# - TMDs -
# a transverse look at hadrons

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UVA/SpinQuest - Nuclear Physics Seminar

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# Outline



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# Quantum Chromodynamics

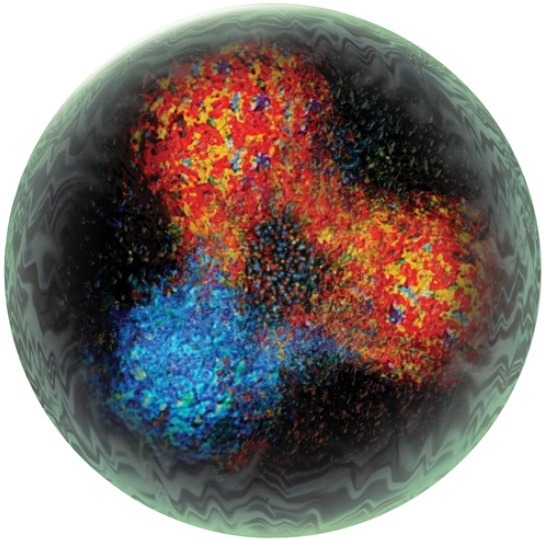
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a,\,\mu\nu} F^{a}_{\mu\nu} + \overline{\psi}(i\not{D} - m)\psi \ , \qquad D_\mu = \partial_\mu - igT^a A^a_\mu$$

QCD: quarks and gluons are the elementary degrees of freedom, but they manifest only in bound states $\rightarrow$ hadrons

Hadrons have different:

- mass
- spin
- size

Can we explain their properties from the QCD Lagrangian $\mathcal{L}_{QCD}$?

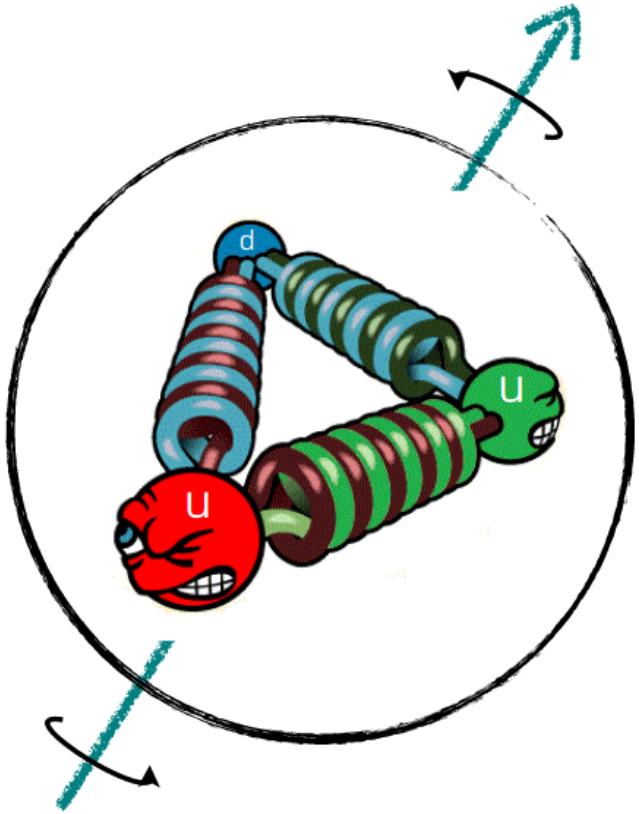


# Confinement


how does confinement work ?

What is the mechanism that keeps quarks and gluons bound inside hadrons?

# Spin and mass

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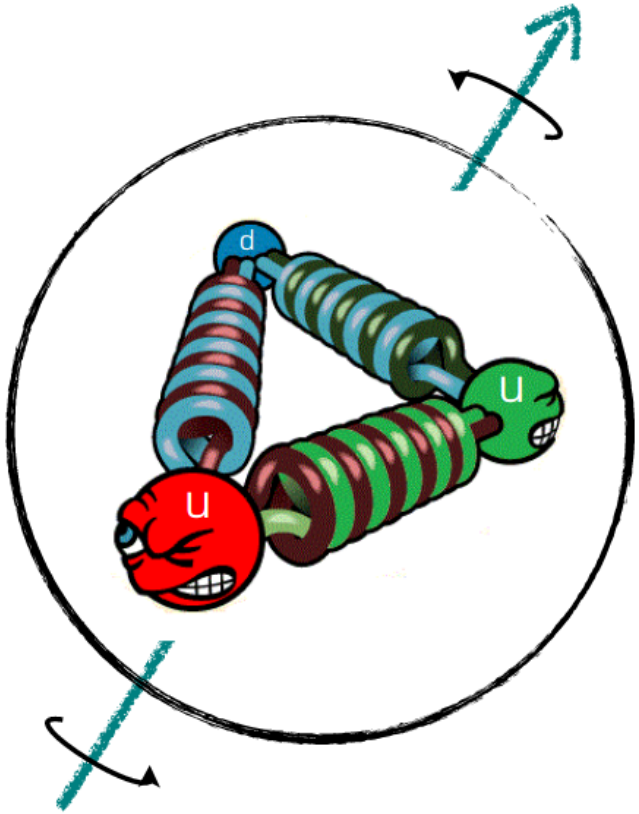


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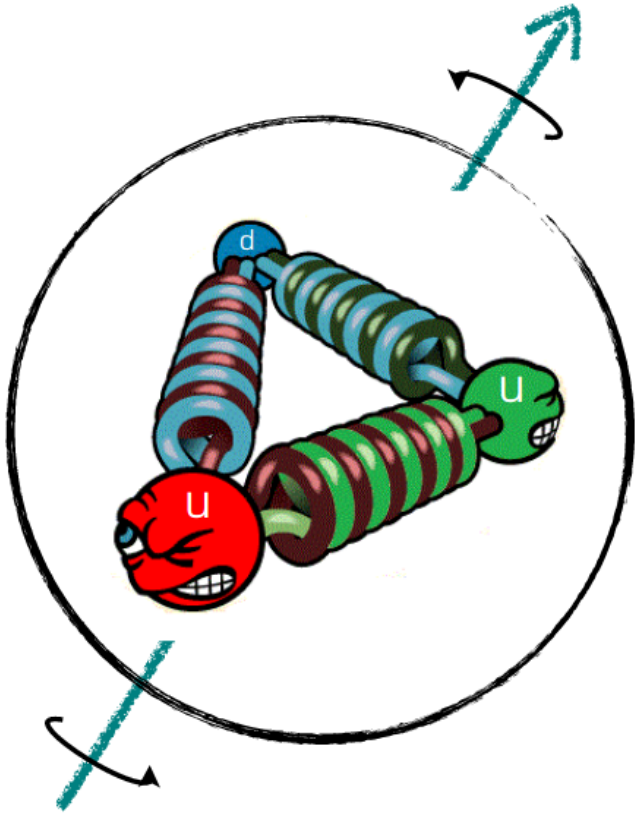


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What about the
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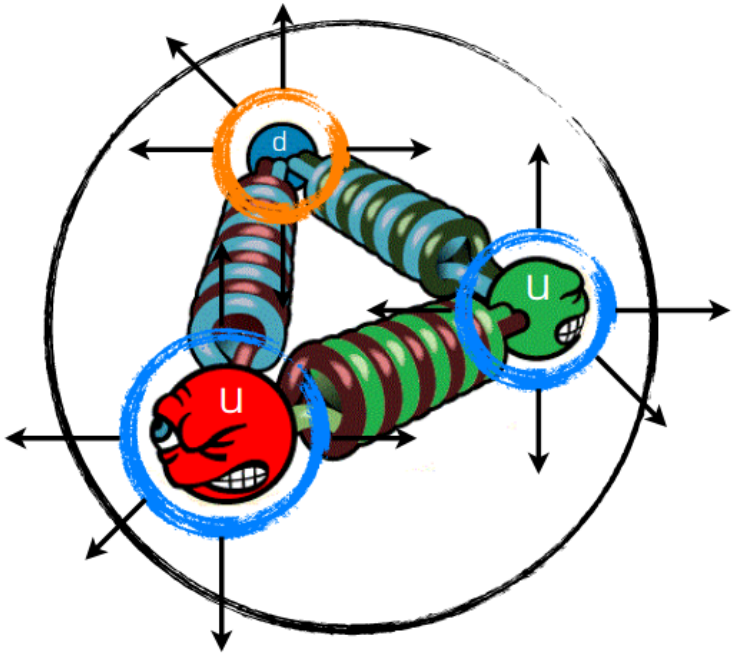
- spin $1/2$ = quark/gluon spin + quark/gluon orbital motion ?
- $M_p \sim 1$ GeV $\gg 3 \times m_q \sim$ a few MeV

# Tomography



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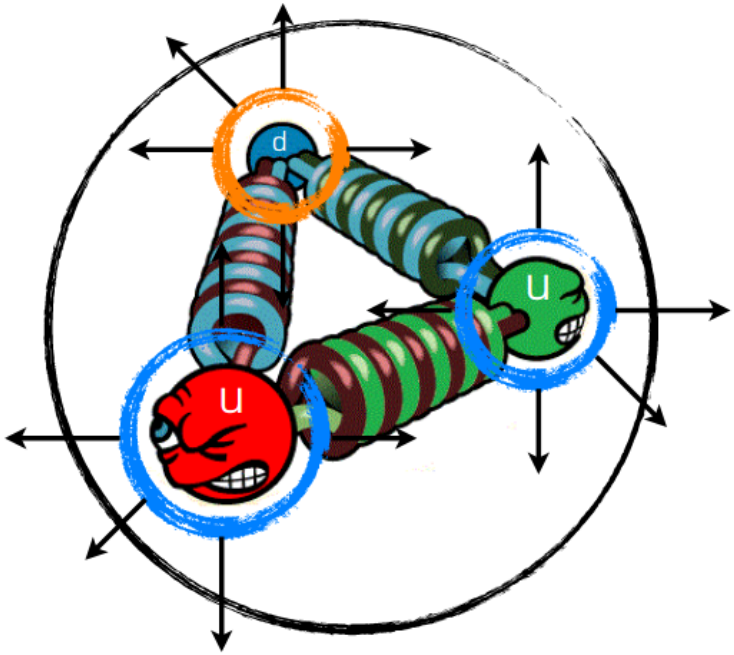


How are quarks and gluons distributed inside hadrons?

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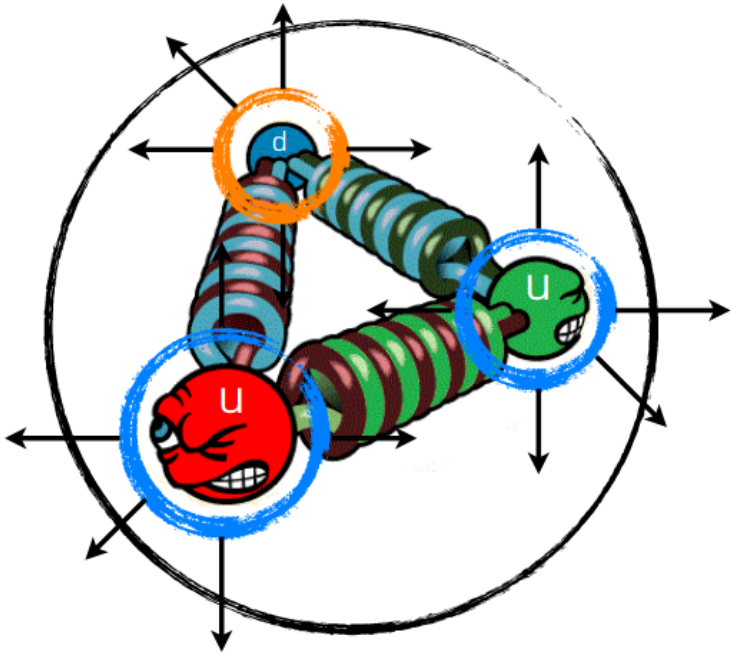


How are quarks and gluons distributed inside hadrons?

What is the impact of 3D hadron structure in high-energy physics?

# How should we "use" QCD?

Expansion of an observable $\mathcal{O}$ in powers of the coupling constant $\alpha_s(Q)$:

$$\mathcal{O}(Q) \sim \mathcal{O}^{(0)} + \alpha_s^1(Q)\,\mathcal{O}^{(1)} + \alpha_s^2(Q)\,\mathcal{O}^{(2)} + \alpha_s^3(Q)\,\mathcal{O}^{(3)} + \ldots = ??$$

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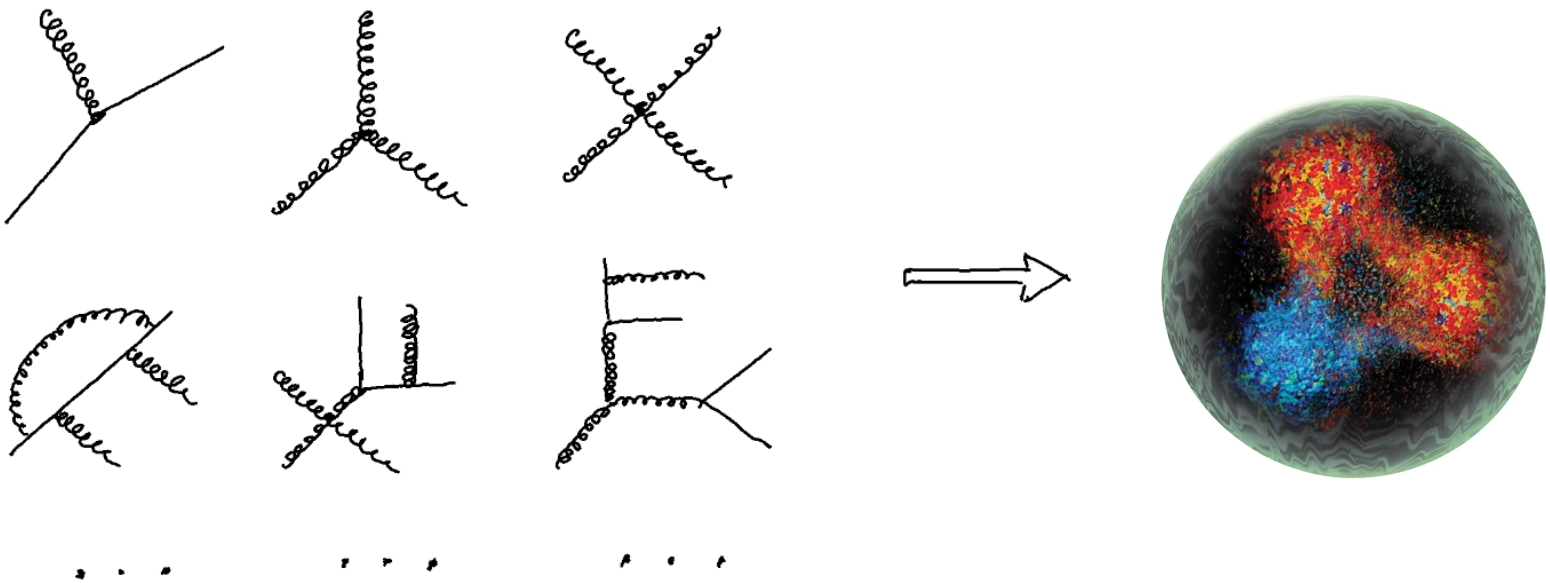

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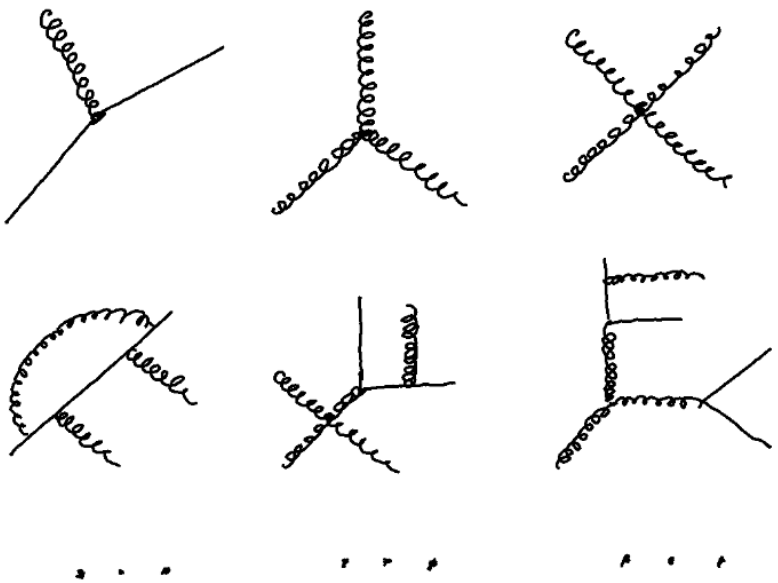

