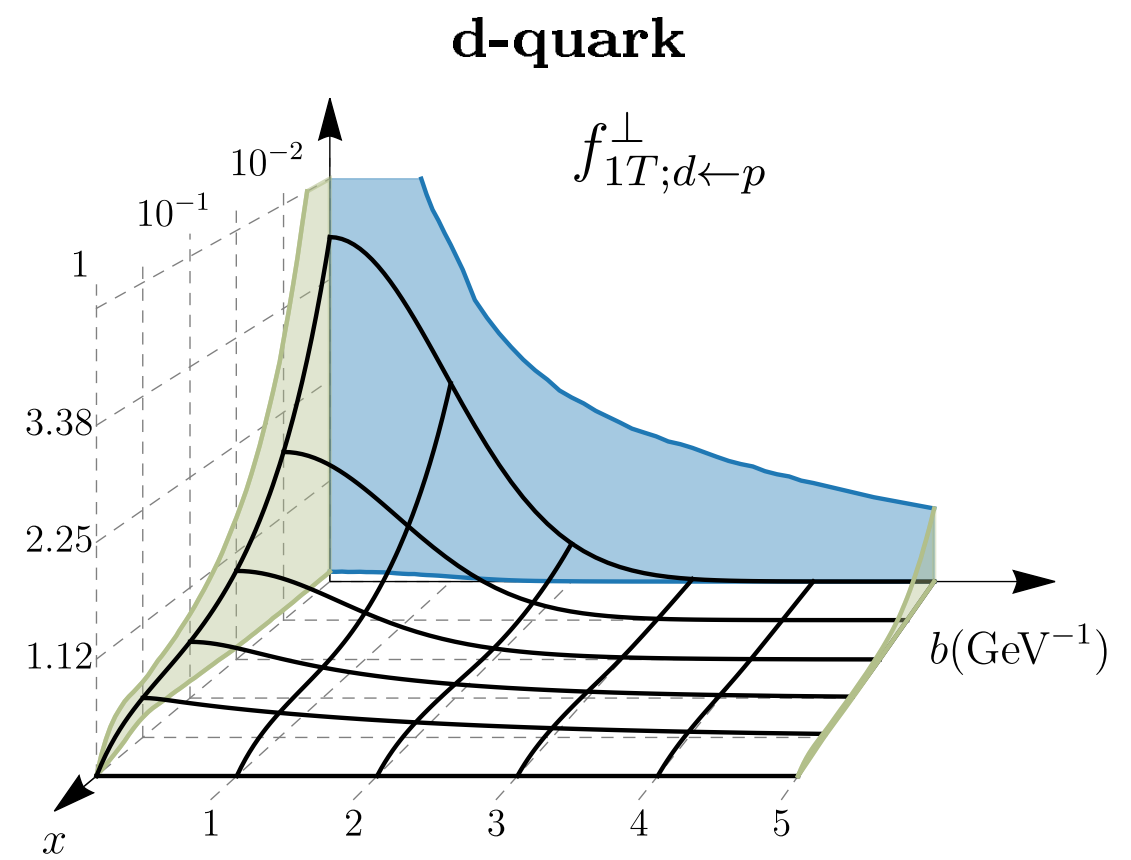
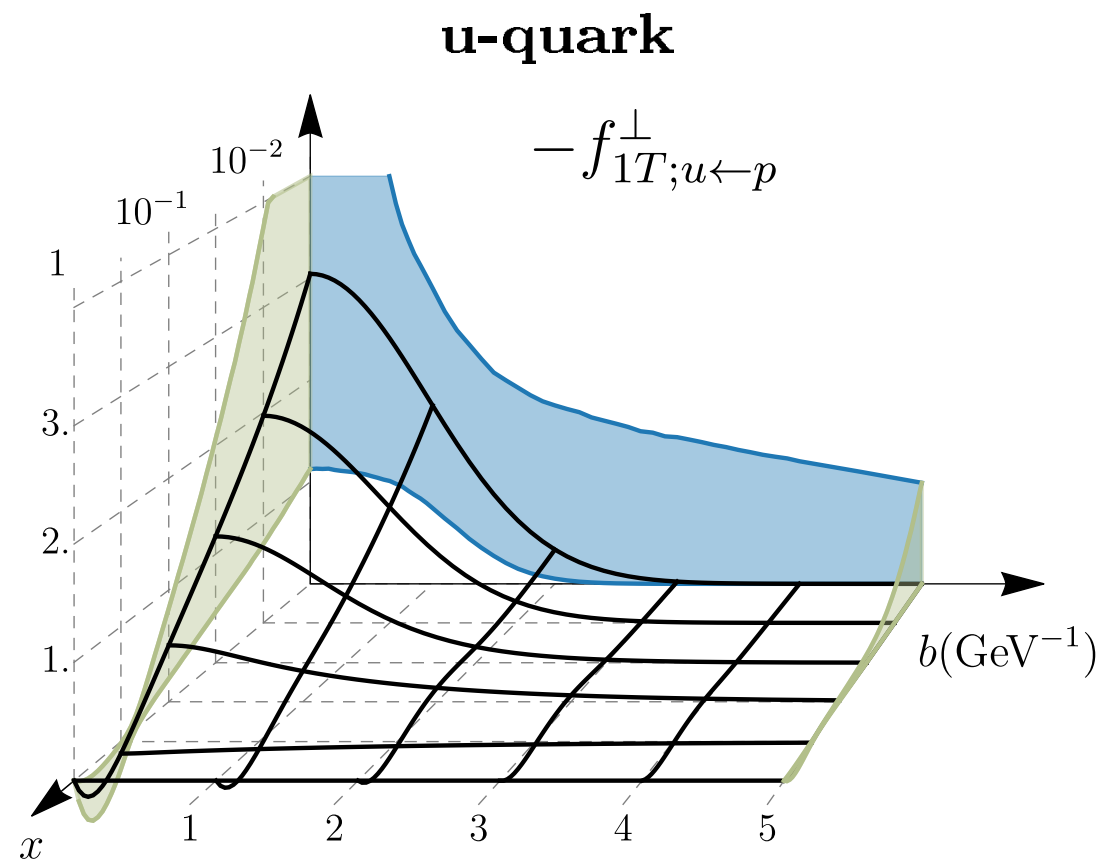
## W/Z production, STAR

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



# SIVERS FUNCTION IN THE POSITION SPACE

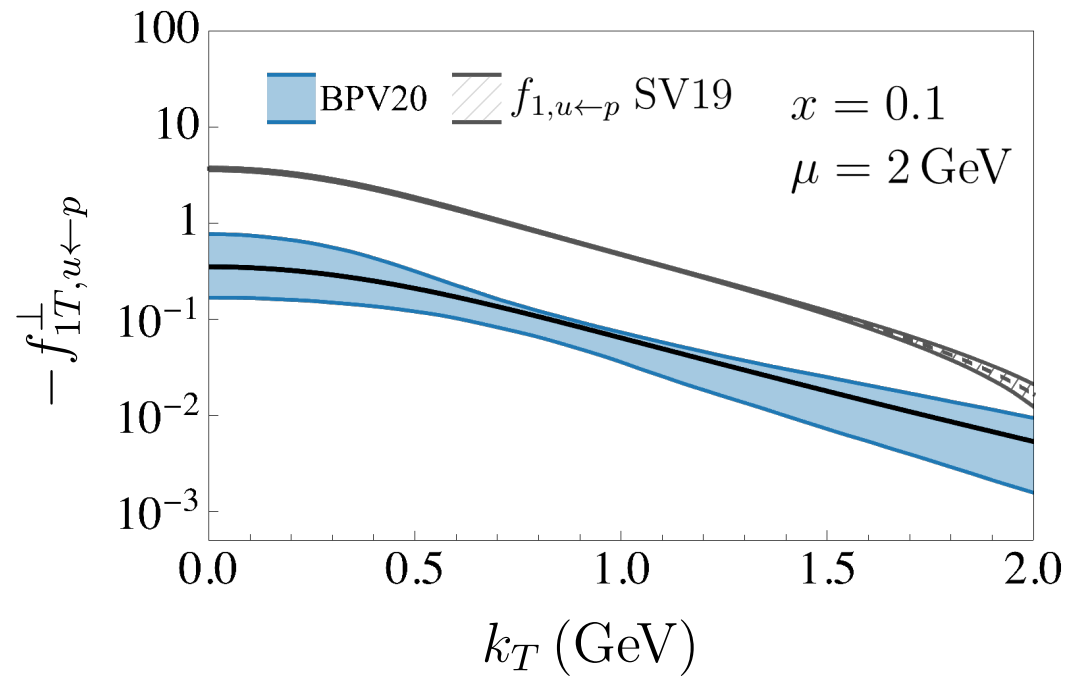
Bury, Prokudin, Vladimirov (2021)