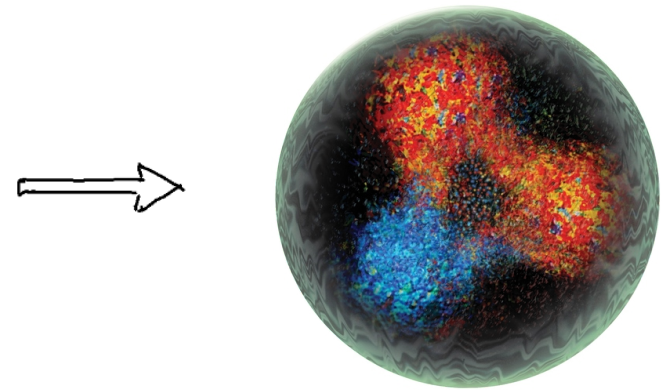

$$Q = M_{p/n} \sim 1 \text{ GeV}$$

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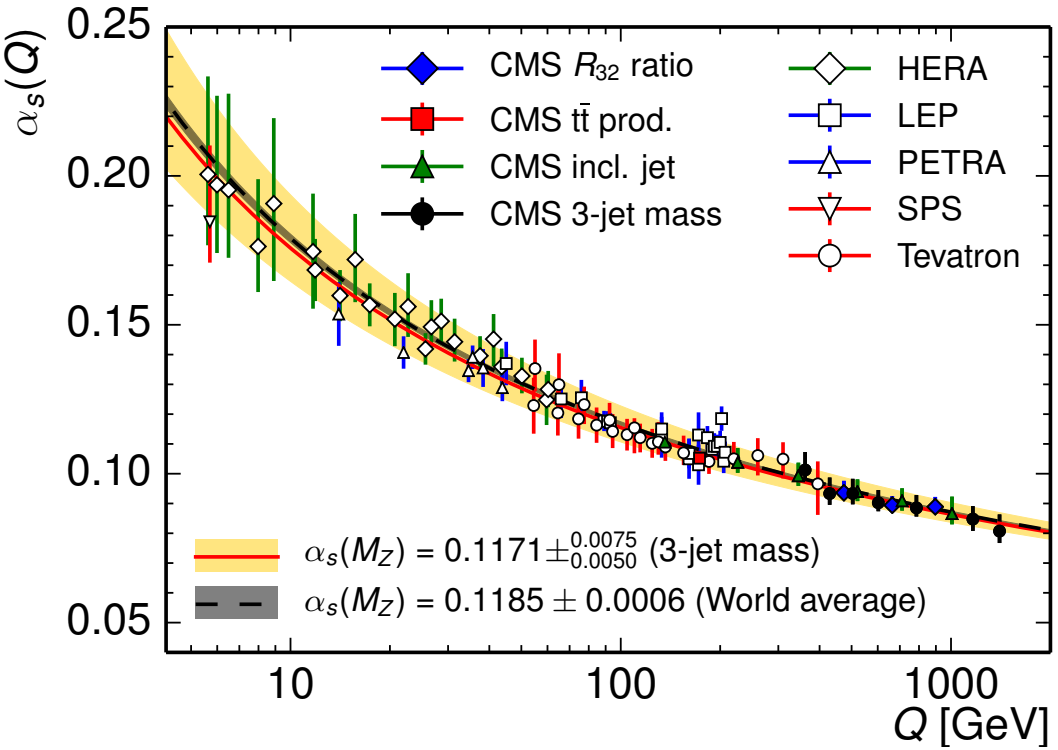

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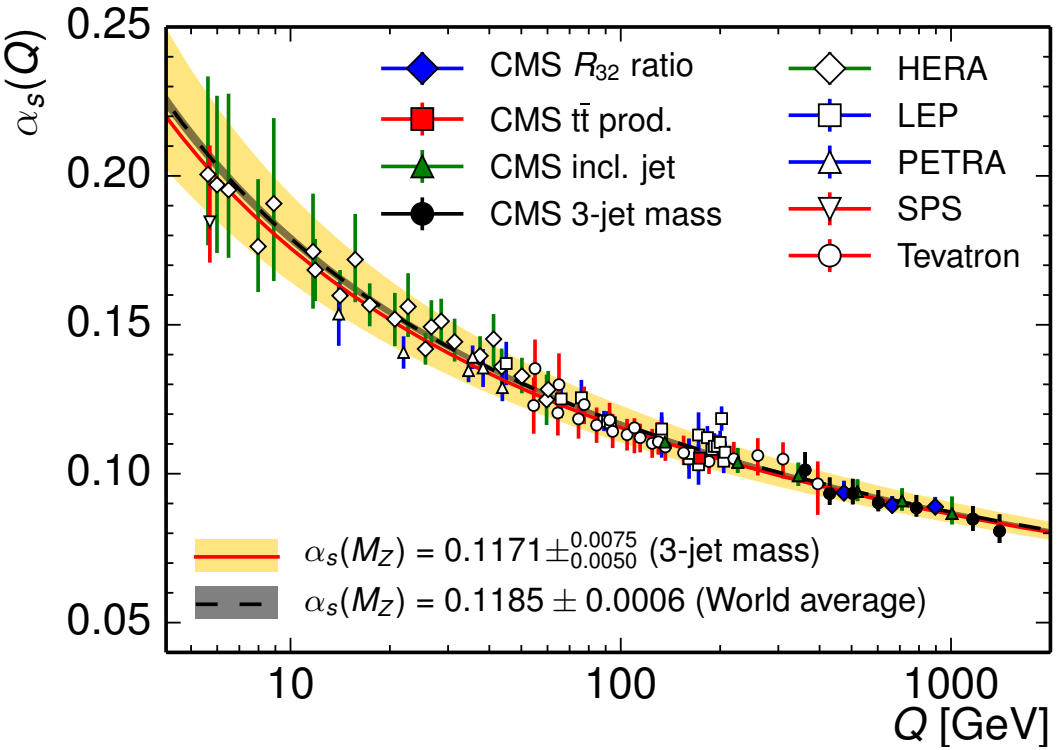


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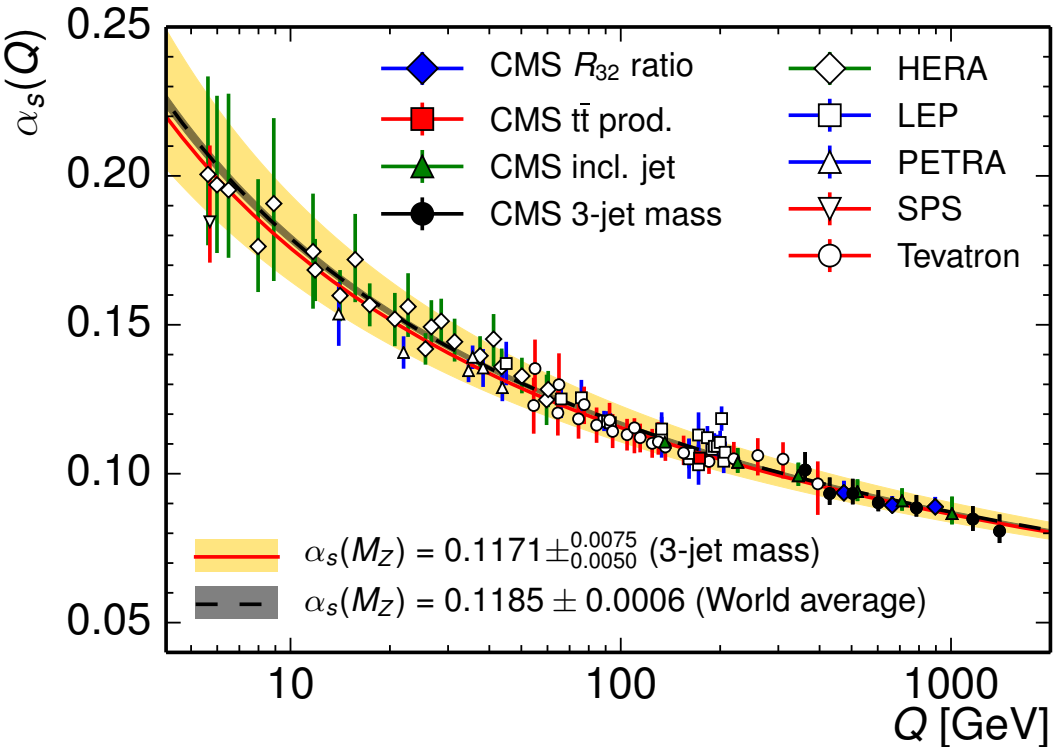


high energy $Q \rightarrow$ small $\alpha_s(Q) \rightarrow$ convergence $\rightarrow$ "perturbative" QCD

low energy $Q \sim M_{p/n} \sim 1\,\text{GeV} \rightarrow$ "non-perturbative" QCD

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# Hadron structure and hadronization

Hadron physics $\leftrightarrow$ (non-)perturbative QCD

Hadron tomography $\rightarrow$ two "macro areas":

- hadron structure: hadron $\rightarrow$ quark/gluon transition
- hadronization: hadron $\leftarrow$ quark/gluon transition

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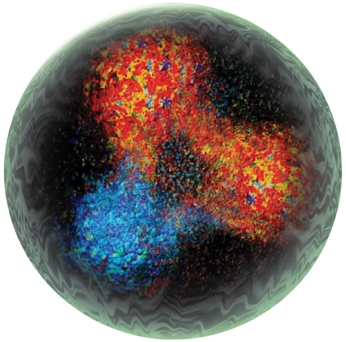
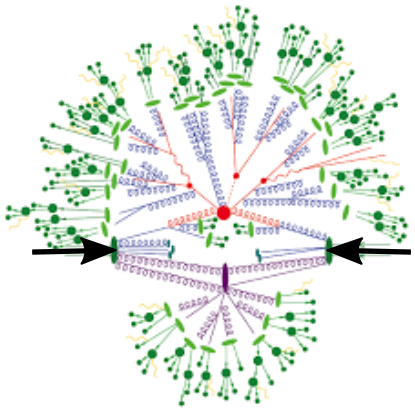
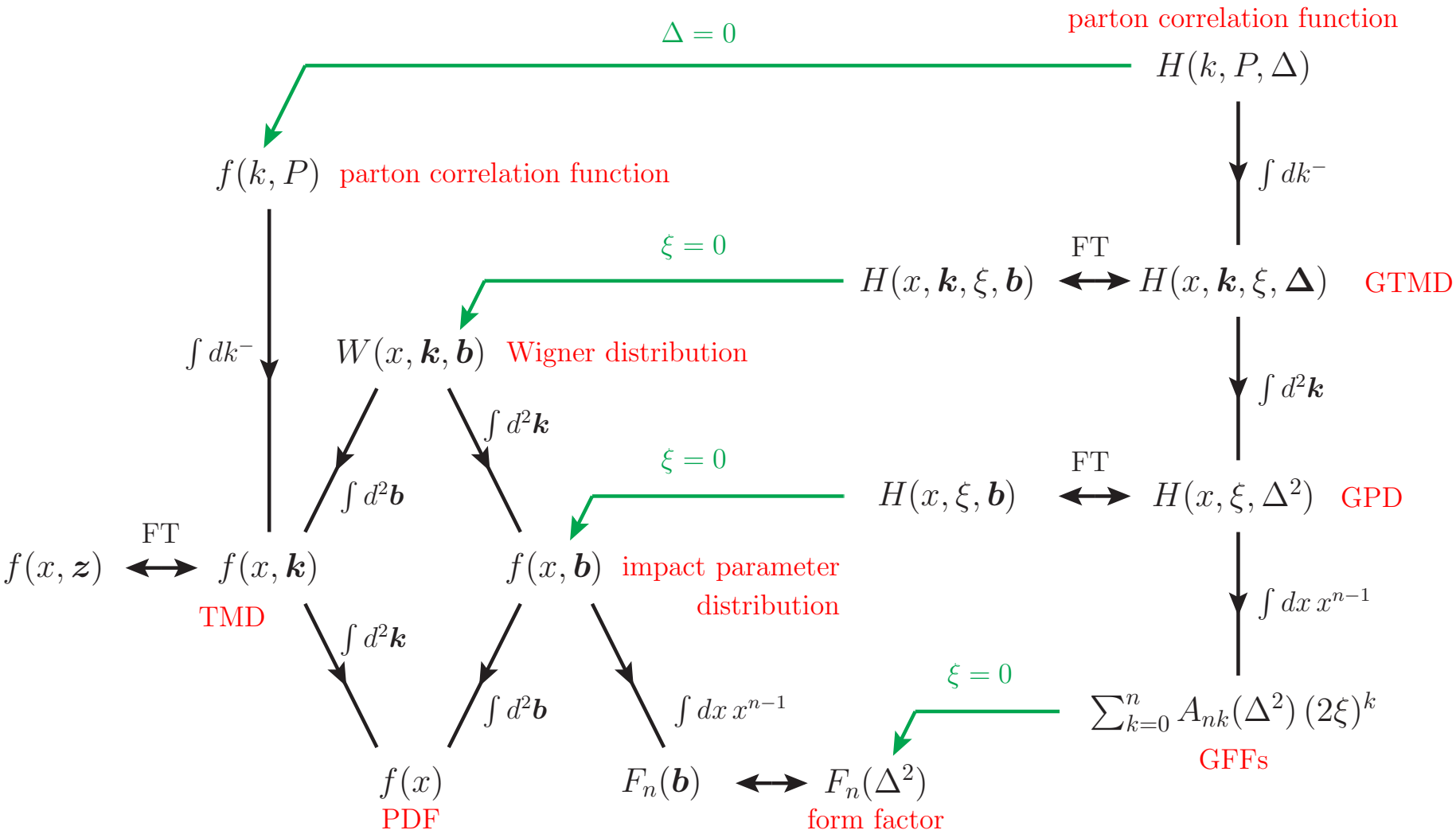
The motivations are:

- conceptual: understand confinement, mass generation, etc.
- practical: improve our understanding of scattering experiments





# Hadron structure: landscape




credit picture: M. Diehl - [1512.01328]

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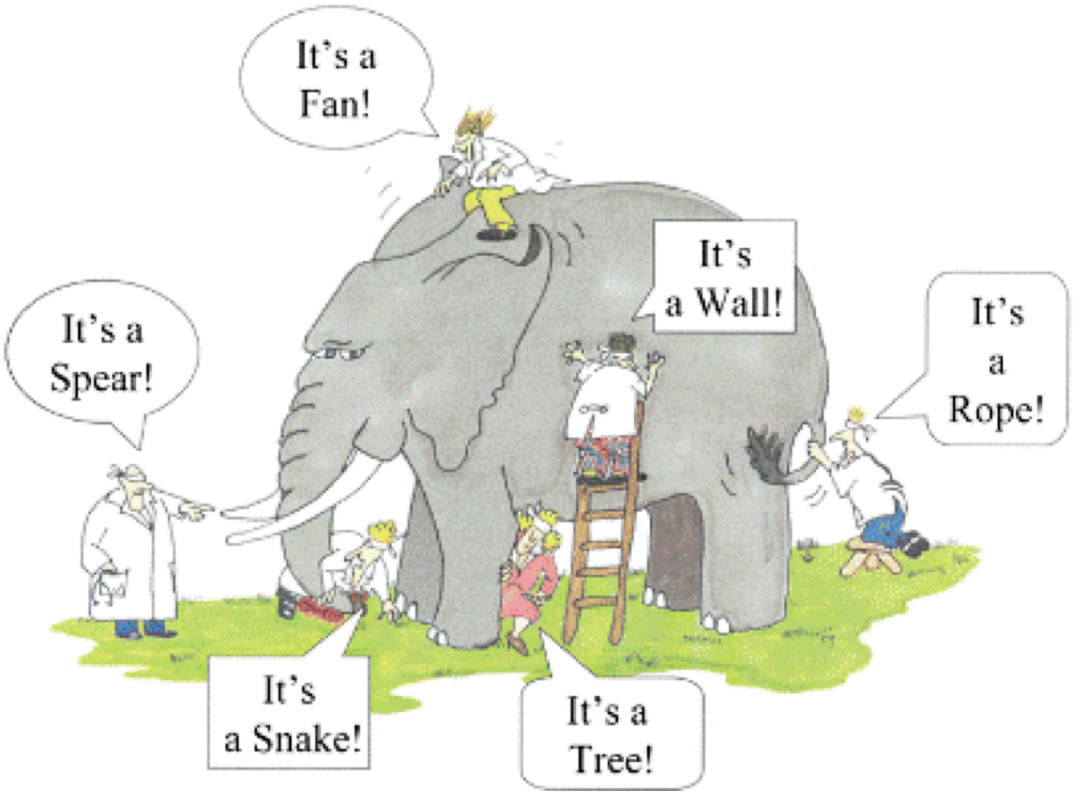
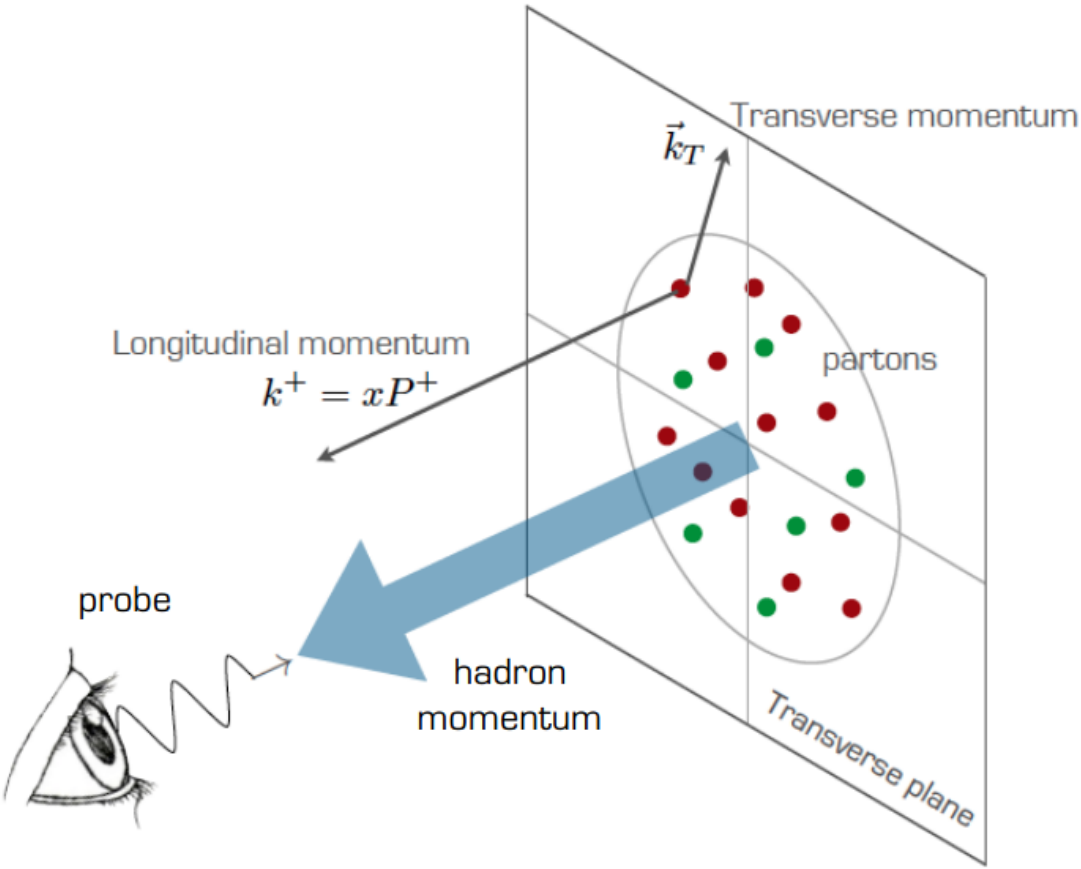