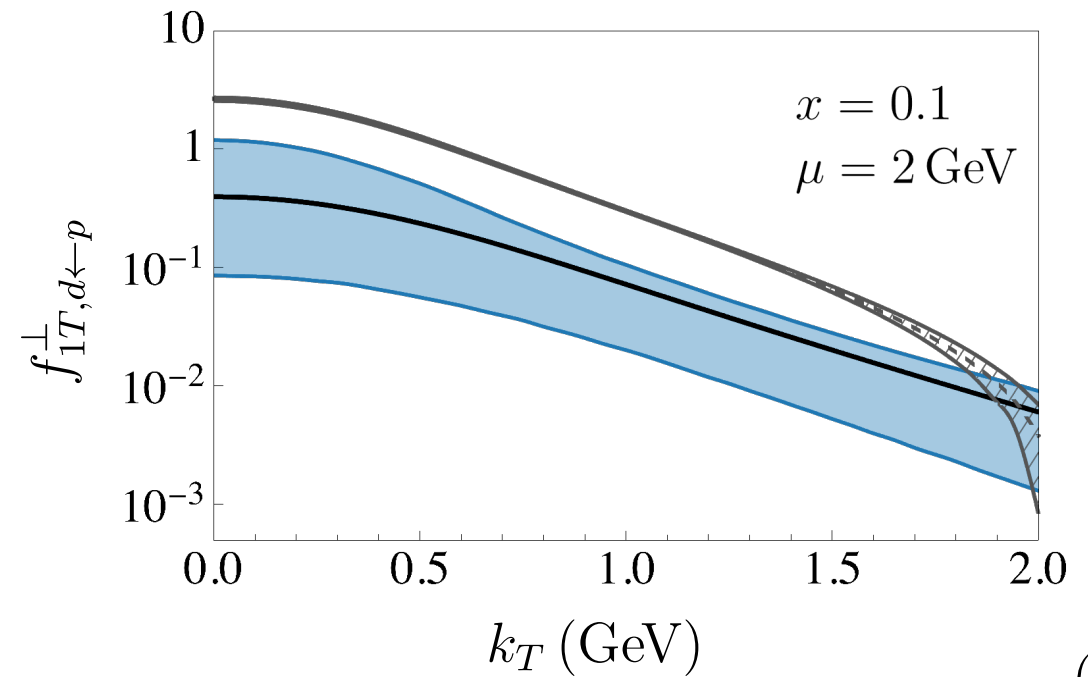
- ▶ Large uncertainties
- ▶ Node for u quark
- ▶ More data needed: EIC, JLab 12, etc

# SIVERS FUNCTION IN THE MOMENTUM SPACE

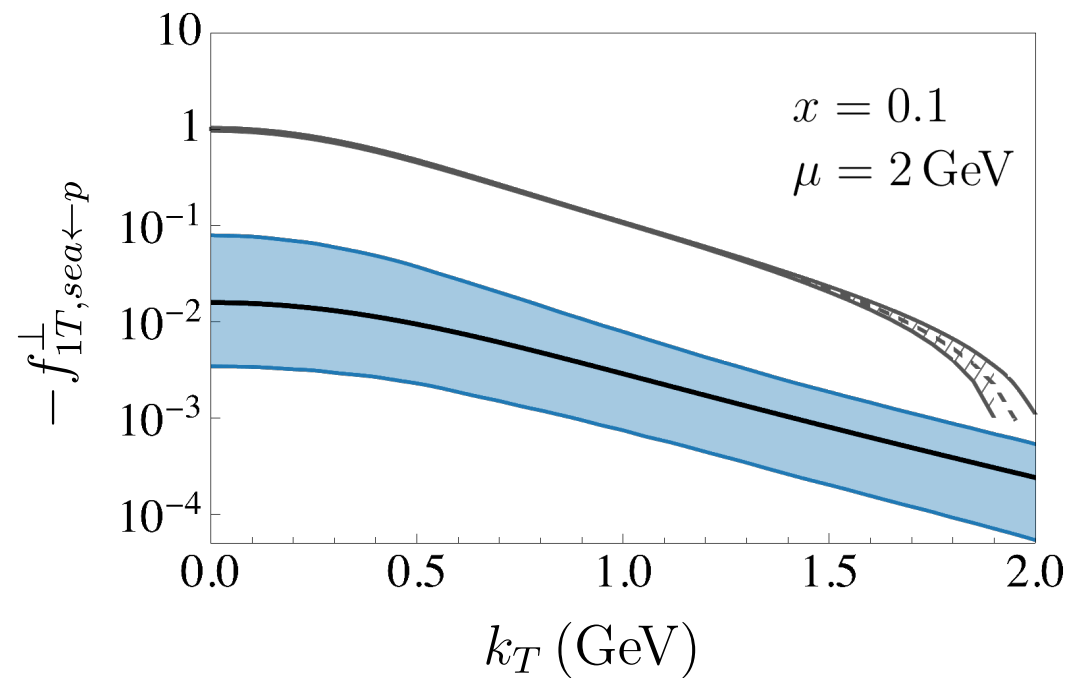
Bury, Prokudin, Vladimirov (2021)



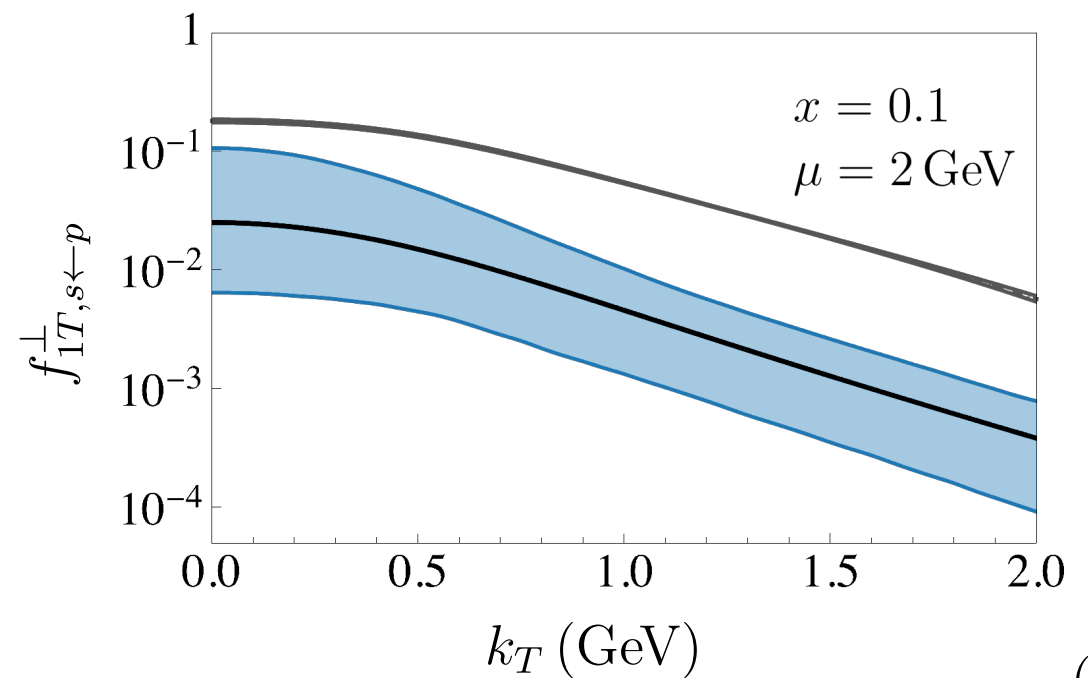
(a)



(b)



(c)

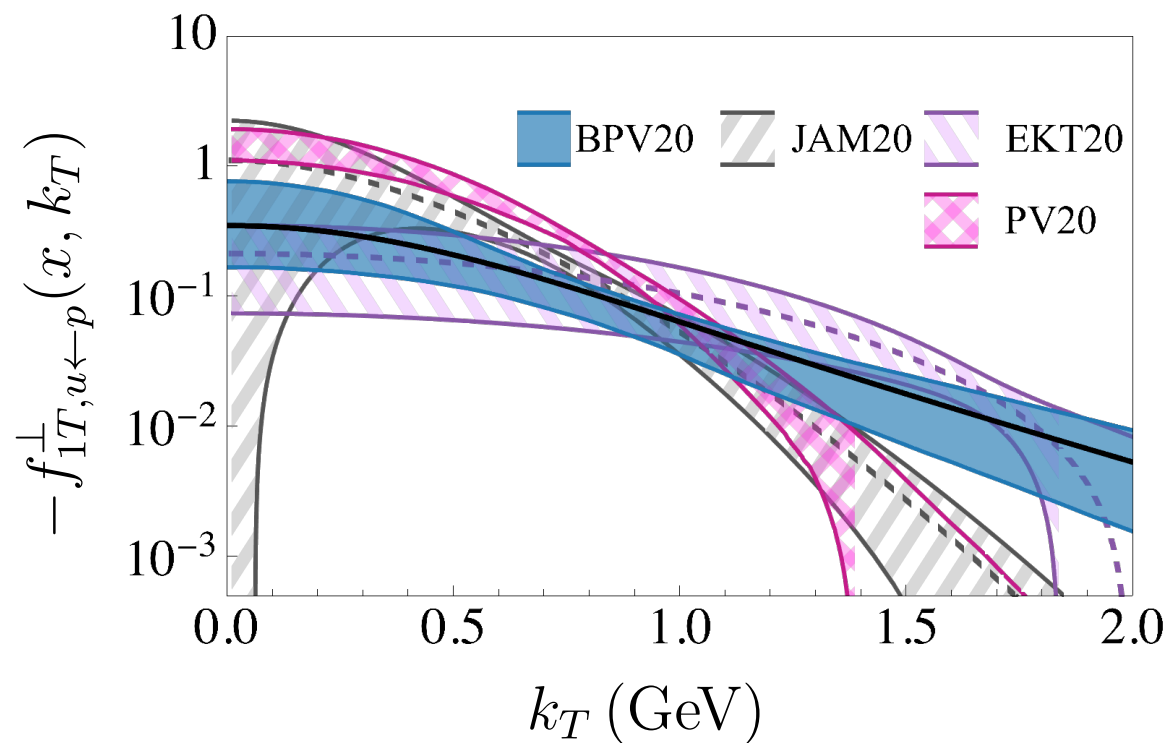


(d)