Global knowledge by combining different "projections"

# Hadron structure: PDFs

Tomography: mapping the interal structure of hadrons in terms of parton distribution functions (PDFs)

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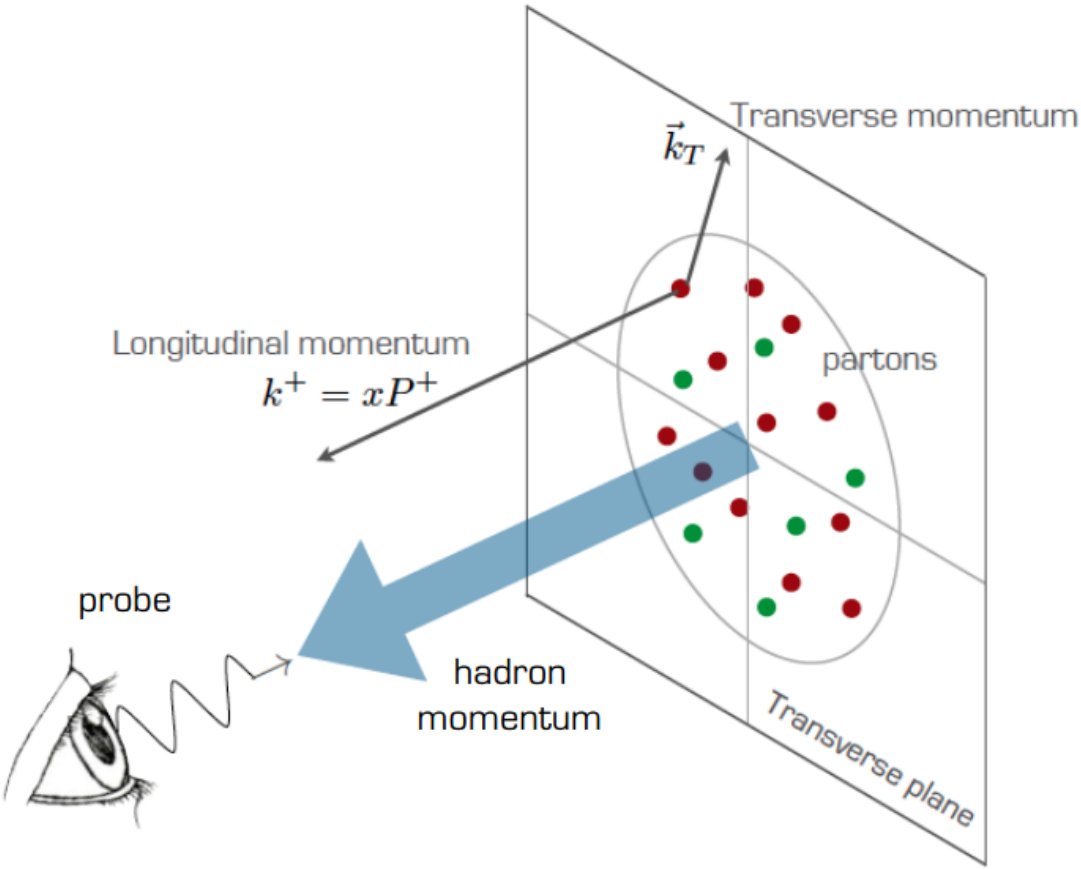


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Momentum distributions:

$f_1^{a/h}(x)$: collinear PDF
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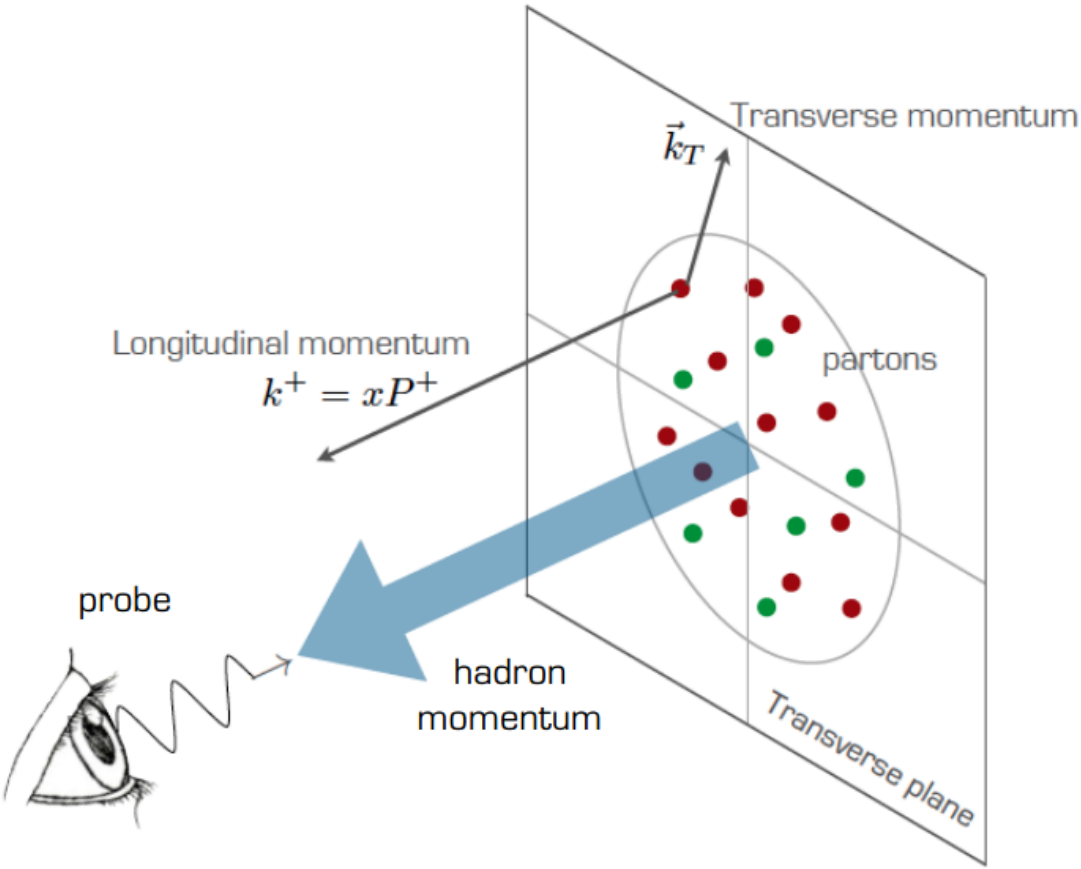


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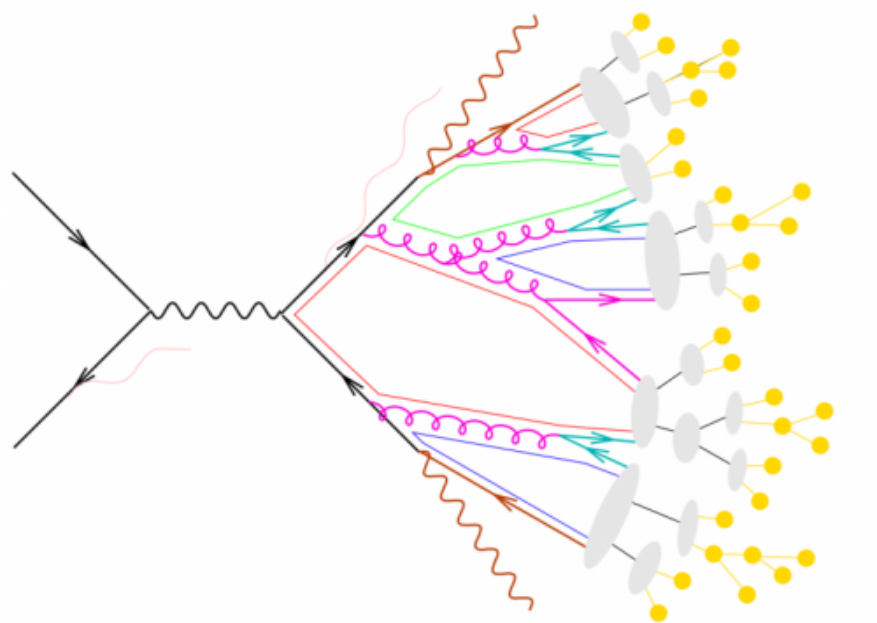
Momentum distributions:

$f_1^{a/h}(x)$: collinear PDF
hadron structure in 1D

$f_1^{a/h}(x, k_T^2)$: TMD PDF
hadron structure in 3D

# Hadronization: FFs

Hadronization: dynamical generation of hadrons from quarks/gluons



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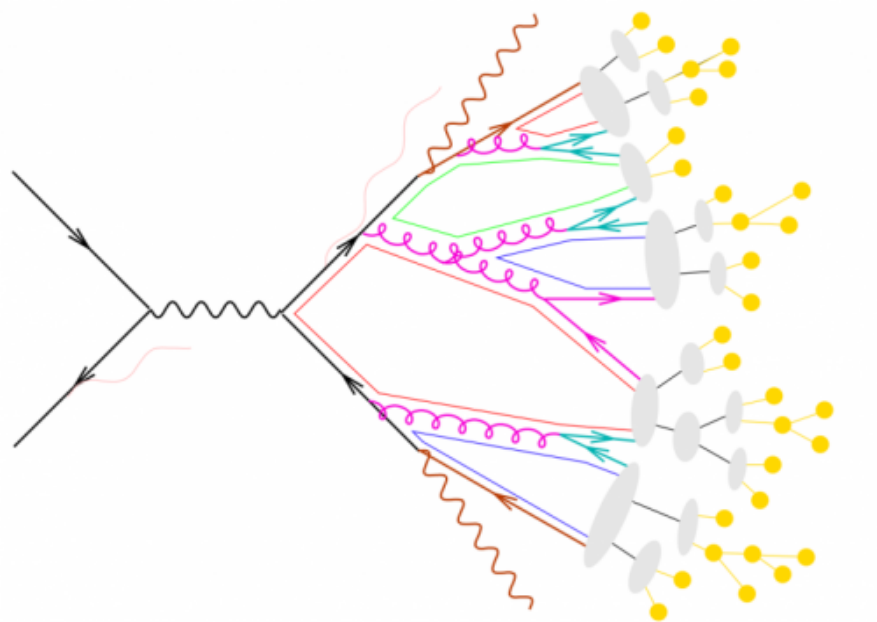
It follows any QCD hard scattering event populating the final states with hadrons.



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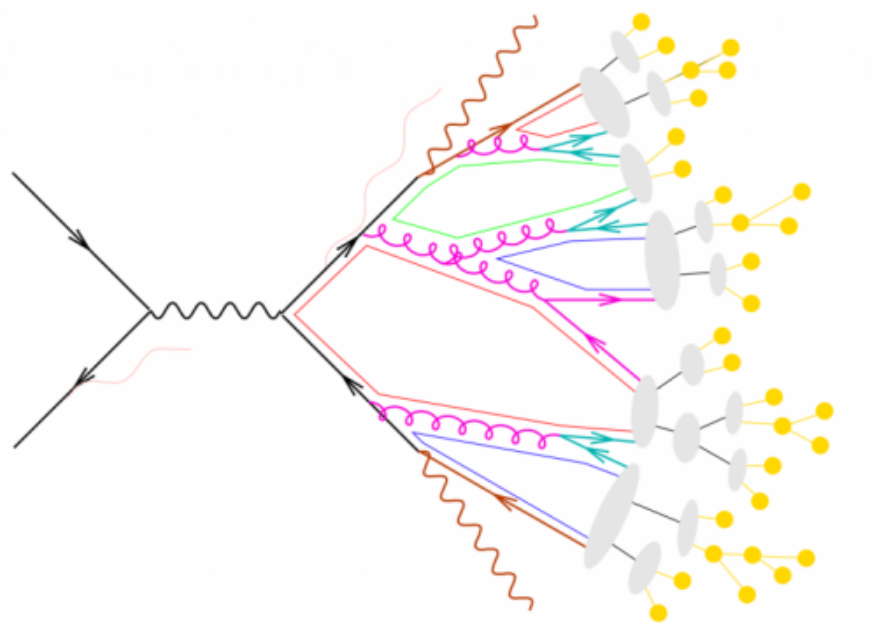
Maps of the hadronization process in momentum space: fragmentation functions (FFs)



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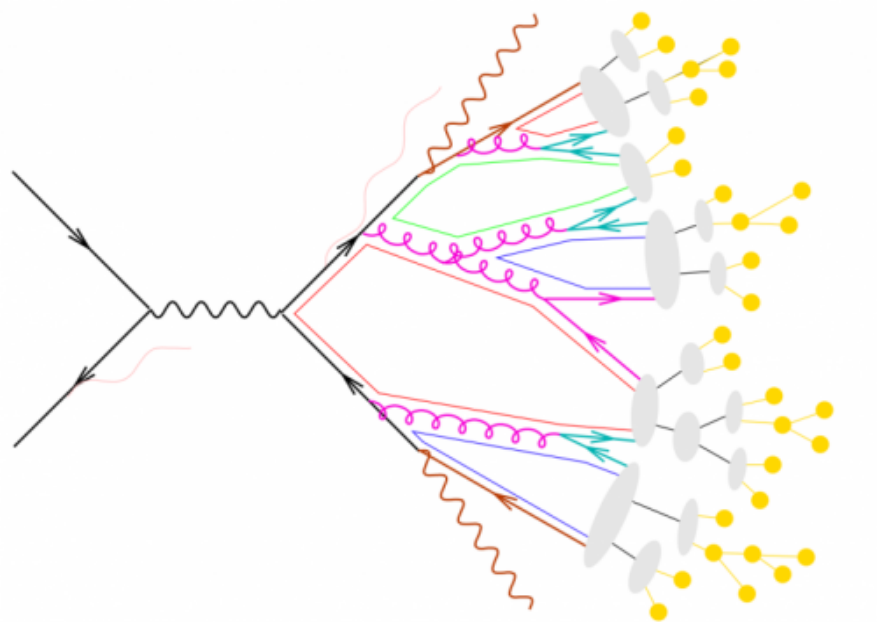

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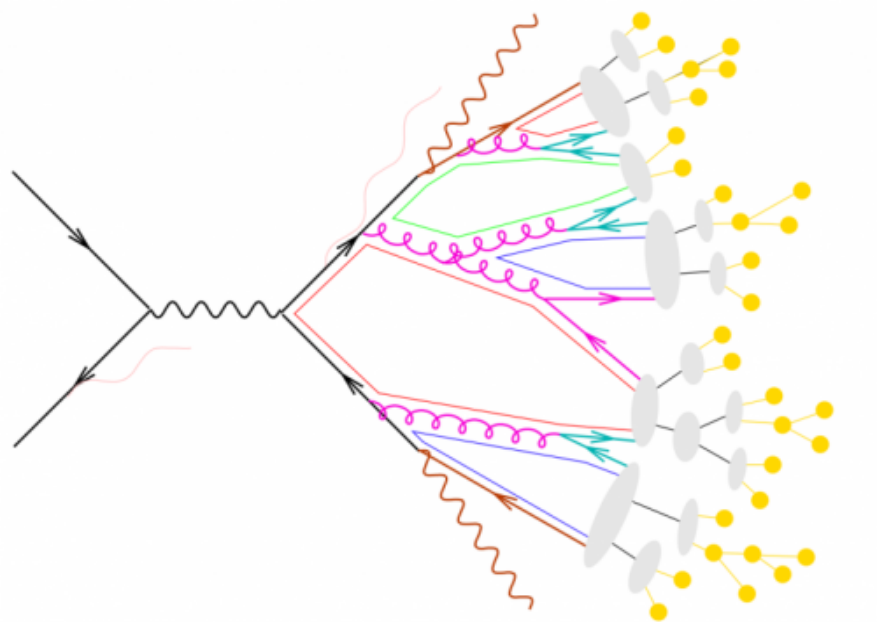
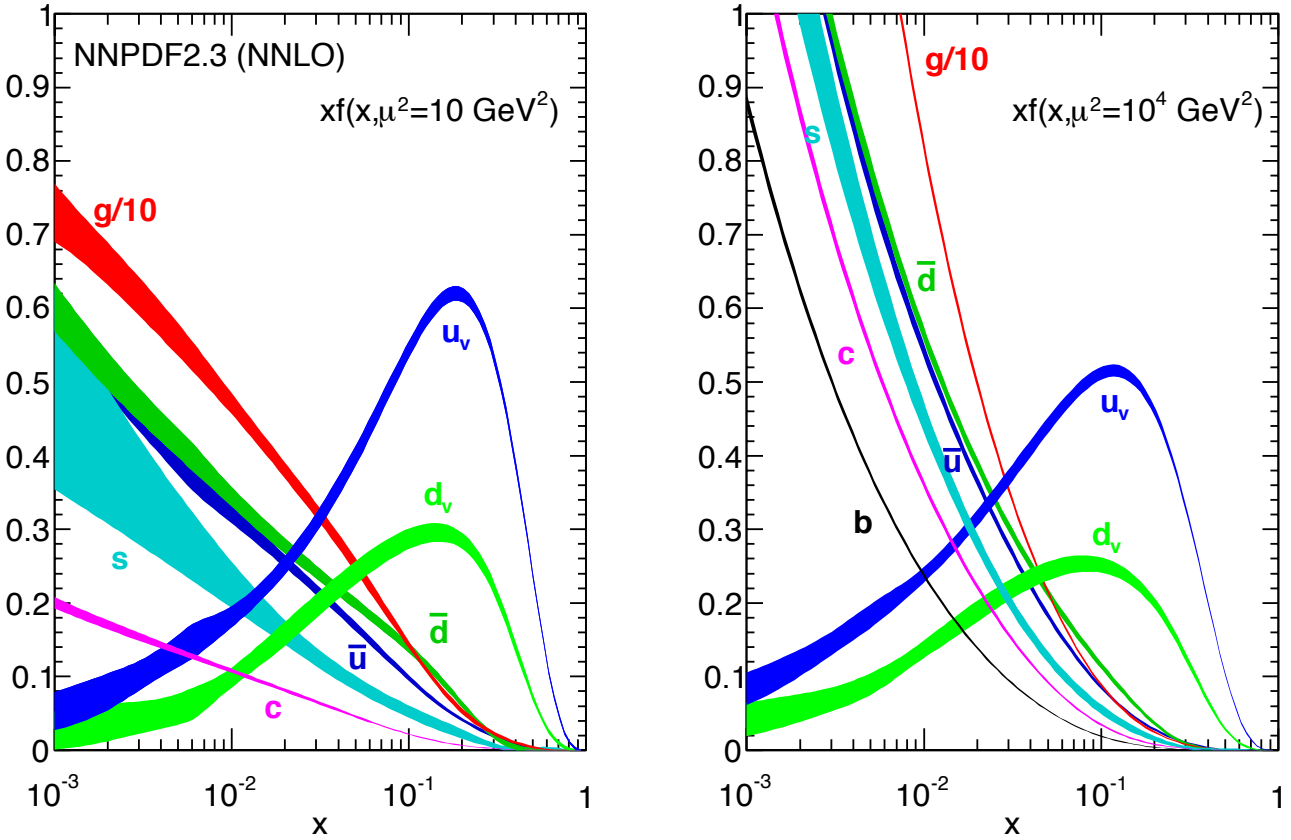


$D_1^{a\to h}(z)$: collinear FF hadronization in 1D

$D_1^{a\to h}(z, P_T^2)$: TMD FF hadronization in 3D

# PDFs: 1D structure

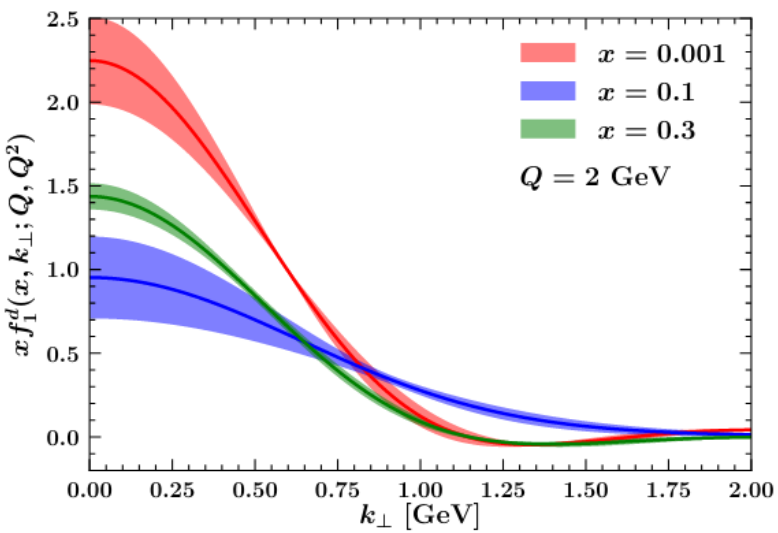
$f_1^{a/h}(x, Q)$: quark/gluon with a momentum fraction $x$ wrt parent hadron at the resolution scale $Q$

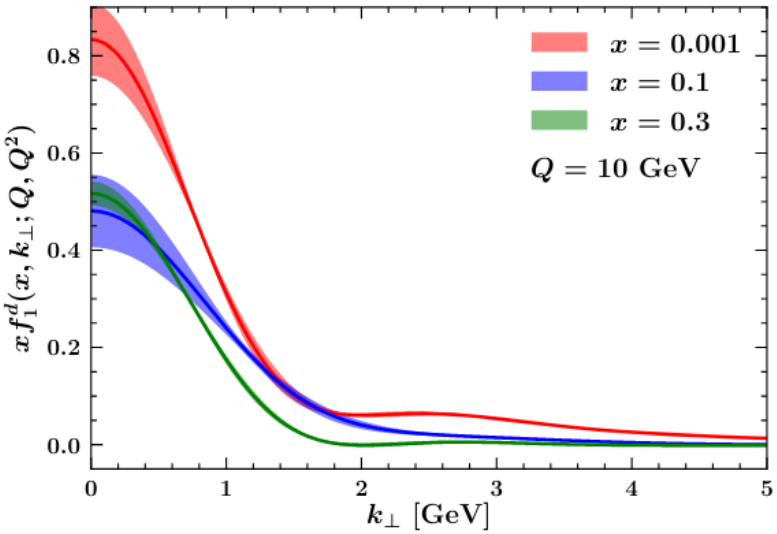



hadron structure in terms of quarks/gluons in 1D momentum space

# TMD PDFs: 3D structure

$f_1^{a/h}(x, k_T^2, Q)$: quark/gluon with a momentum fraction $x$ and transverse momentum $k_T$ at the resolution scale $Q$




hadron structure in terms of quarks/gluons in 3D momentum space
[from PV19 extraction - 1912.07550]

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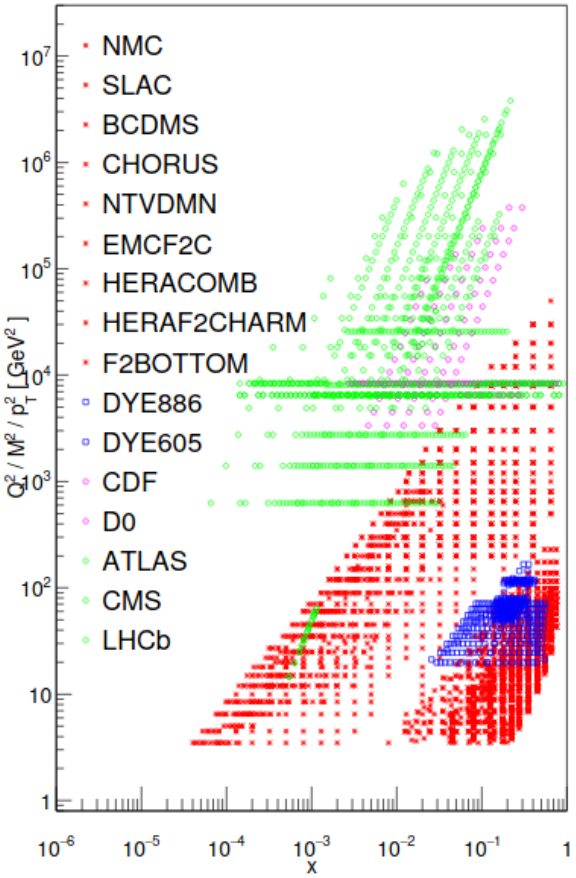


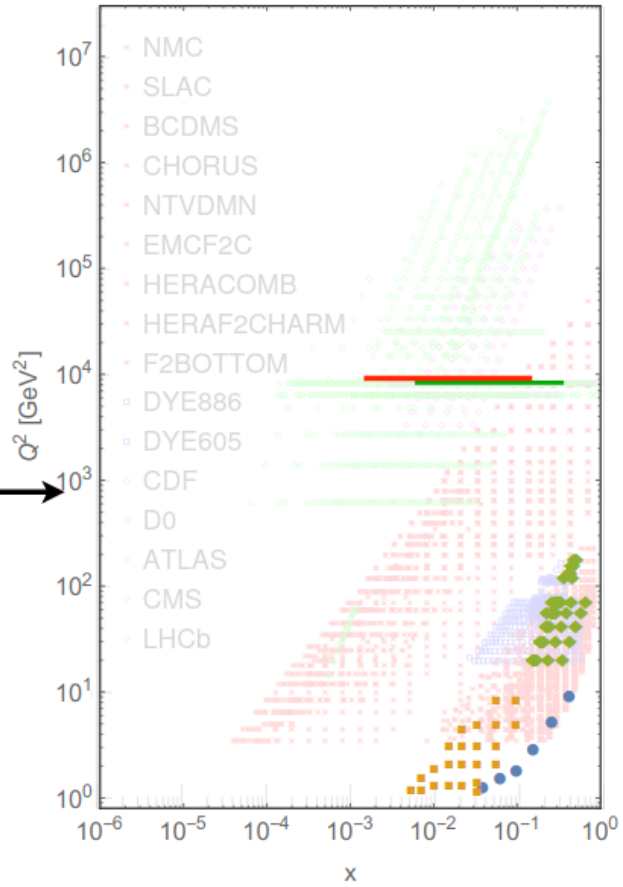

data driven science

**data sets available:**

**collinear PDFs**

**vs**

**TMD PDFs**



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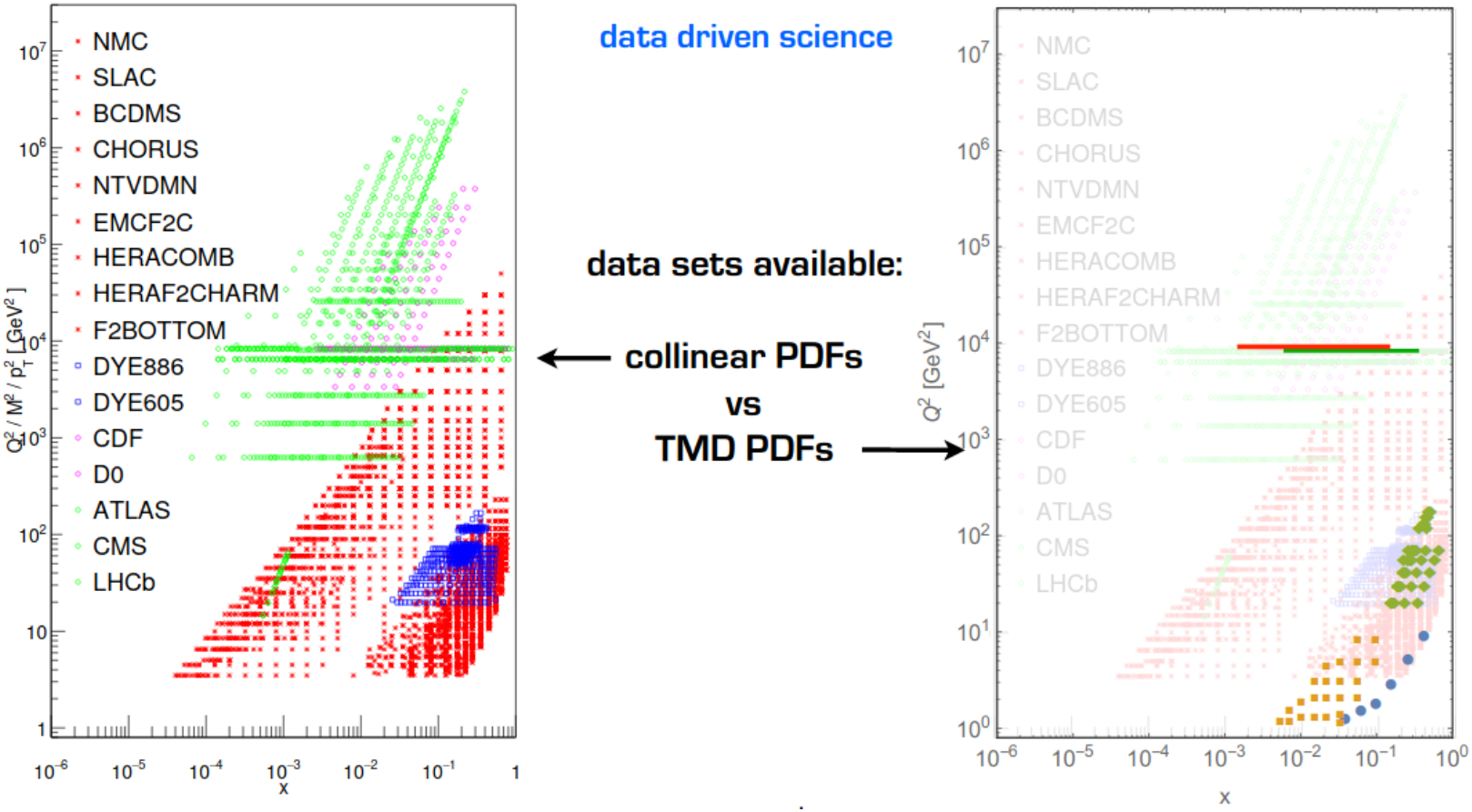


we definitely need more experimental information → need for new experiment(s)! credit picture: E. Nocera

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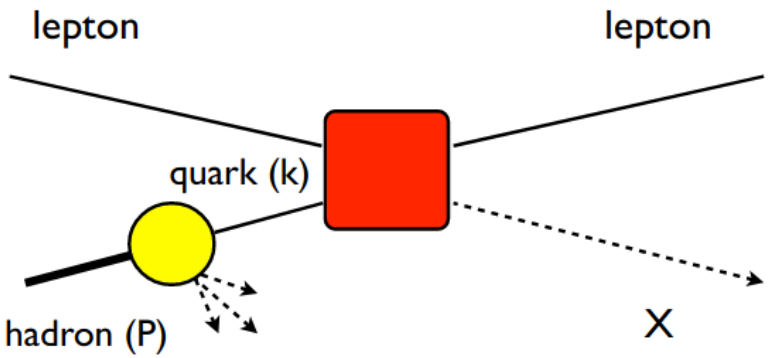
# Operator definition

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Parton Distribution Function - PDF