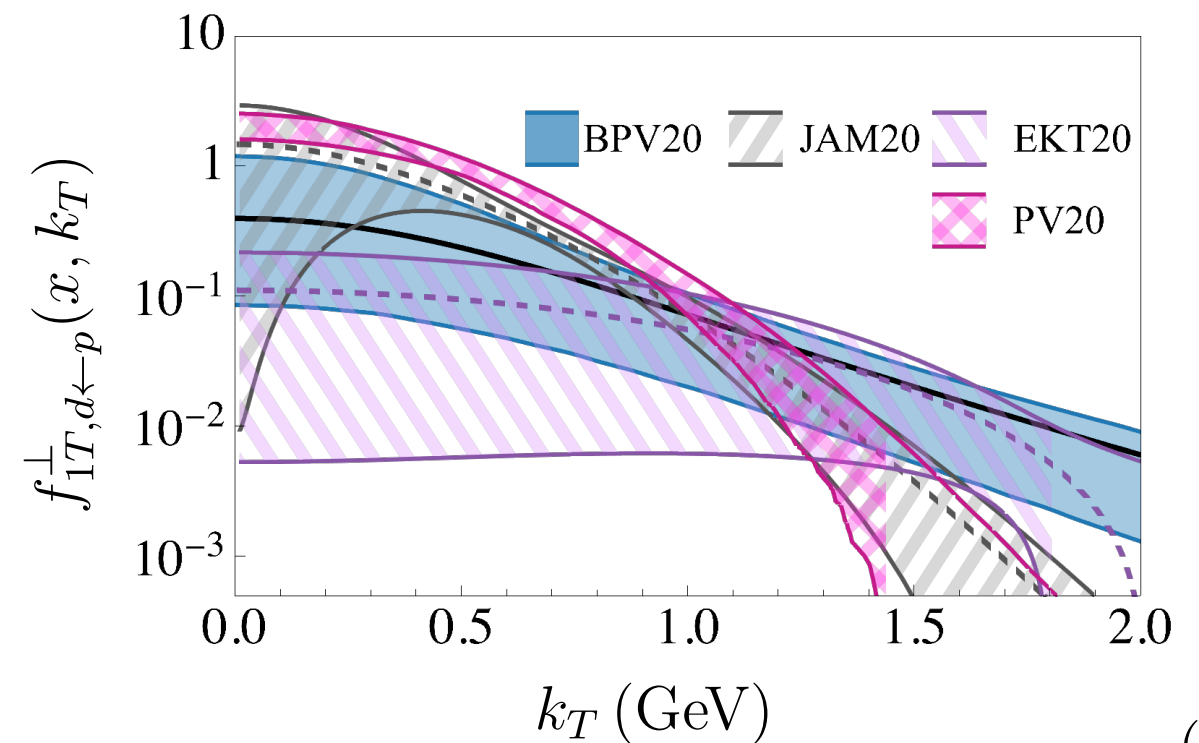


# SIVERS FUNCTION IN THE MOMENTUM SPACE

Bury, Prokudin, Vladimirov (2021)



(a)



(b)

## ► Comparison to JAM20 (LO) analysis

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

## ► PV20, NLL analysis

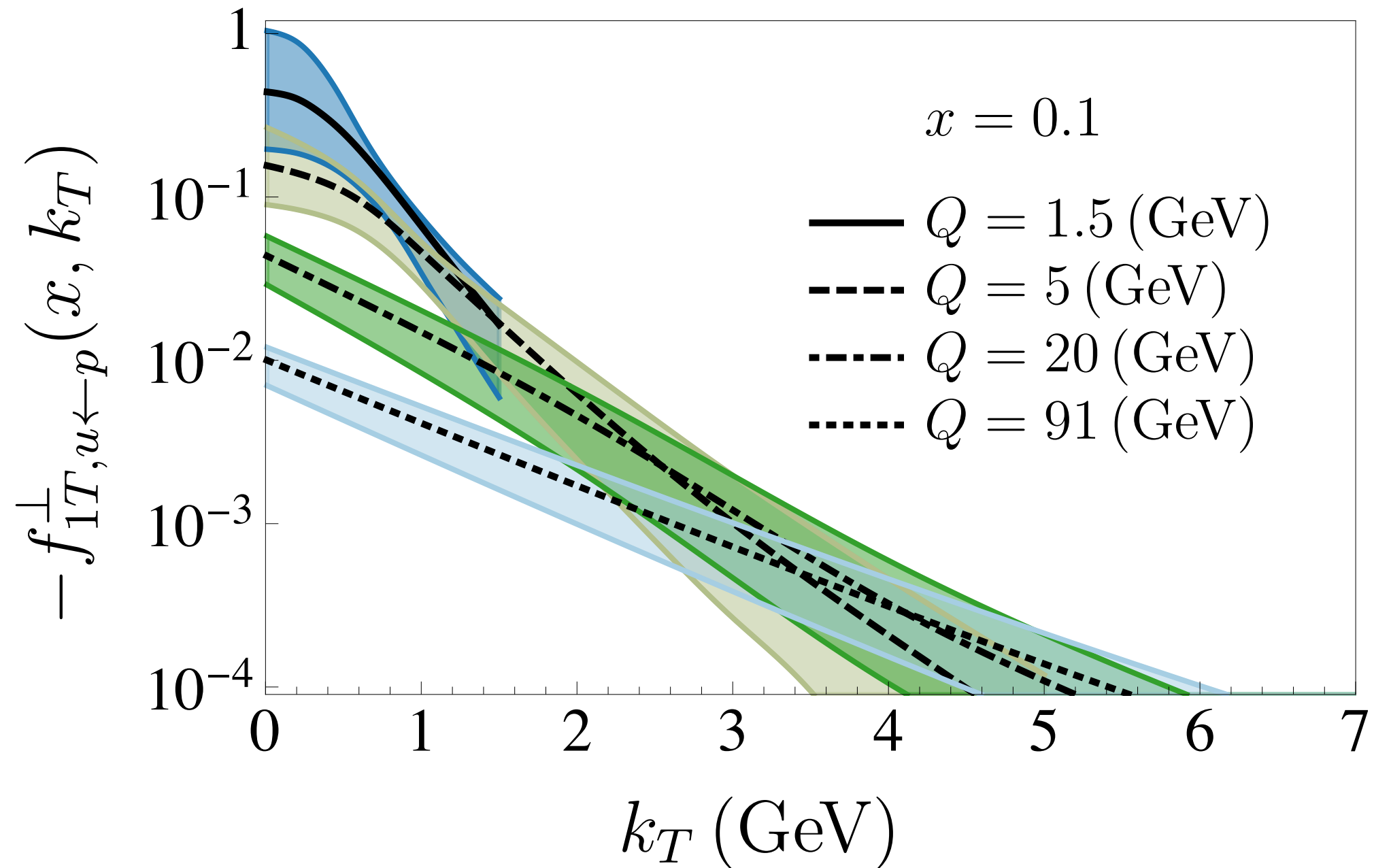
Bacchetta, Delcarro, Pisano, Radici (2020)

## ► EKT20, NNLL analysis

Echevarria, Terry, Kang (2020)

# THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

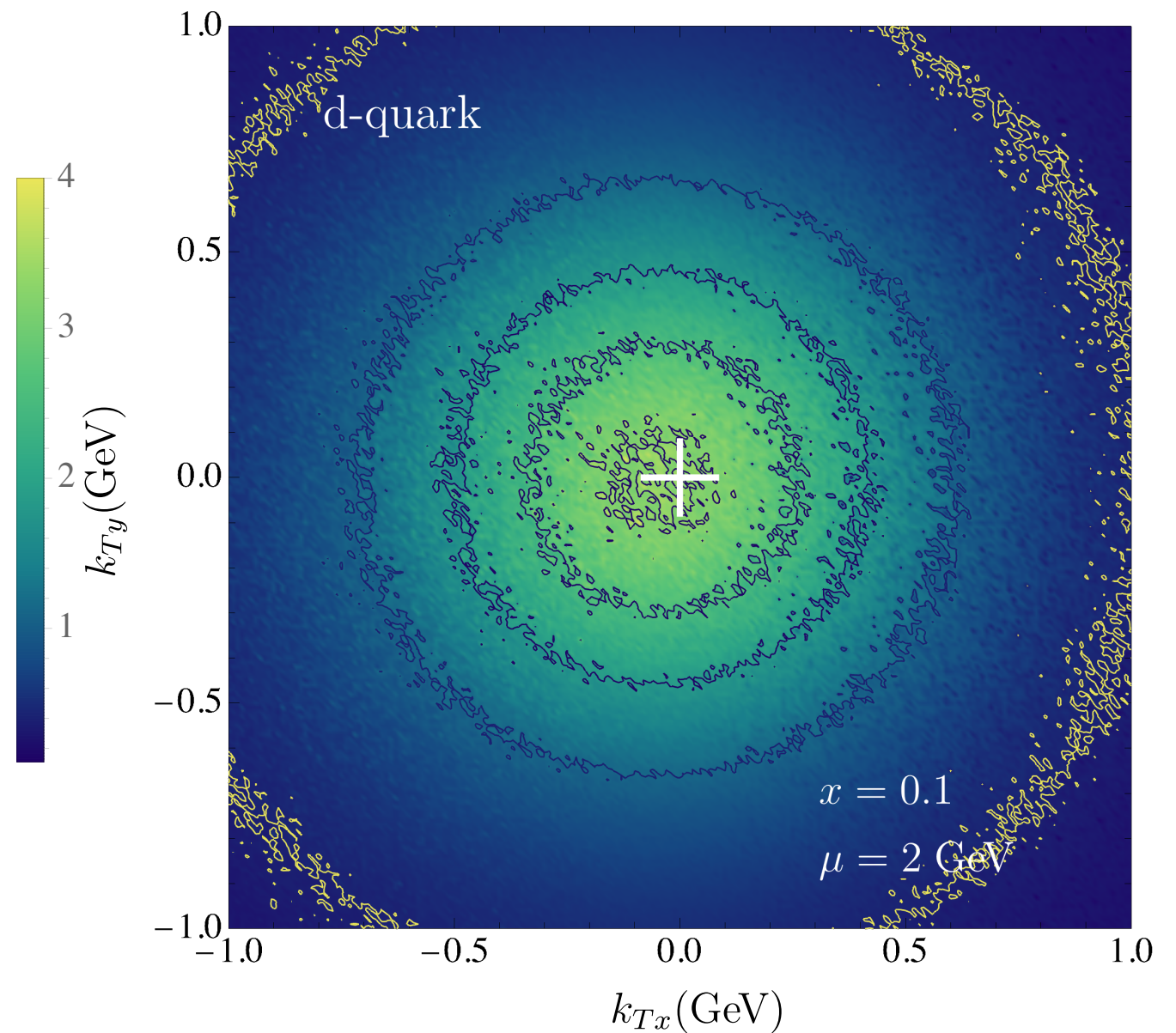
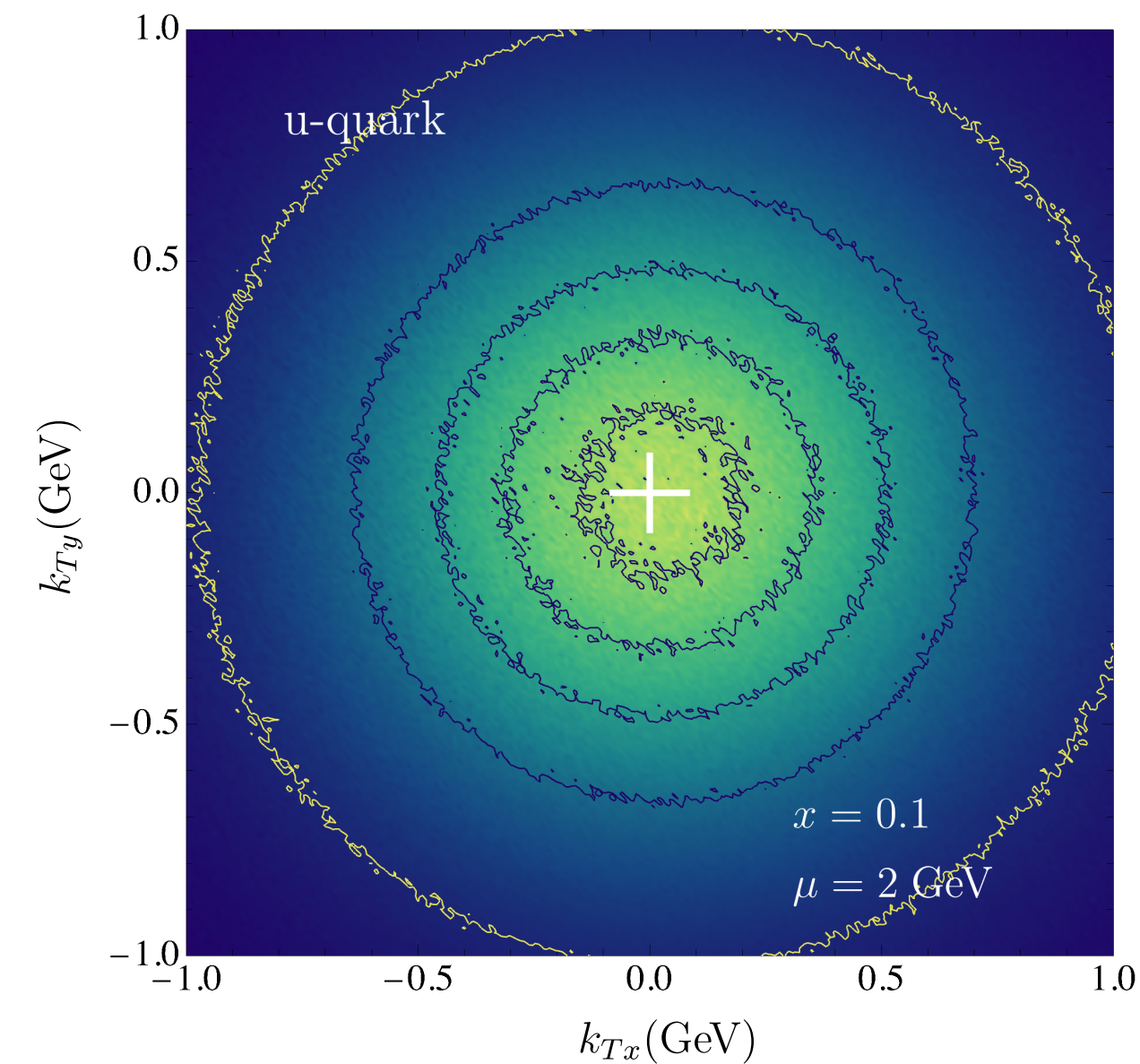


# TOMOGRAPHY

# NUCLEON TOMOGRAPHY

Bury, Prokudin, Vladimirov (2021)

$$\rho_{1;q\leftarrow h^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T, \mu) = f_{1;q\leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q\leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$

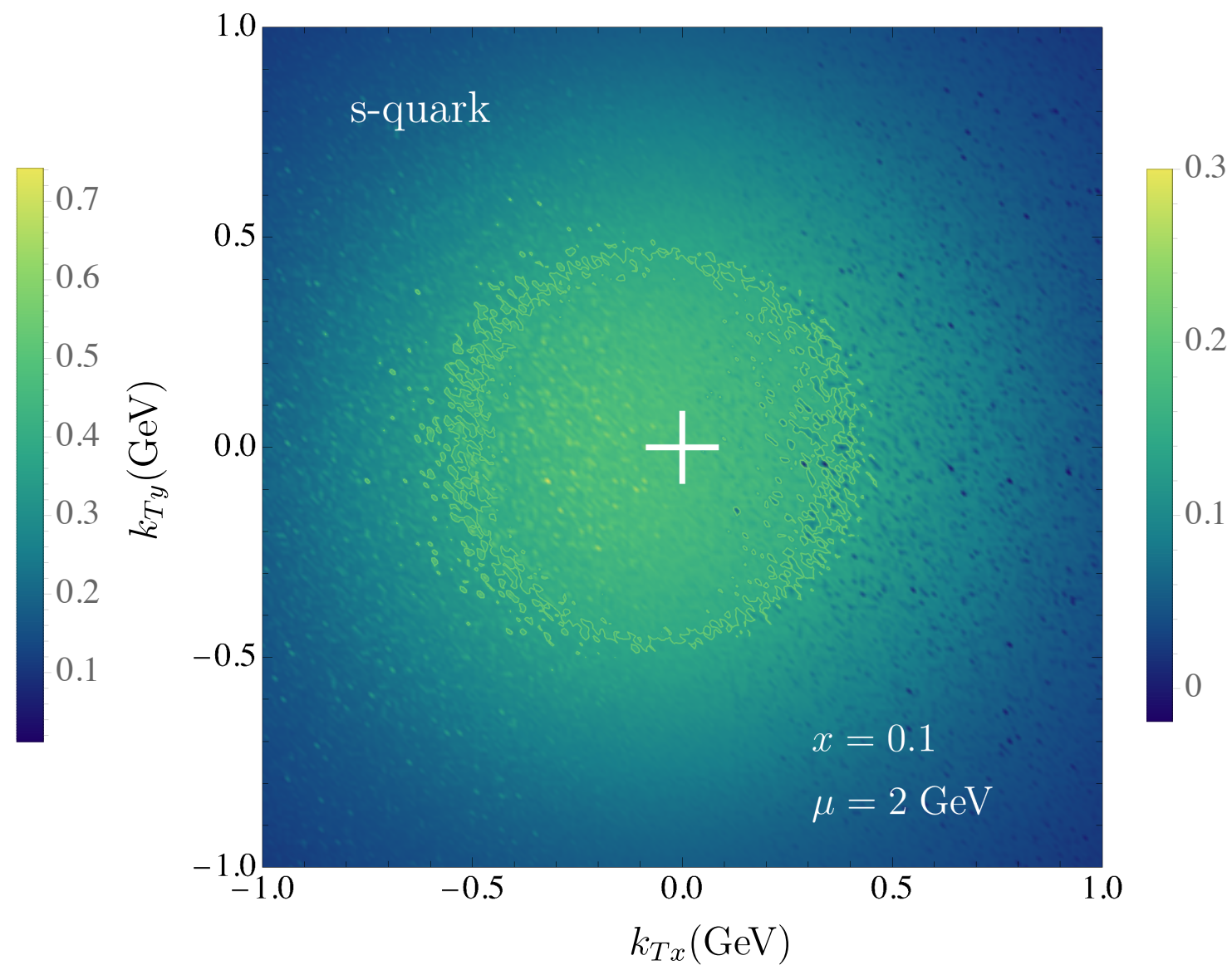
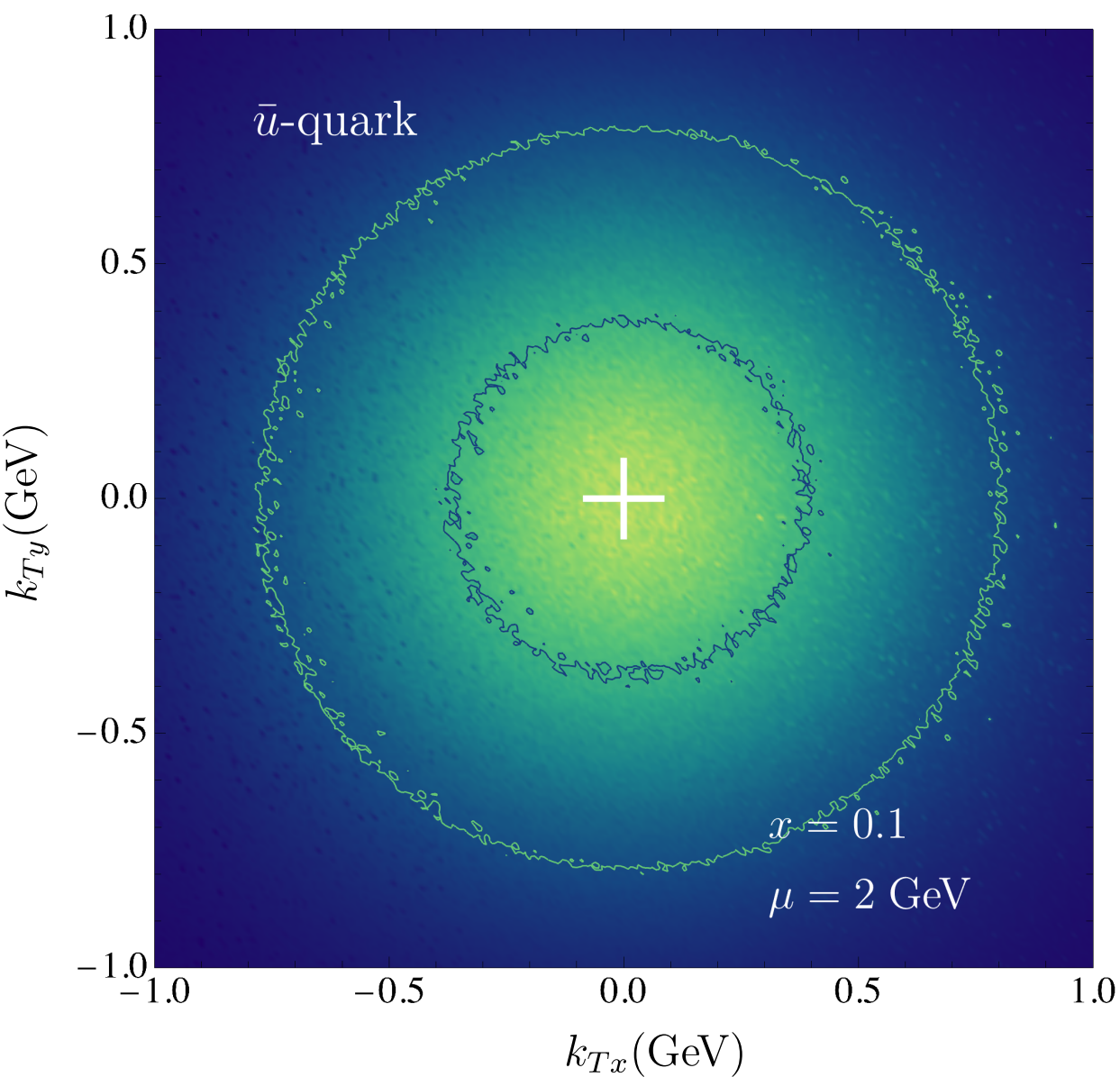