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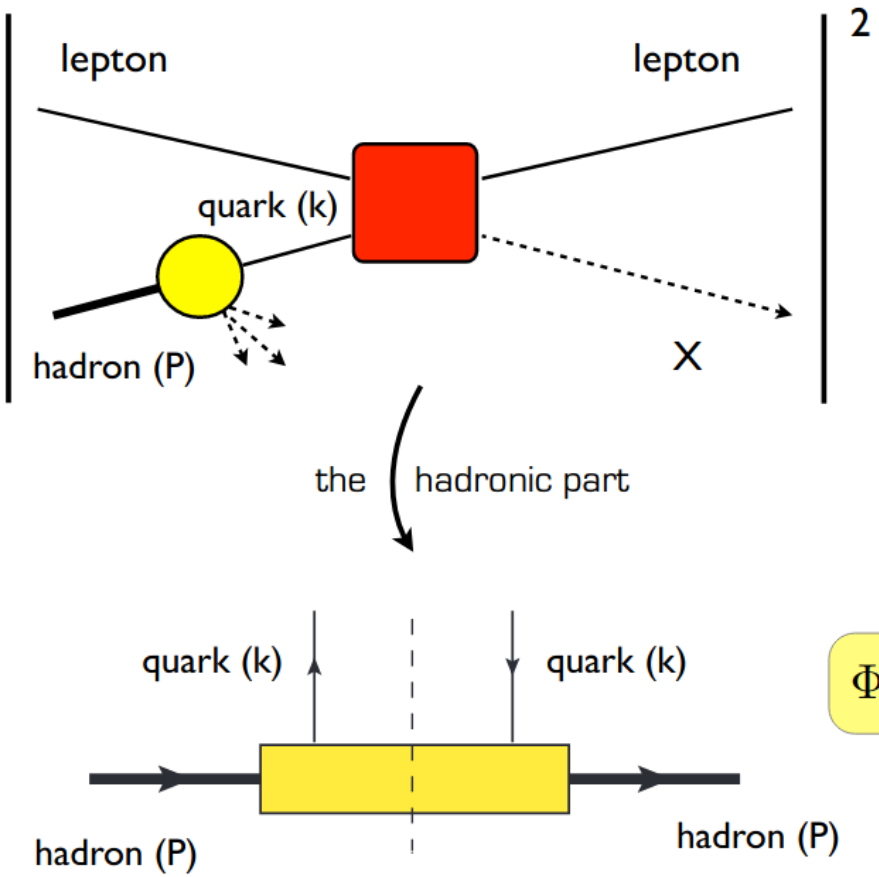


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Parton Distribution Function - PDF

hadronic part
described as a **quark-quark correlation** in space-time

$$\Phi(k,P) = \mathrm{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle$$

$\xi$ is the Fourier-conjugated of
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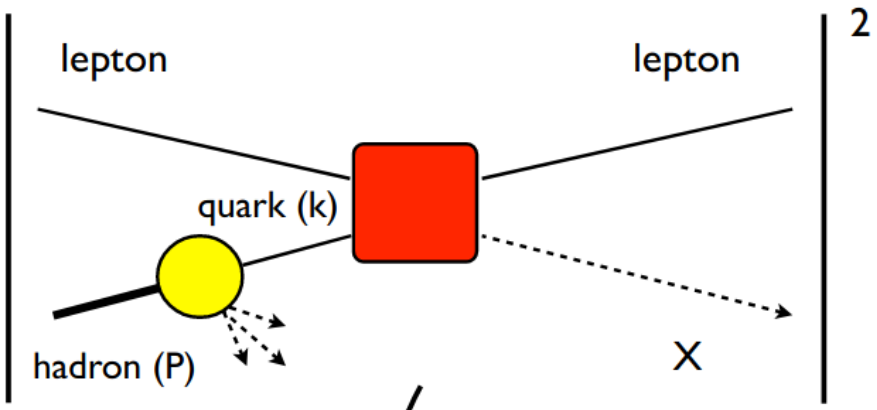


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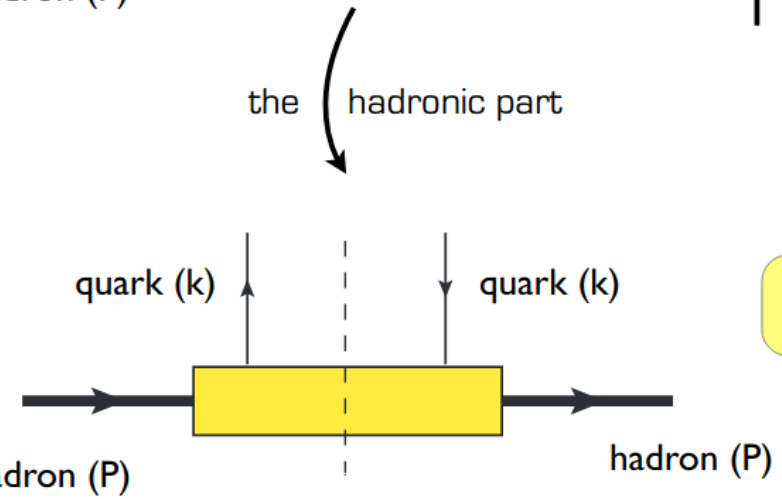


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Parton distributions as traces of $\Phi$: $f_{\dots}^{[U]}(x,k_T^2) \sim \mathrm{Tr}\big[\Phi\,\Gamma\big]\ ,\ \Gamma = \gamma^+\ ,\dots$

# TMD quark distribution functions

TMD PDFs for a quark in a spin 1/2 hadron (twist 2)

| nucleon pol. \ quark pol. | U | L | T |
|---|---|---|---|
| U | $f_1$ | | $h_1^\perp$ |
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Spin-1 target: additional distributions

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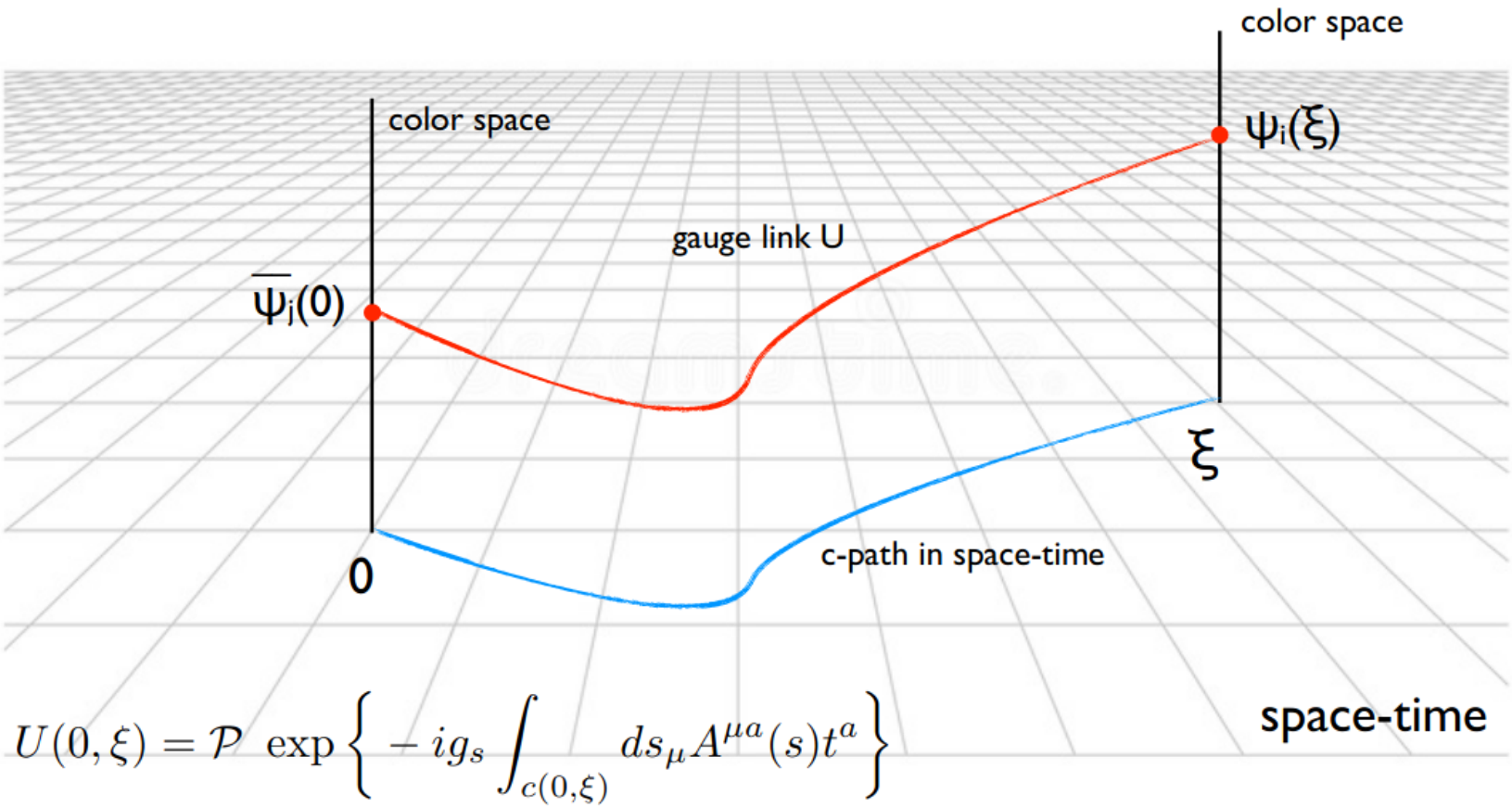
Spin-1 target: additional distributions
A similar table for gluons and for fragmentation functions

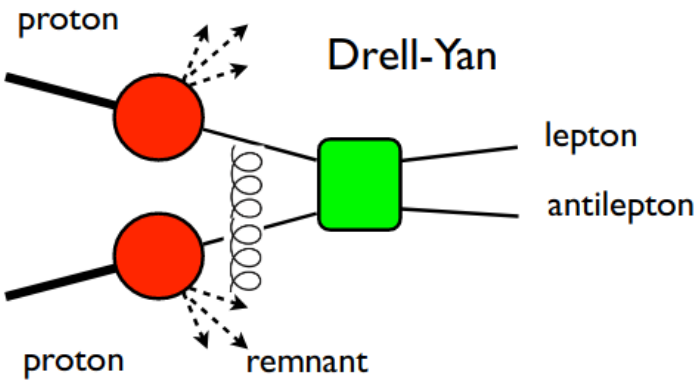
# Process dependence

$$\Phi(k,P) = \text{F.T.}\langle P|\overline{\psi_j}(0)\ U\ \psi_i(\xi)|P\rangle \longrightarrow f_1^{a\ [U]}(x,k_T^2)\ \not{P} + \cdots$$




$$U(0,\xi) = \mathcal{P}\ \exp\left\{-ig_s\int_{c(0,\xi)} ds_\mu A^{\mu a}(s)t^a\right\}$$

# Process dependence




In **Drell-Yan** the **remnant** of the proton feels the color force of a **quark** in the **initial state**

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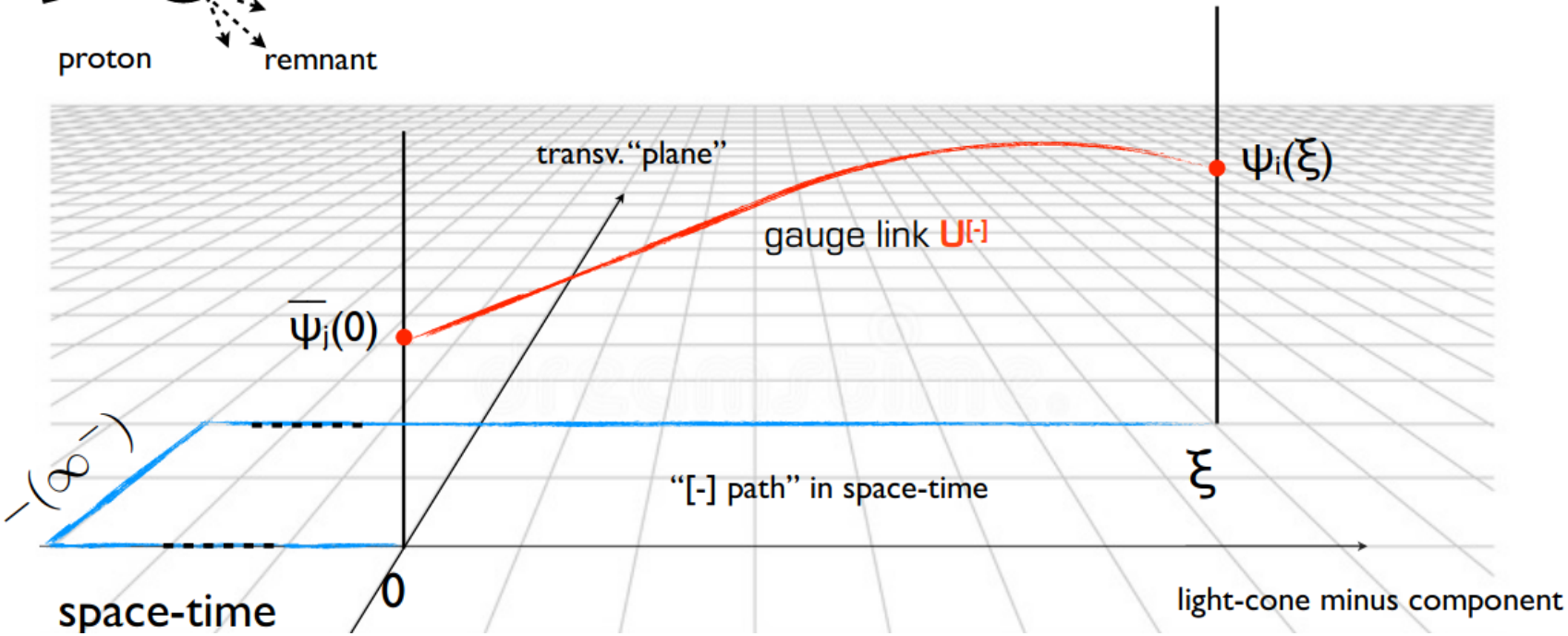

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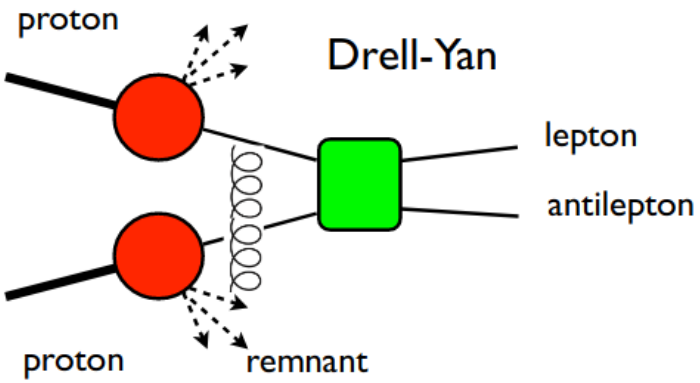


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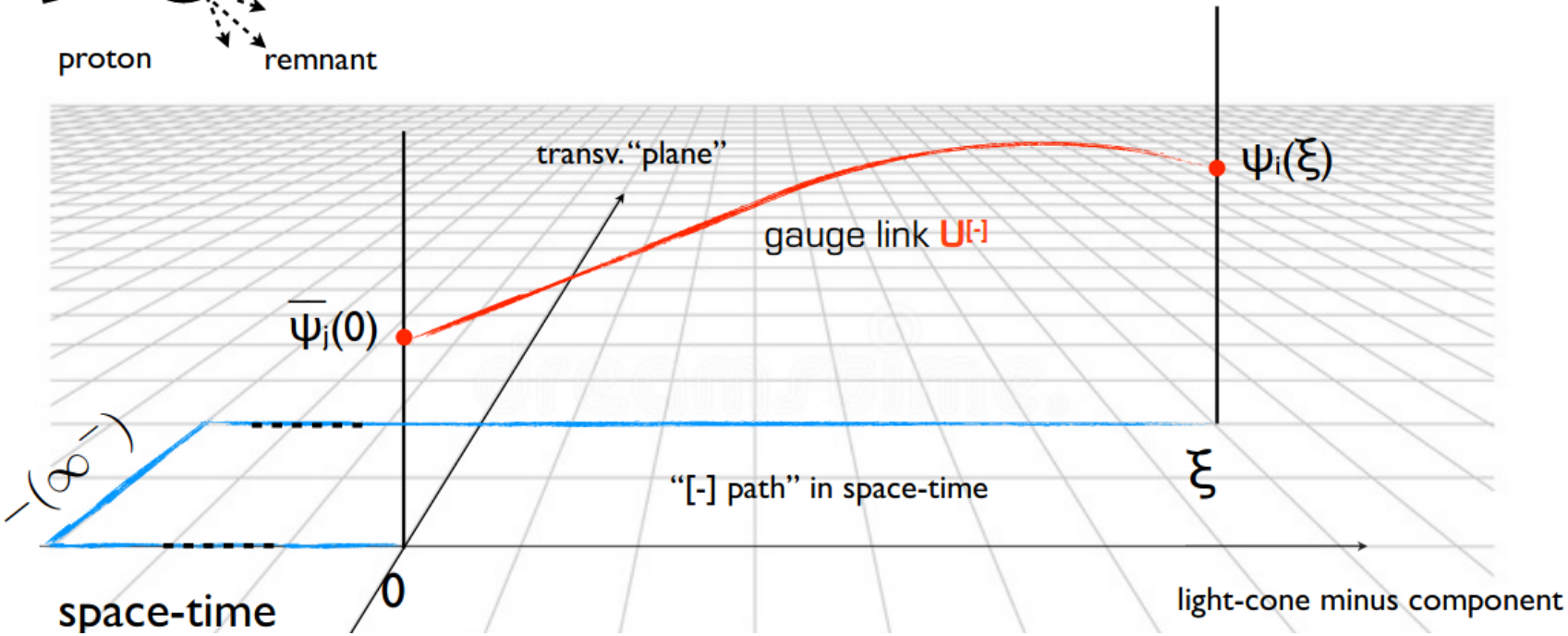
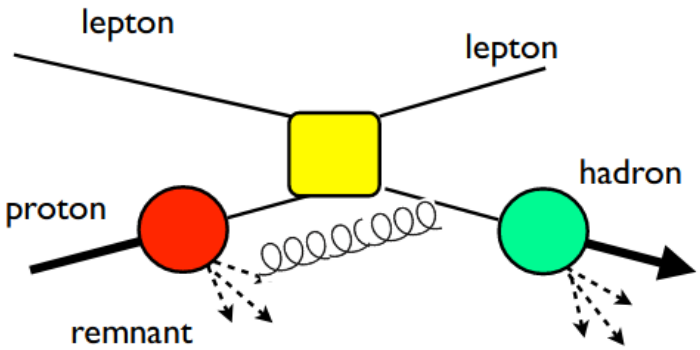