# NUCLEON TOMOGRAPHY

Bury, Prokudin, Vladimirov (2021)

$$\rho_{1;q\leftarrow h^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T, \mu) = f_{1;q\leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q\leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$



# QS FUNCTIONS



# THE QIU-STERMAN MATRIX ELEMENT

► At small  $b_T$  the Sivers function is related to the twist-3 function

Scimemi, Tarasov, Vladimirov (19)

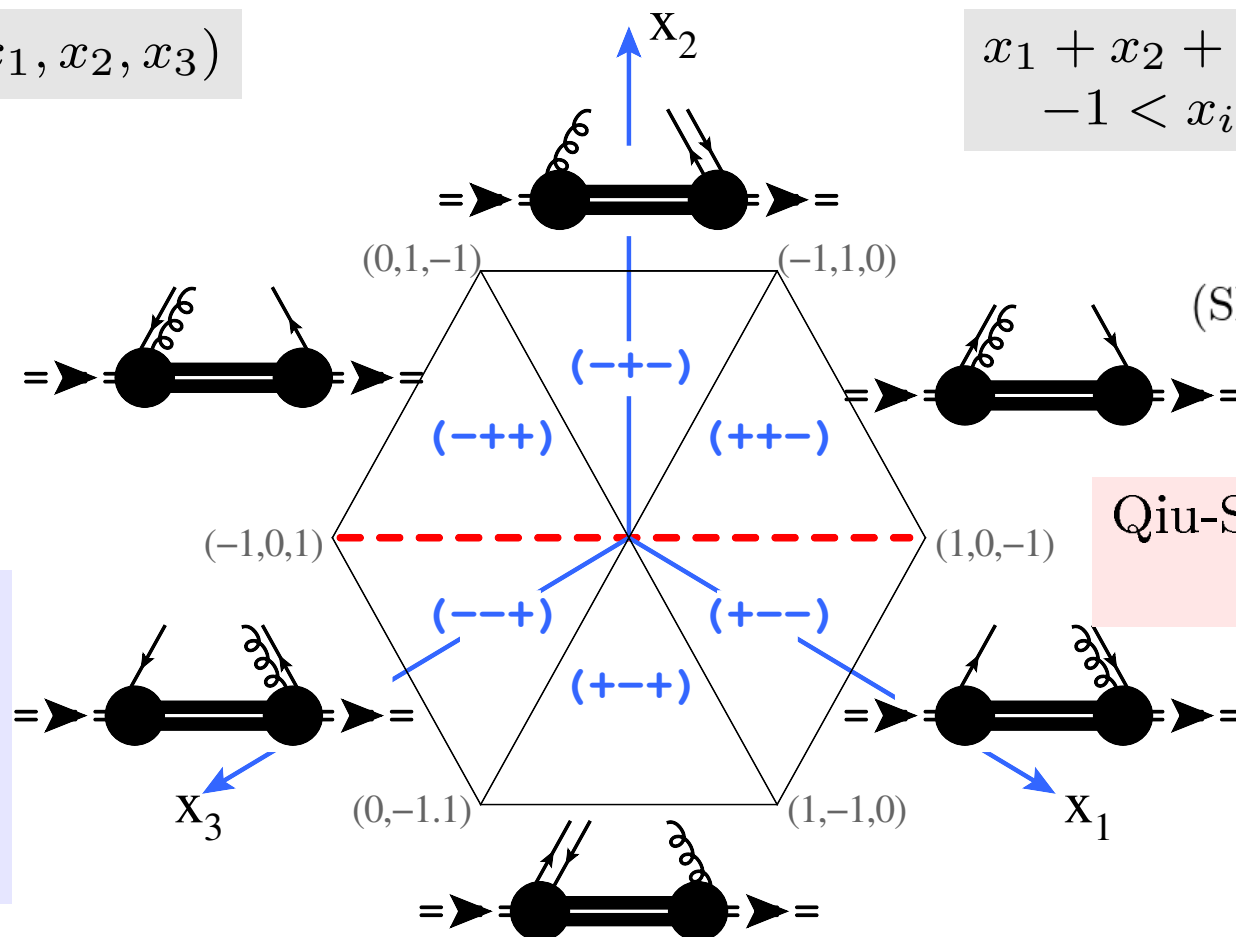
$$\langle p, s | g\bar{q}(z_1 n) [z_1 n, z_2 n] \not{n} F_{\mu+}(z_2 n) [z_2 n, z_3 n] q(z_3 n) | p, s \rangle \quad (4.9)$$

$$= 2\epsilon_T^{\mu\nu} s_\nu (np)^2 M \int_{-1}^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3) e^{-i(np)(x_1 z_1 + x_2 z_2 + x_3 z_3)} T_q(x_1, x_2, x_3),$$

$T(x_1, x_2, x_3)$

$$x_1 + x_2 + x_3 = 0$$

$$-1 < x_i < 1$$



(DY)  $f_{1T}^\perp(x, \mathbf{b}) = \pi T(-x, 0, x) + O(\mathbf{b}^2),$

(SIDIS)  $f_{1T}^\perp(x, \mathbf{b}) = -\pi T(-x, 0, x) + O(\mathbf{b}^2).$

Qiu-Sterman function  
 $T(-x, 0, x)$

Each region  
 $x_i \leq 0$   
has its own  
partonic  
interpretation

Important for  
description of  
asymmetries in PP, etc

# THE QIU-STERMAN MATRIX ELEMENT

► Beyond LO it is complicated

Scimemi, Tarasov, Vladimirov (19)

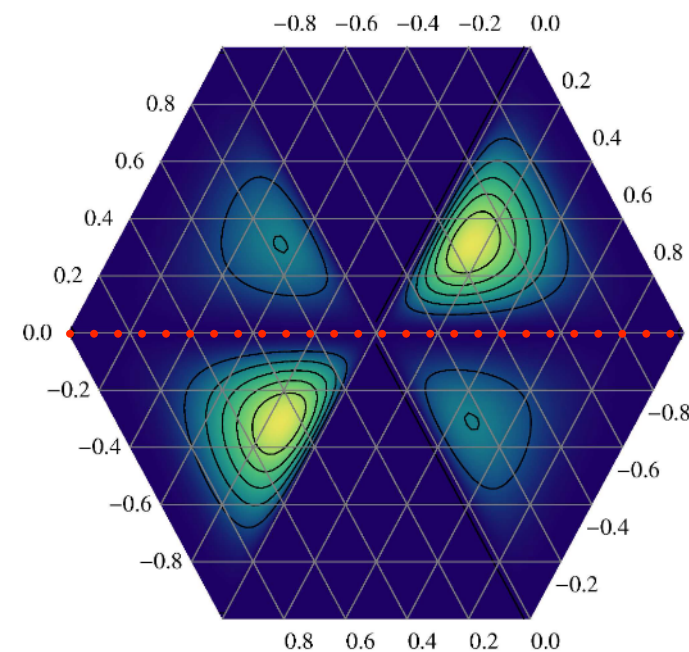
$$f_{1T;q\leftarrow h;DY}^\perp(x, \mathbf{b}; \mu, \zeta) = \pi T(-x, 0, x) + \pi a_s(\mu) \left\{ \begin{aligned} & -2\mathbf{L}_\mu P \otimes T + C_F \left( -\mathbf{L}_\mu^2 + 2\mathbf{l}_\zeta \mathbf{L}_\mu + 3\mathbf{L}_\mu - \frac{\pi^2}{6} \right) T(-x, 0, x) \\ & + \int d\xi \int_0^1 dy \delta(x - y\xi) \left[ \left( C_F - \frac{C_A}{2} \right) 2\bar{y} T(-\xi, 0, \xi) + \frac{3y\bar{y}}{2} \frac{G_+(-\xi, 0, \xi) + G_-(-\xi, 0, \xi)}{\xi} \right] \right\} \\ & + O(a_s^2) + O(\mathbf{b}^2), \end{aligned} \right.$$