Parton distributions with $U^-$ gauge link: $f_1^{[-]}(x, k_T^2)$

# Process dependence

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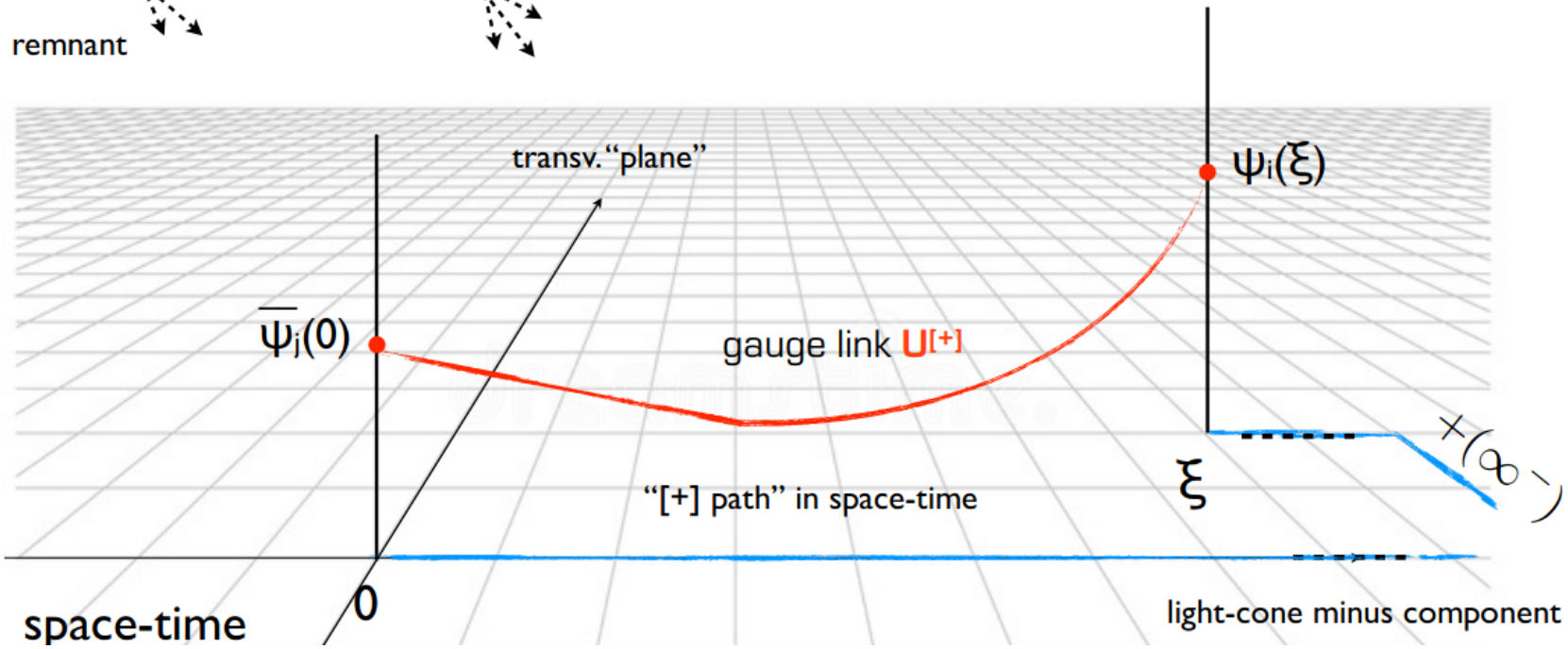

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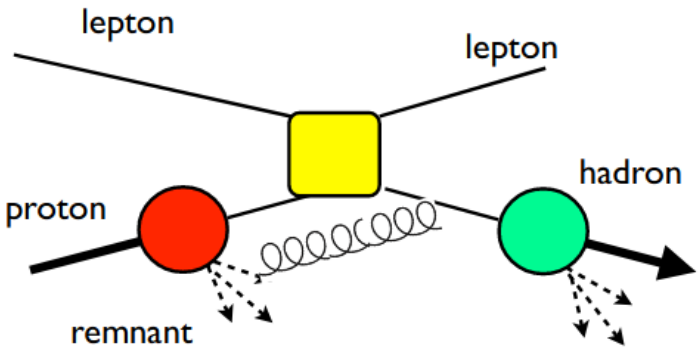


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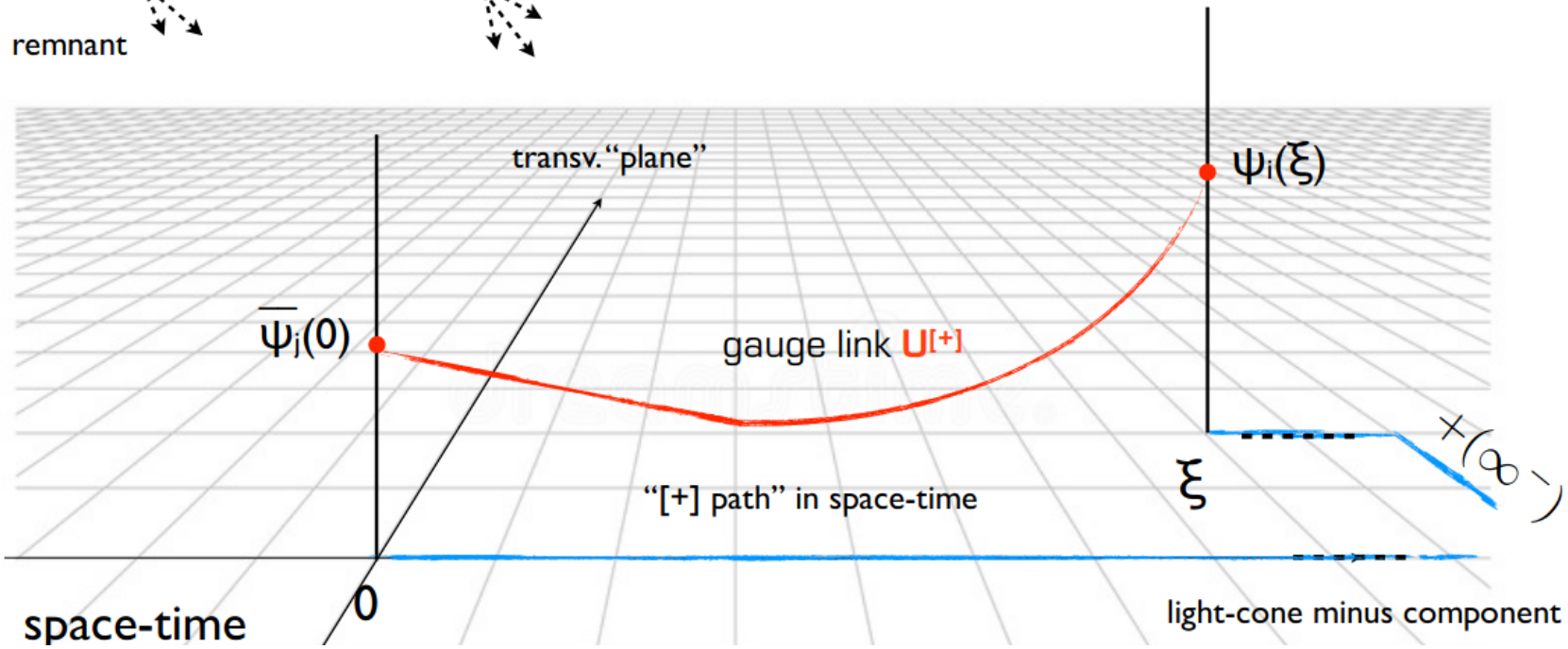



Parton distributions with $U^+$ gauge link: $f_1^{[+]}(x, k_T^2)$

# Process dependence

The hard process determines the direction of the gauge link
Thus **the distributions depend on the process**

**What happens to the concept of hadron structure?**

# Process dependence

The interplay between **time-reversal** symmetry and **gauge** symmetry generates relations between the two different gauge link configurations.
For example:

$$f_1^{a\ [+]}(x, k_T^2) = f_1^{a\ [-]}(x, k_T^2)$$

**T-even distribution**

**striking consequence of the symmetries of QCD**

$$f_{1T}^{a\perp\ [+]}(x, k_T^2) = -f_{1T}^{a\perp\ [-]}(x, k_T^2)$$

**T-odd distribution**

The "sign-change" relation for T-odd TMD PDFs,
such as the Sivers function, is **yet to be proved experimentally.**

# Outline



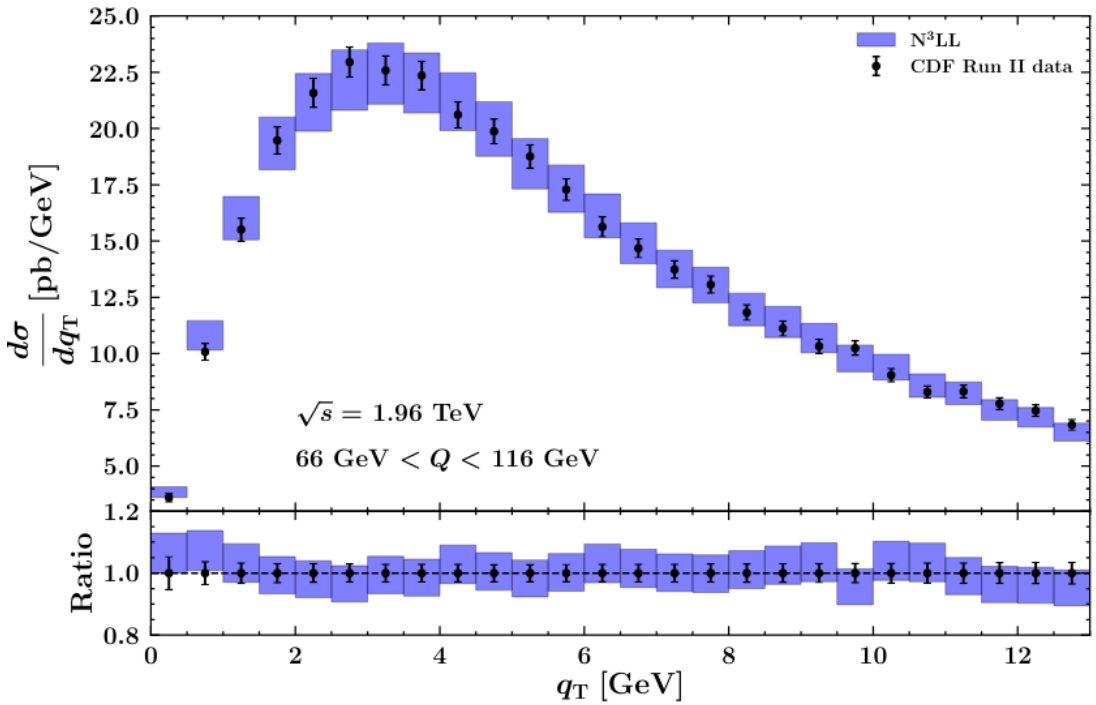
# TMD factorization

Cross section for the Drell-Yan process (schematically):

$$\frac{d\sigma}{dq_T} \sim \mathcal{H}\, f_1(x_a, k_{T\,a}, Q, Q^2)\, f_1(x_b, k_{T\,b}, Q, Q^2)\, \delta^{(2)}\left(q_T - k_{T\,a} - k_{T\,b}\right)$$

[TMD region, $q_T \ll Q$]

$$+ \mathcal{O}(q_T/Q) + \mathcal{O}(\Lambda/Q)$$ [large $q_T$ and low $Q$ corrections]




[from PV19 extraction - 1912.07550]

# Evolution

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The QCD evolution equations govern the scale change

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The QCD evolution equations govern the scale change
direct interplay with factorization theorems

# TMD evolution


Fourier transform of a TMD distribution ($b_T$ space):

$$\tilde{f}_1^a(x, b_T^2;\, \mu_f, \zeta_f) = \tilde{f}_1^a(x, b_T^2;\, \mu_i, \zeta_i)$$

$$\times \exp\left\{ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F \left[ \alpha_s(\mu), \frac{\zeta_f}{\mu^2} \right] \right\} \longrightarrow \mu \text{ evolution}$$

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The input TMD: expanded at low $b_T$ on the collinear distributions:

$$\tilde{f}_1^a(x, b_T^2;\, \mu_i, \zeta_i) = \sum_b C_{a/b}(x, b_T^2; \mu_i, \zeta_i) \otimes f_1^b(z, \mu_i)$$

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$\gamma_F, K, C_{a/b}$ : calculable in perturbation theory

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$f_1, F_{NP}, g_K$ : non-perturbative contributions - fits to data

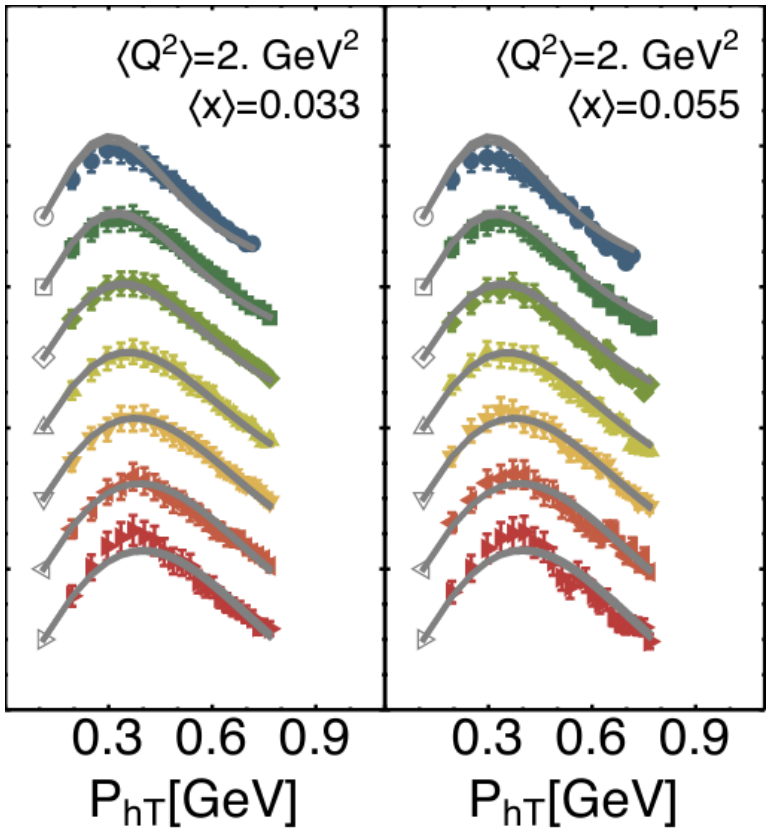
# Outline



# SIDIS

Semi-Inclusive Deep-Inelastic Scattering (SIDIS): $\ell(l)\, N(P) \to \ell(l')\, h(P_h)\, X$

One of the richest process for TMD physics:
sensitivity to structure and to hadronization