► Evolution of T is complicated and non closed

Braun et al (11), Kang, Qiu (09), Vogelsang

1/N<sub>c</sub>-suppressed

The only QS-term



$$\mu^2 \frac{d}{d\mu^2} T(-x, 0, x) = 2a_s(\mu) P \otimes T = 2a_s \int d\xi \int_0^1 dy \delta(x - y\xi) \left\{ \begin{aligned} & \left( C_F - \frac{C_A}{2} \right) \left[ \left( \frac{1+y^2}{1-y} \right)_+ T(-\xi, 0, \xi) \right] + (2y-1)_+ T(-x, \xi, x-\xi) - \Delta T(-x, \xi, x-\xi) \\ & + \frac{C_A}{2} \left[ \left( \frac{1+y}{1-y} \right)_+ T(-x, x-\xi, \xi) + \Delta T(-x, x-\xi, \xi) \right] \\ & + \frac{1-2y\bar{y}}{4} \frac{G_+(-\xi, 0, \xi) + Y_+(-\xi, 0, \xi) + G_-(-\xi, 0, \xi) + Y_-(-\xi, 0, \xi)}{\xi} \end{aligned} \right\},$$

# THE QIU-STERMAN MATRIX ELEMENT

---

► Invert the formula for Operator Product Expansion of Sivers via the QS functions

Bury, Prokudin, Vladimirov (2020)

$$T_q(-x, 0, x; \mu_b) = -\frac{1}{\pi} \left( 1 + C_F \frac{\alpha_s(\mu_b)}{4\pi} \frac{\pi^2}{6} \right) f_{1T;q \leftarrow h}^\perp(x, b) - \frac{\alpha_s(\mu_b)}{4\pi^2} \int_x^1 \frac{dy}{y} \left[ \frac{\bar{y}}{N_c} f_{1T;q \leftarrow h}^\perp\left(\frac{x}{y}, b\right) + \frac{3y^2 \bar{y}}{2x} G^{(+)}\left(-\frac{x}{y}, 0, \frac{x}{y}; \mu_b\right) \right] + \mathcal{O}(\alpha_s^2, b^2)$$

Choose the scale to eliminate logs  $\mu_b = \frac{2e^{-\gamma_E}}{b}$

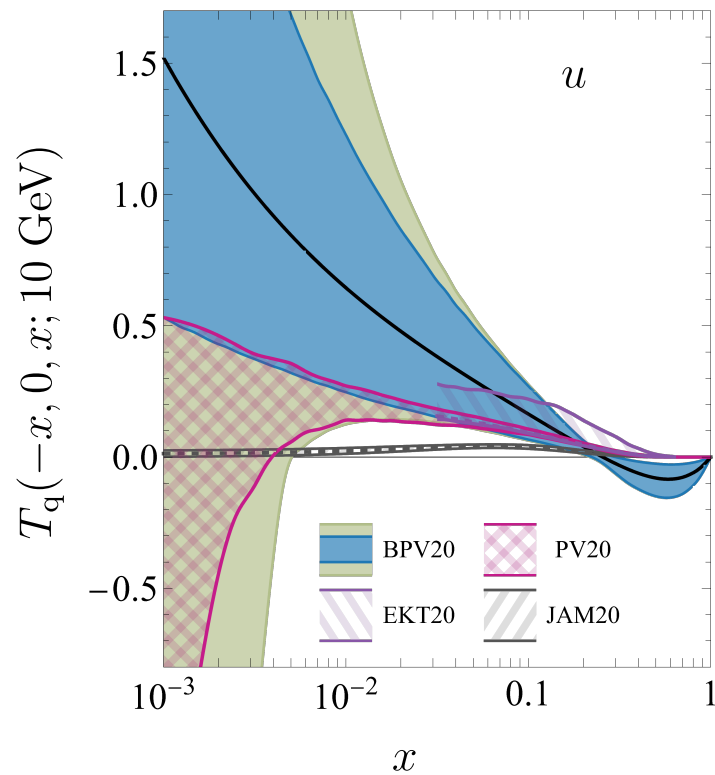
We choose  $b = 0.11 \text{ (GeV}^{-1}\text{)}$ ,  $\mu_b = 10 \text{ (GeV)}$

and estimate gluon contribution  $G^{(+)} = \pm(|T_u| + |T_d|)$

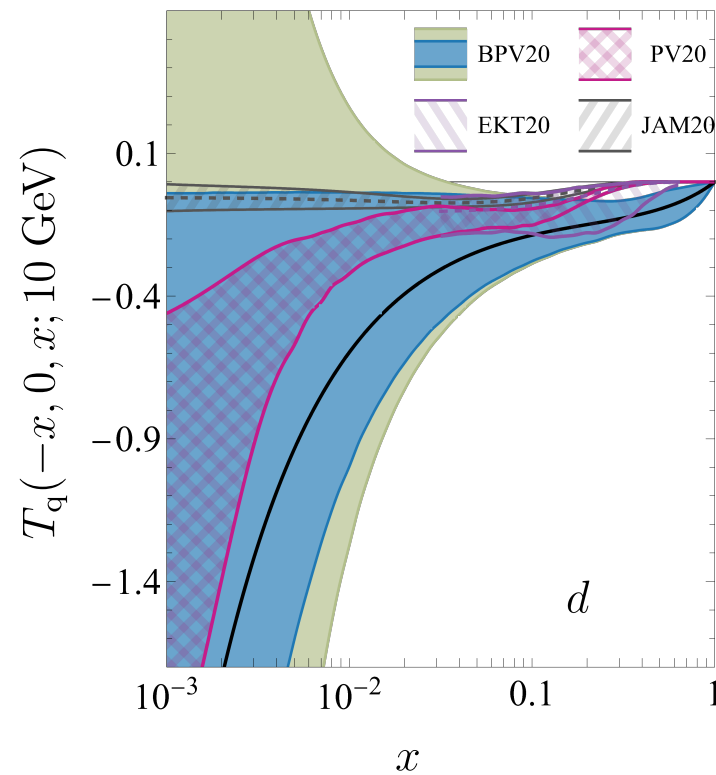
► Exact model independent relation!

# THE QIU-STERMAN MATRIX ELEMENT

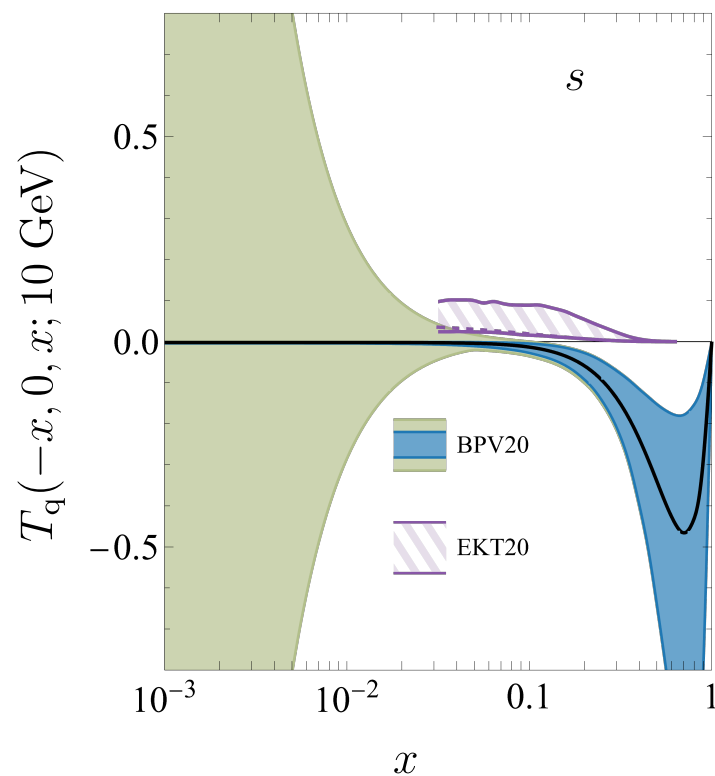
Bury, Prokudin, Vladimirov (2020)



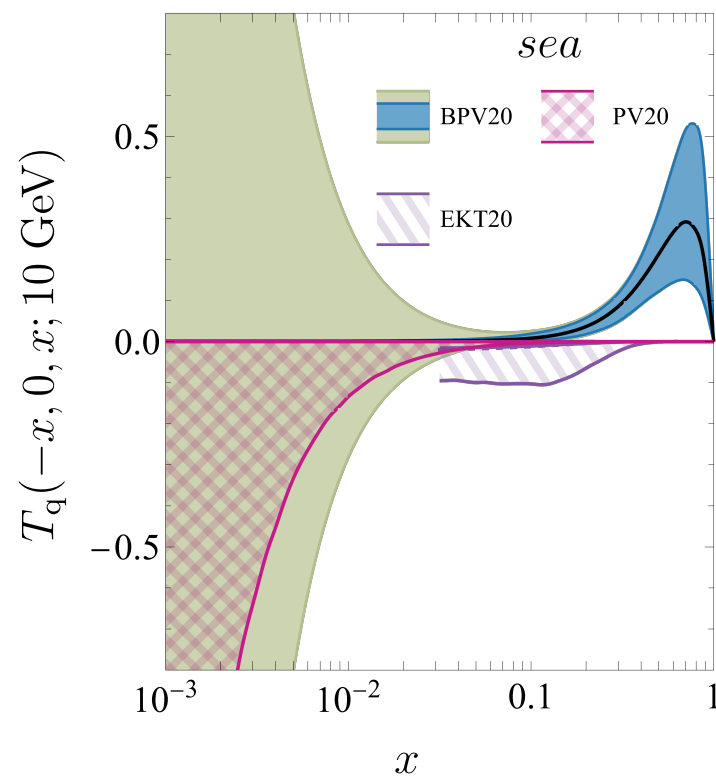
(a)



(b)



(c)



(d)

Compares well with  
Jam 20 (LO)

Jam20: Cammarota, Gamberg, Kang,  
Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

PV20 (NLL)

Bacchetta, Delcarro, Pisano, Radici (2020)

EKT20 (NNLL)

Echevarria, Kang, Terry (2020)

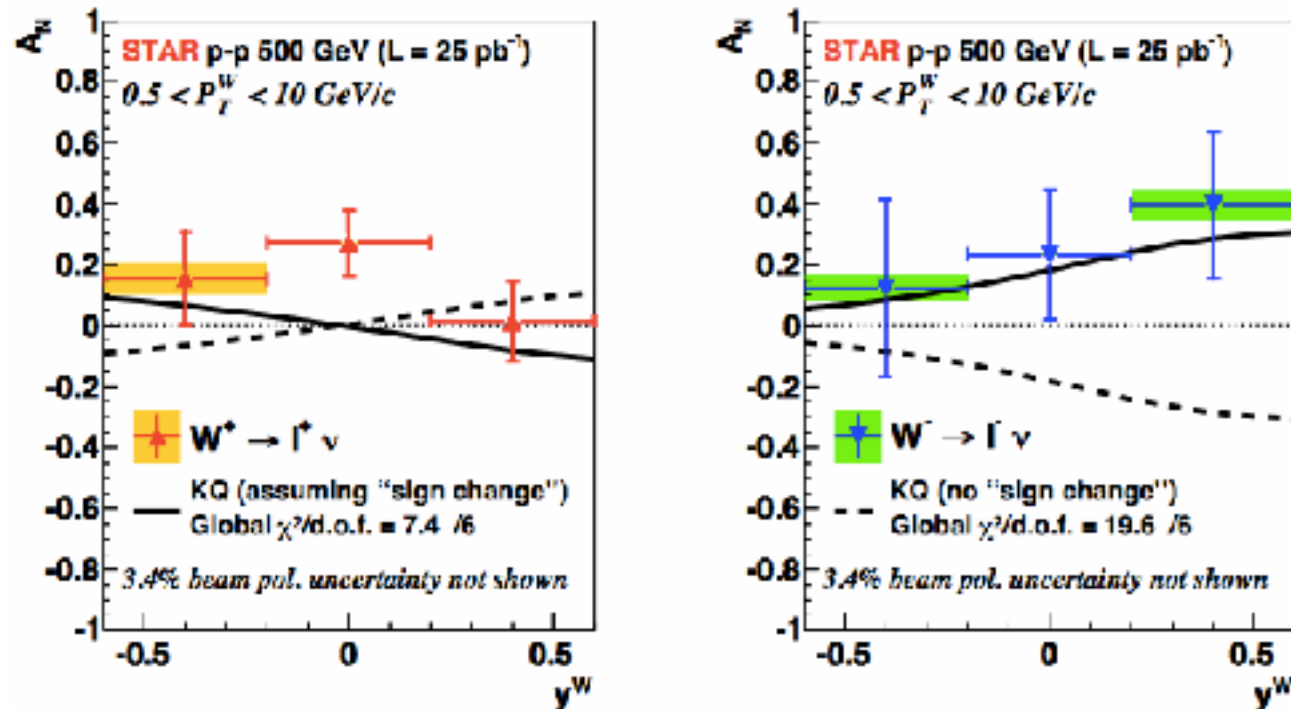
Sea quark functions  
is still a mystery to explore

# PROCESS DEPENDENCE

# PROCESS DEPENDENCE OF THE SIVERS FUNCTION

- First experimental hints on the sign change, W/Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



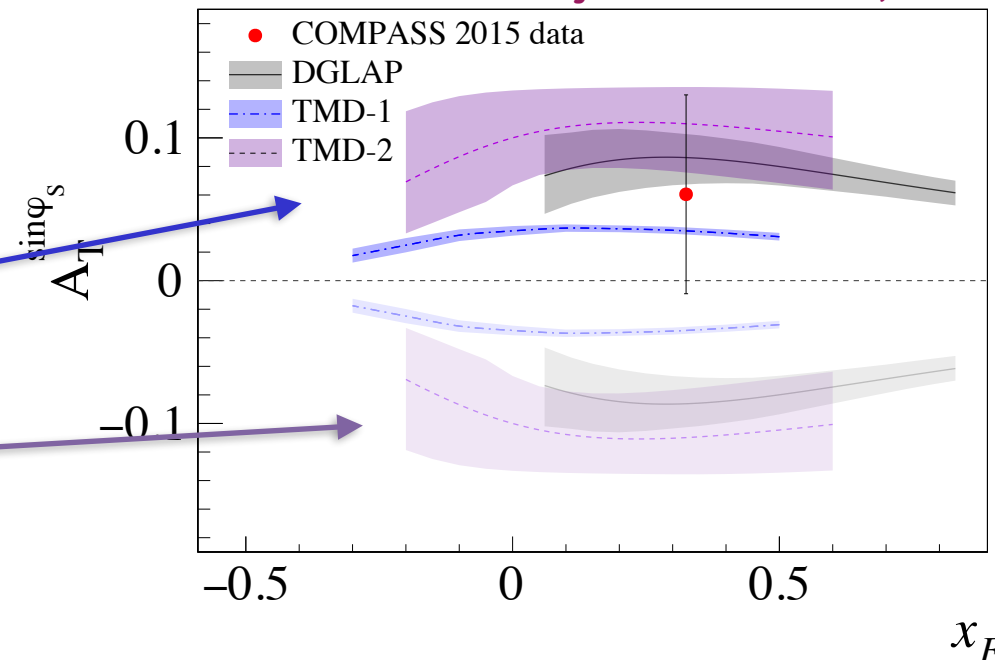
$$p^\uparrow p \rightarrow W^\pm X$$

KQ → Kang, Qiu '09

- Pion induced Drell-Yan

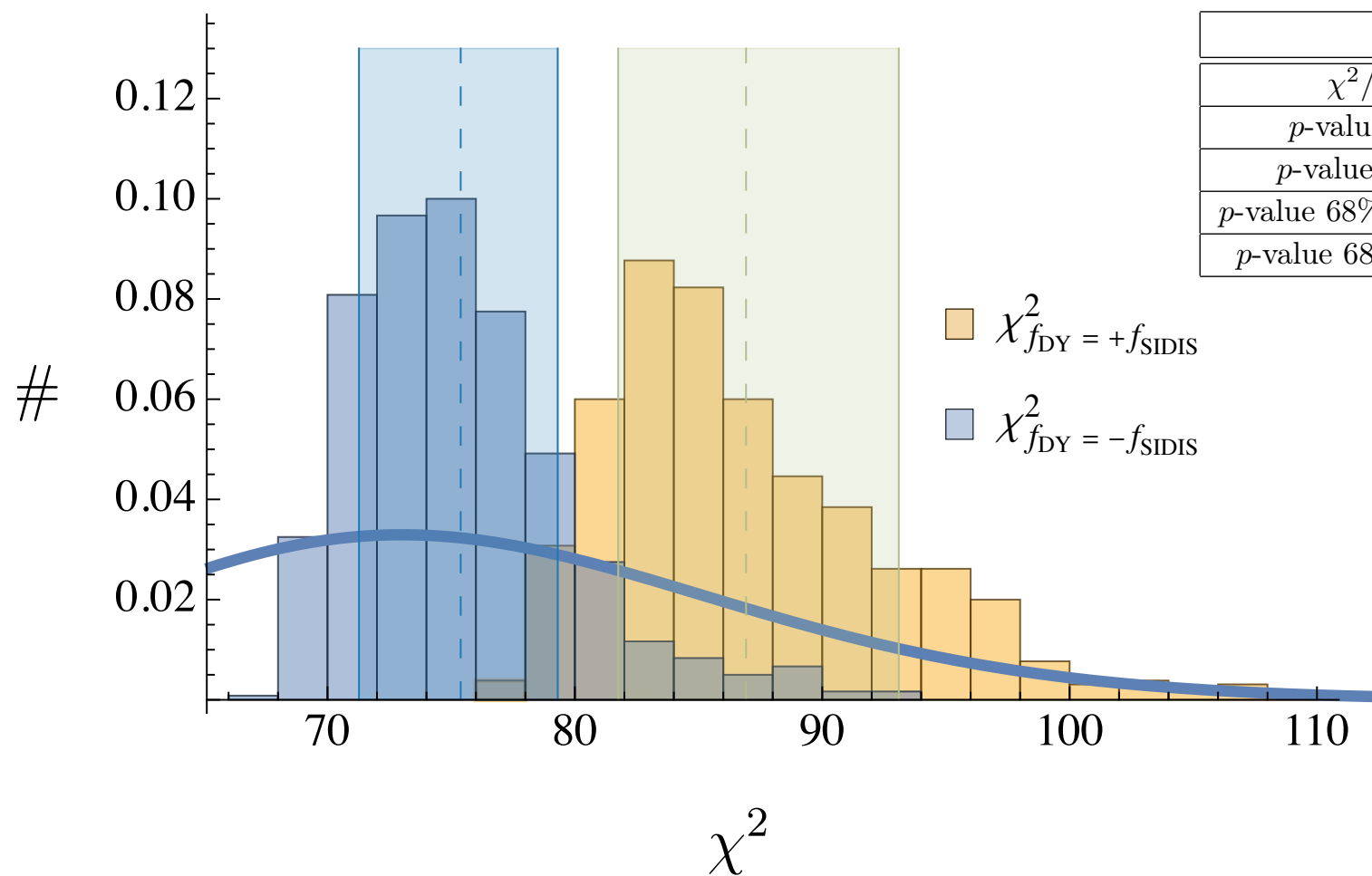
Sign change  
 No sign change

COMPASS Collab. Phys. Rev. Lett. 119, 112002 (2017)