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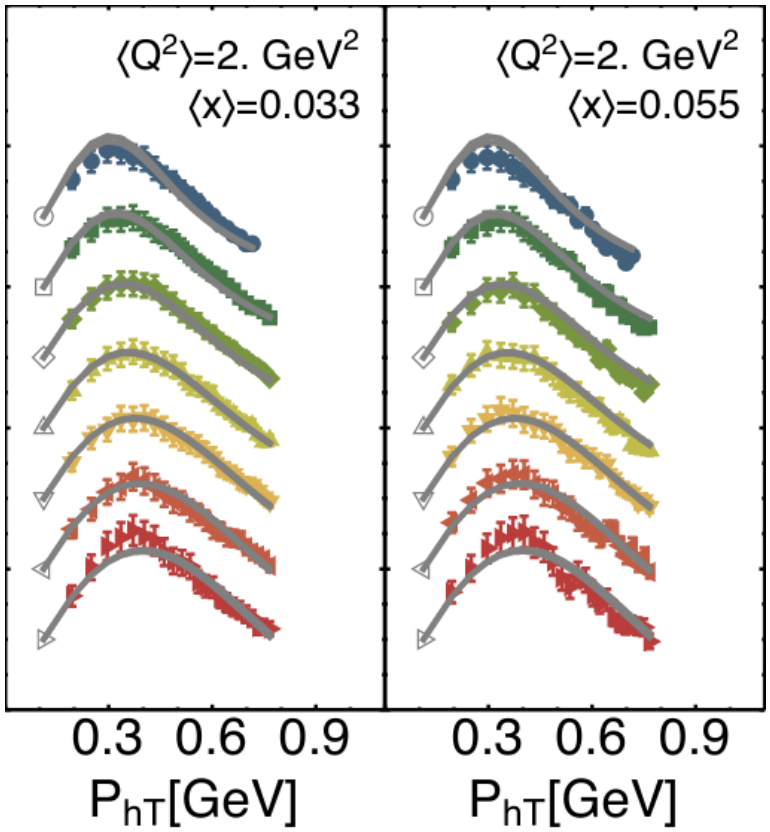
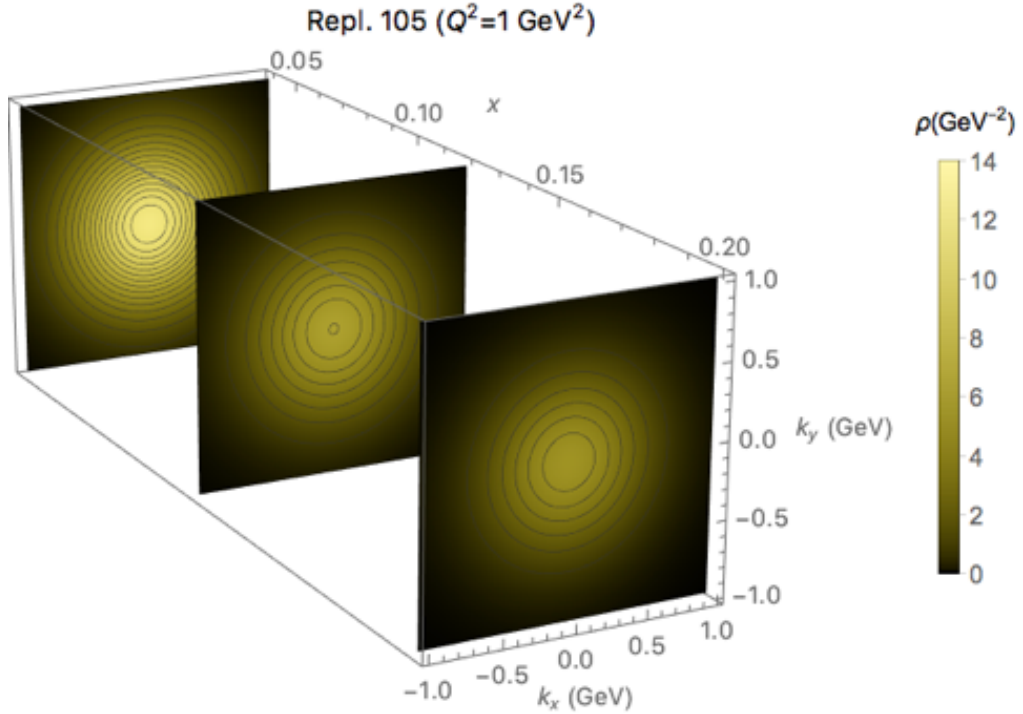


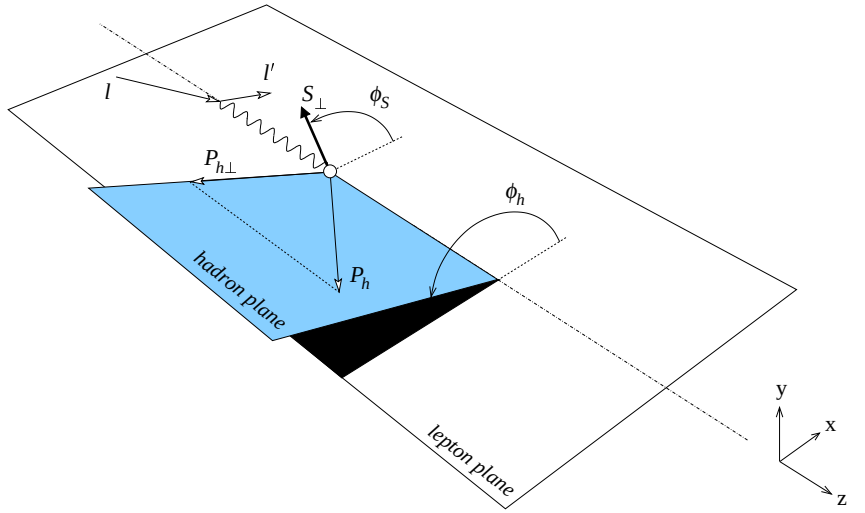

Quark TMD PDF $f_1^{a/P}(x, k_T^2, Q_0)$

# SIDIS: Structure Functions

$$\frac{d\sigma}{dx\,dy\,dz\,d\psi\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{$$

$F_{UU,T}(x,z,P_{h\perp}^2,Q^2)$ $\rightarrow$ 4D quantities

$+\,\epsilon\,F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h)\,F_{UU}^{\cos\phi_h} + \epsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$ $\rightarrow$ unpolarized



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$+\,\lambda_e\left[\ldots 1\,\text{SF}\ldots\right]$ $\rightarrow$ polarized terms

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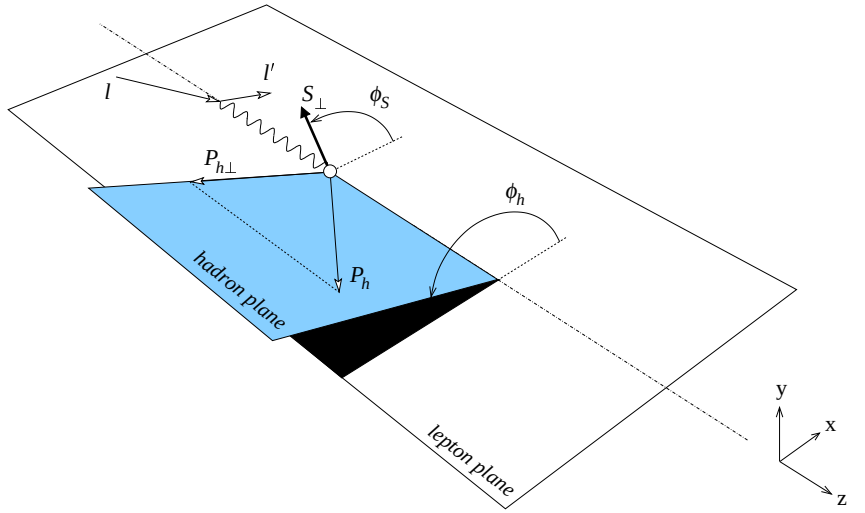

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18 structure functions in total

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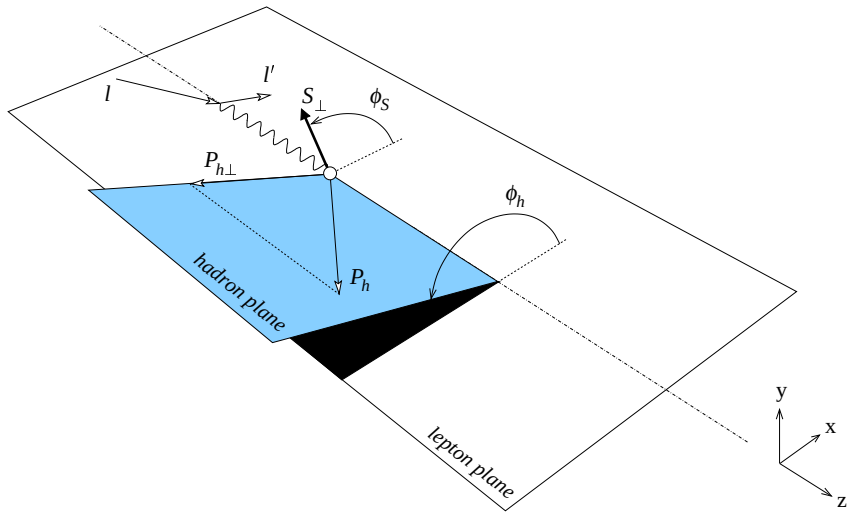

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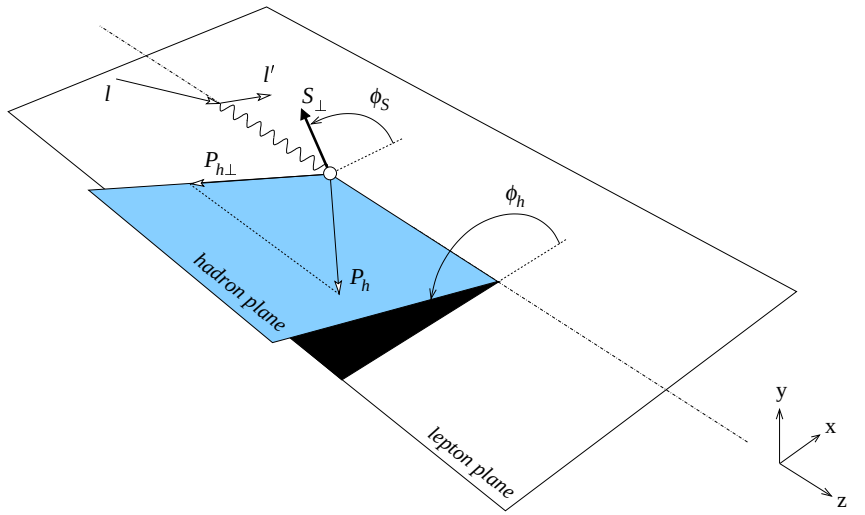
Similar classification for Drell-Yan



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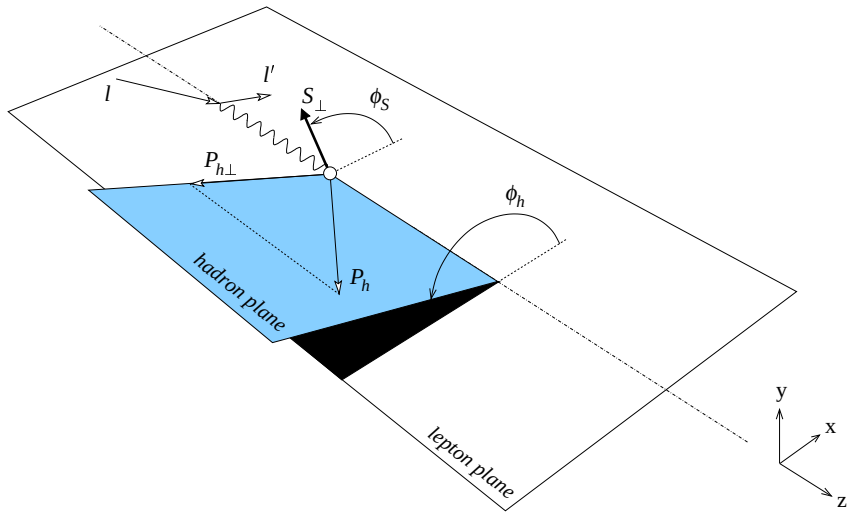
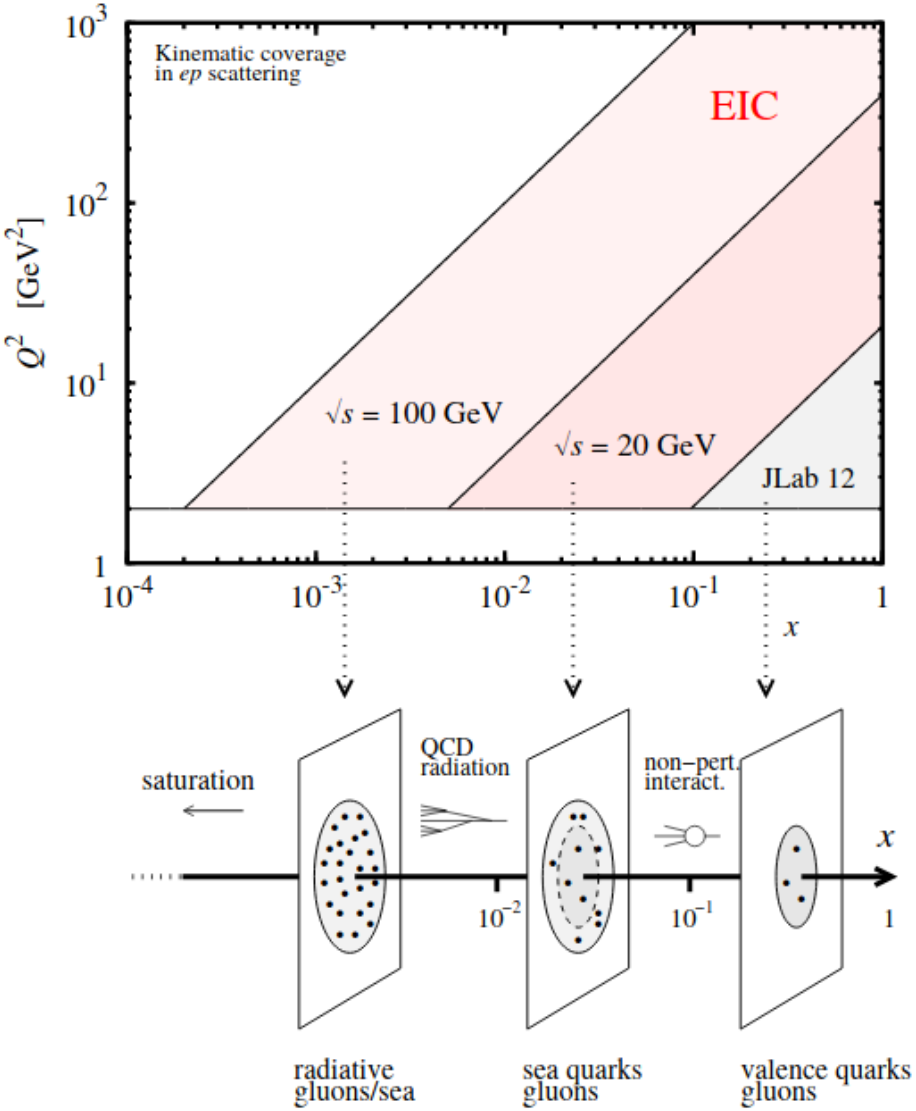
see e.g. Arnold et al. - 0809.2262



# SIDIS: from JLab to the EIC

credit picture: C. Weiss



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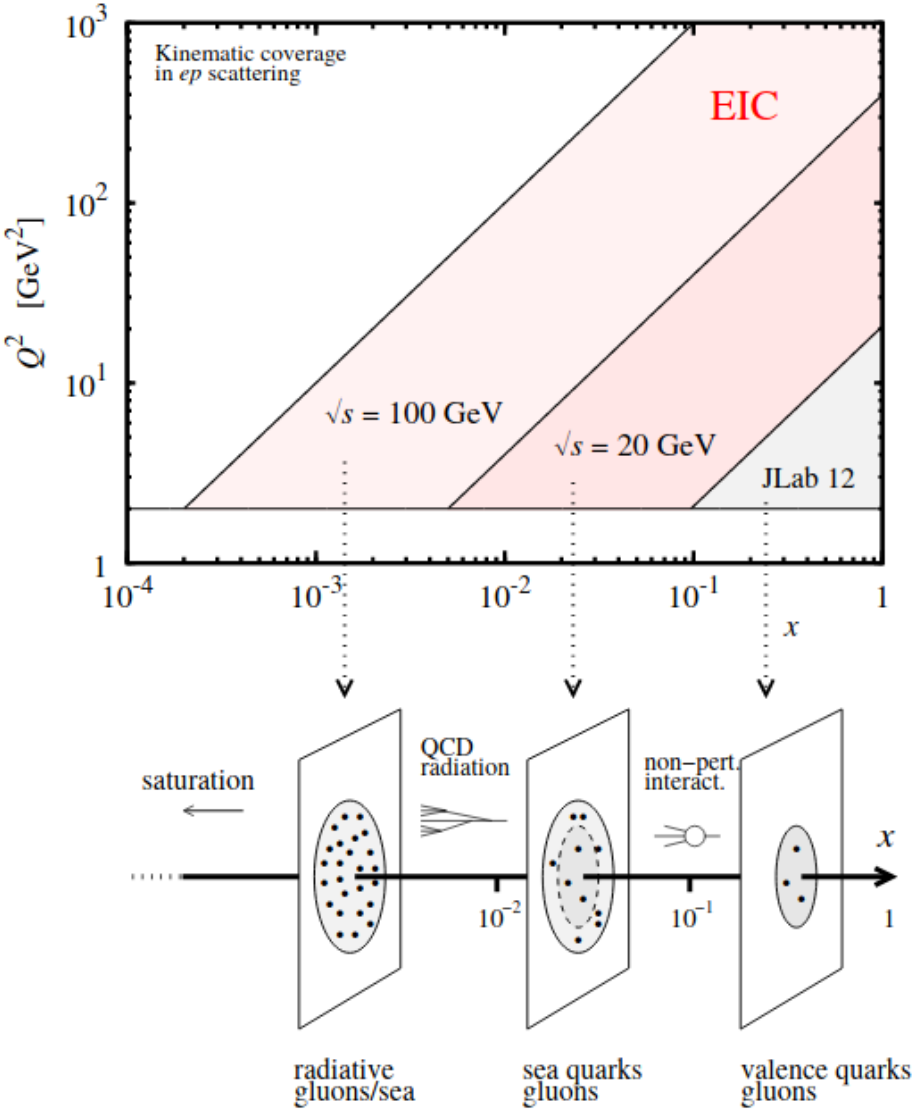