# SIGN CHANGE

Bury, AP, Vladimirov (2021)



	$f_{1T[DY]}^\perp = -f_{1T[SIDIS]}^\perp$	$f_{1T[DY]}^\perp = +f_{1T[SIDIS]}^\perp$
$\chi^2/N_{pt}$	$0.88^{+0.16}_{+0.06}$	$1.00^{+0.22}_{+0.08}$
$p$ -value (CF)	0.74	0.44
$p$ -value 68%CI	[0.60, 0.34]	[0.28, 0.08]
$p$ -value 68%CI (SIDIS)	[0.67, 0.42]	[0.53, 0.11]
$p$ -value 68%CI (DY)	[0.56, 0.17]	[0.68, 0.02]

Large contribution from antiquark Sivers functions to DY makes it possible to describe data without the sign change

$$f_{1T}^{\perp sea} \rightarrow -f_{1T}^{\perp sea}$$

SpinQuest data may prove important to constraint sea-quark functions

# CONCLUSIONS

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- We have extracted Sivers function from the first global fit of SIDIS, pion-induced Drell-Yan and  $W^\pm/Z$  production experimental data at N3LO precision
- Conservative data cuts are used to ensure validity of factorization and unbiased parametrization
- Good agreement between SIDIS and DY data in an analysis with TMD evolution is achieved for the first time
- The Qiu-Sterman functions are extracted in a model independent way
- Our results set a new benchmark and the standard of precision for studies of TMD polarized functions

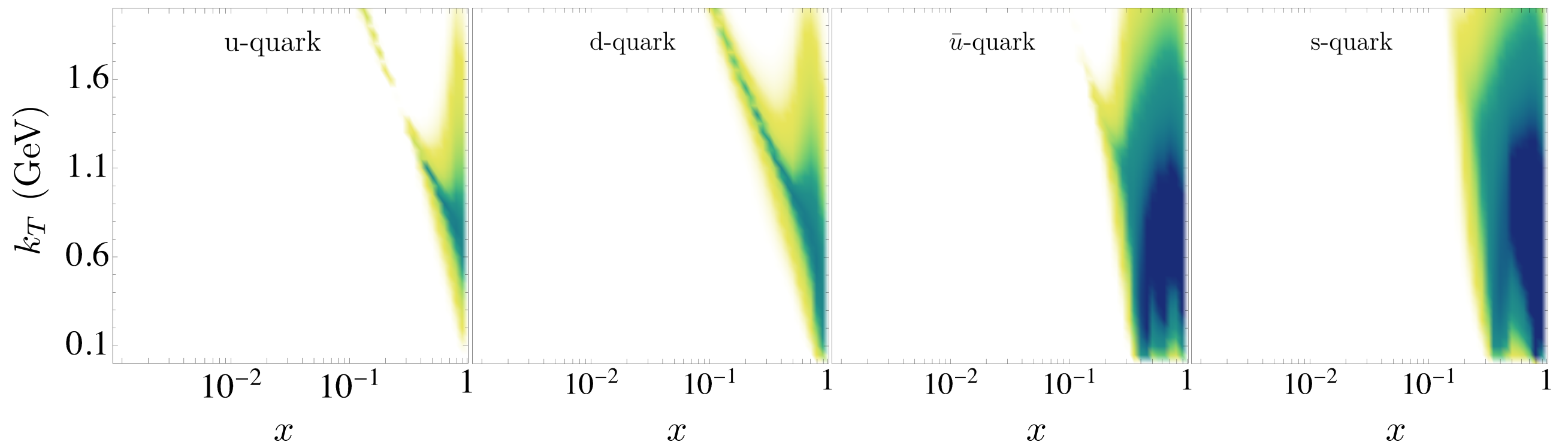


# BACKUP

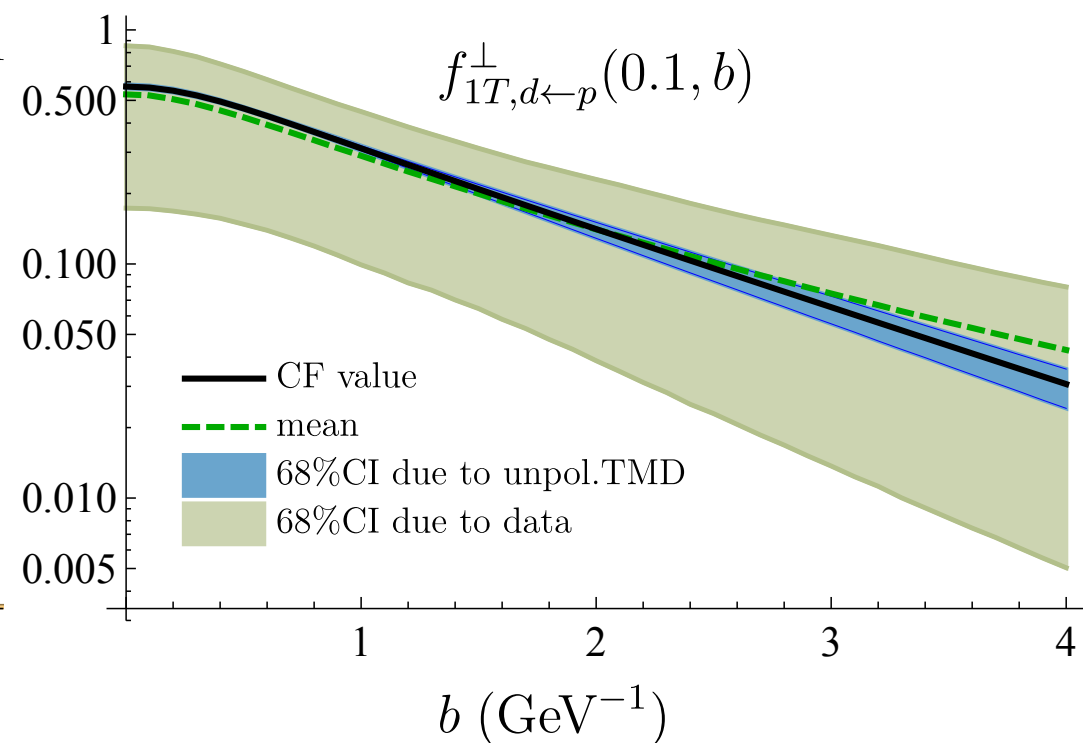
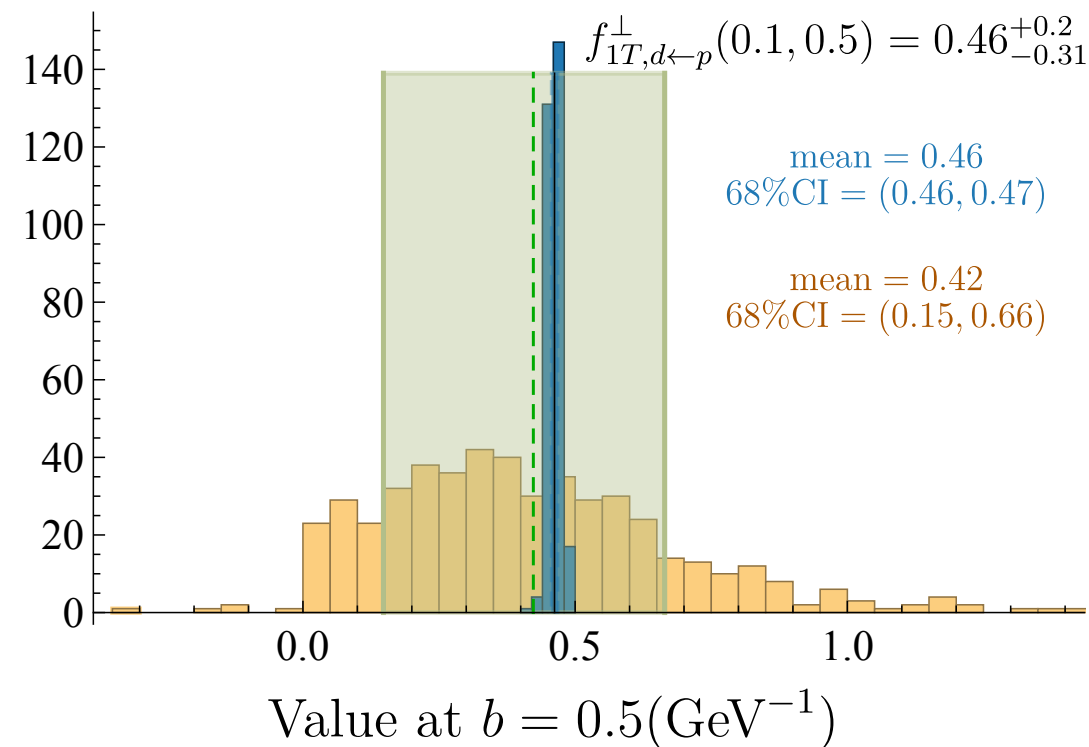


# POSITIVITY

$$\frac{k_T^2}{M^2} (g_{1T}(x, k_T)^2 + f_{1T}^\perp(x, k_T)^2) \leq f_1(x, k_T)^2,$$



# ERROR PROPAGATION



- ▶ Uncertainties estimated by replica method
  - ▶ Fitting 300 replicas of pseudo data
- ▶ Large and (often) asymmetric uncertainties
- ▶ Uncertainty due to unpol.TMD are non-negligible but much smaller than due to data