- CLAS12/HERMES: valence structure

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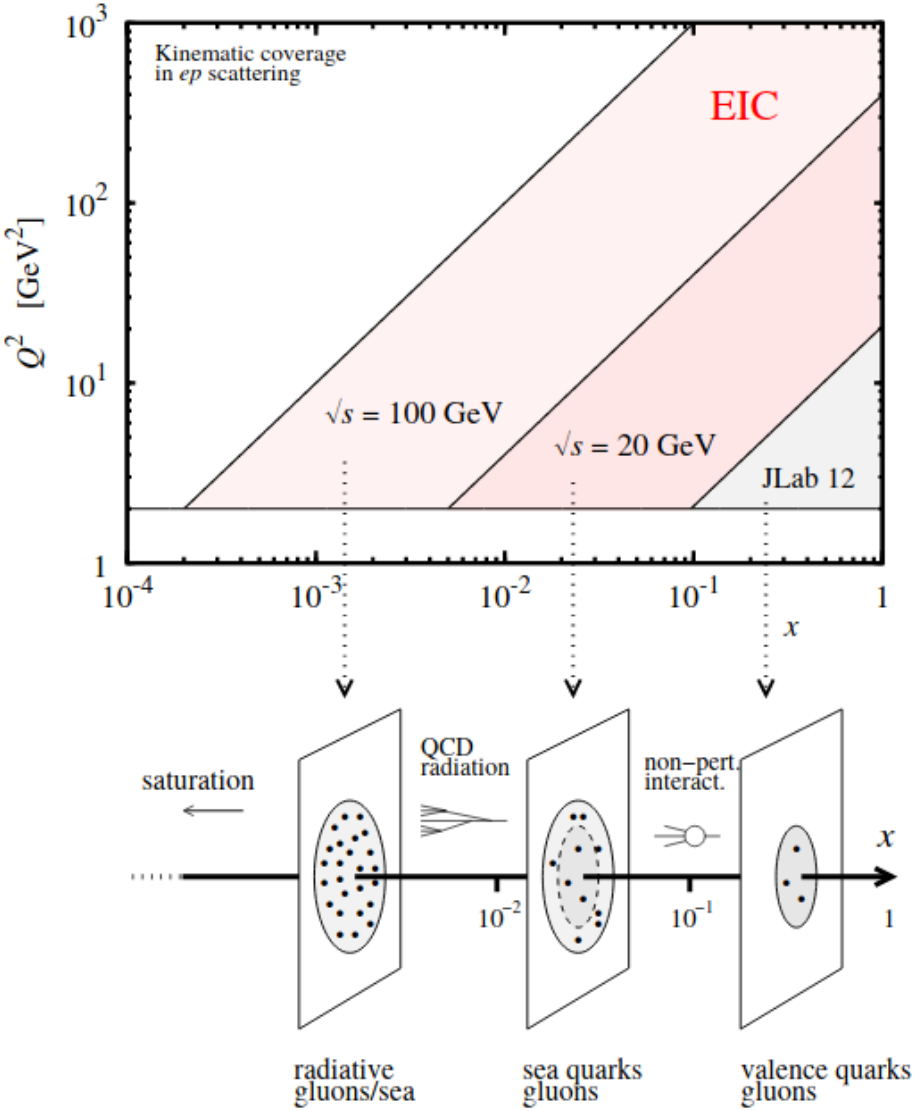


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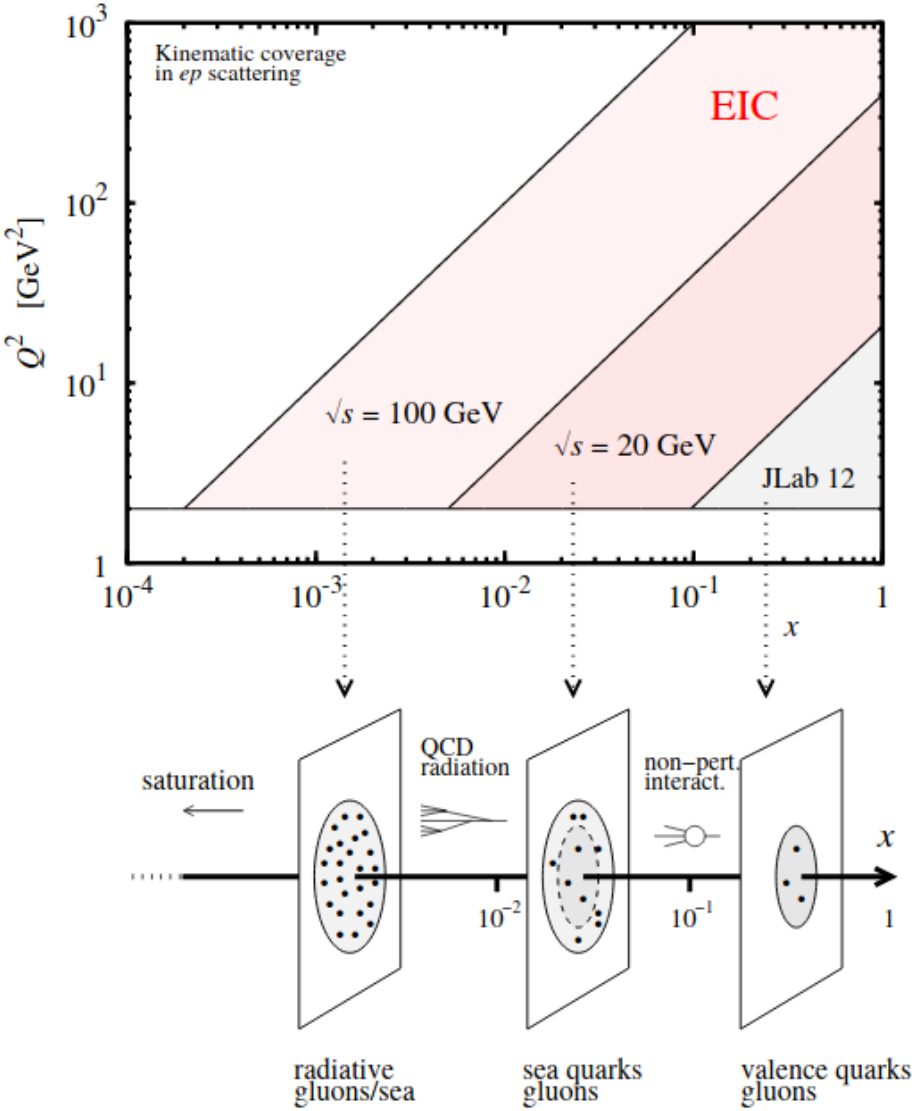


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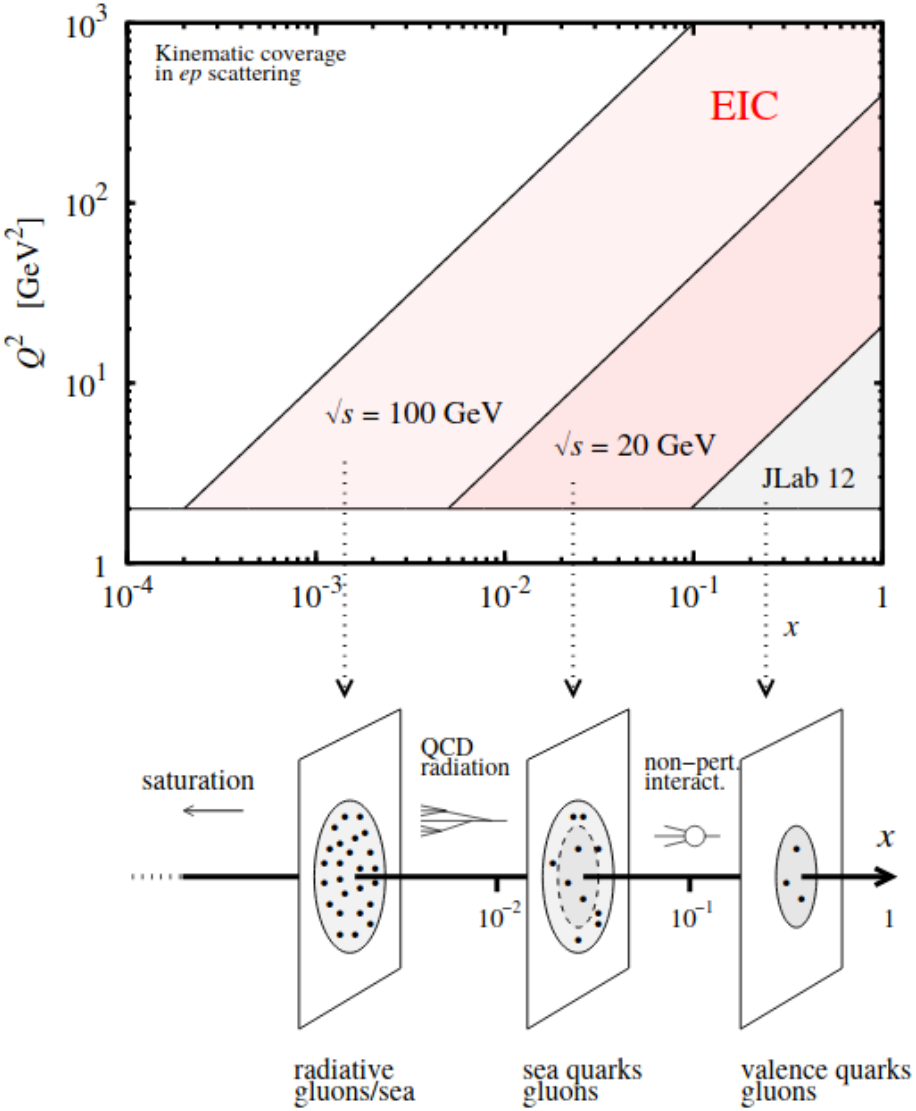


The Electron-Ion Collider (EIC) will greatly extend the kinematic reach of exisisting facilities for SIDIS.

A goldmine for TMD extractions.

- CLAS12/HERMES: valence structure
- COMPASS: valence/sea quarks
- EIC: sea quarks and (radiative) glue

# Extraction of unpolarized TMDs

*First* global fit of TMDs [PV17 - 1703.10157]:

- SIDIS: $lN \to lhX$ - $\sigma = \mathcal{H} \otimes f_1 \otimes D_1 +$ power corrections
- Drell-Yan: $pp \to (\gamma^*) \to ll$ - $\sigma = \mathcal{H} \otimes f_1 \otimes f_1 +$ p.c.
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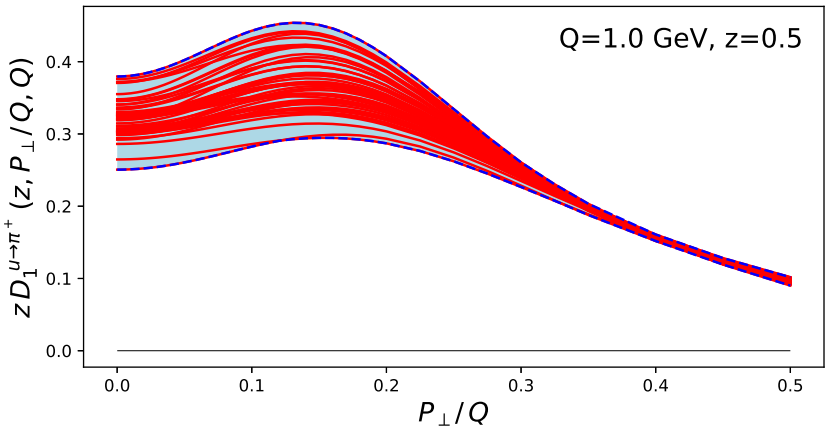
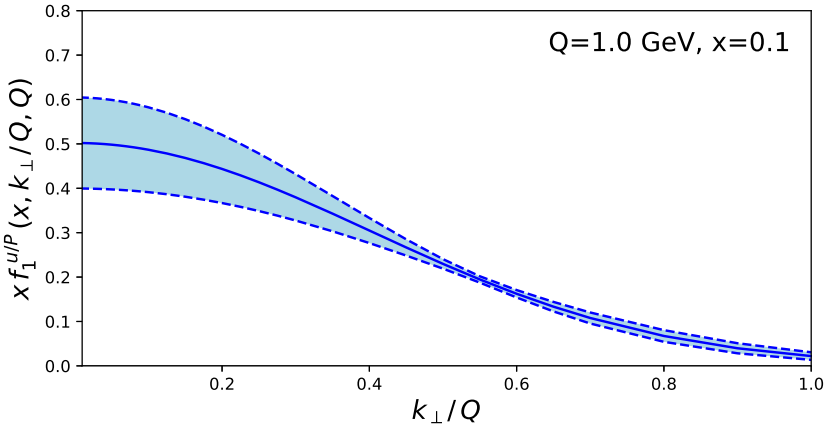



Extraction of TMD FF $D_1$ and PDF $f_1$ at NLL accuracy with bootstrap technique: reliable estimate of statistical uncertainties

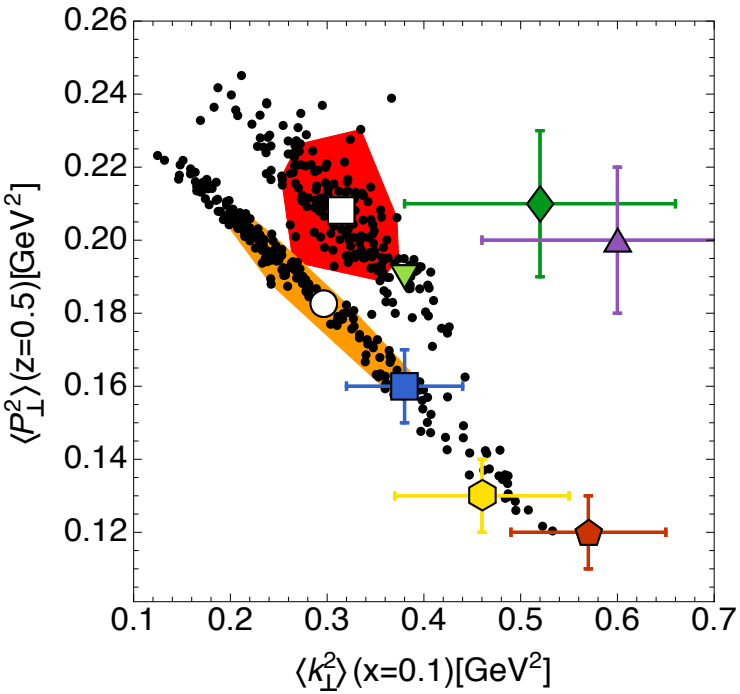
$D_1$ for $u \to \pi^+$: replicas

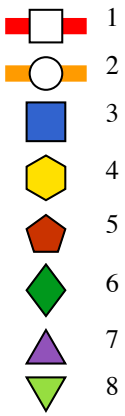
$f_1$ for $u$ in proton: statistical band

# Extraction of unpolarized TMDs

*First* global fit of TMDs [PV17 - 1703.10157]:

- SIDIS: $lN \to lhX$ - $\sigma = \mathcal{H} \otimes f_1 \otimes D_1 +$ power corrections
- Drell-Yan: $pp \to (\gamma^*) \to ll$ - $\sigma = \mathcal{H} \otimes f_1 \otimes f_1 +$ p.c.
- $Z$-production: $pp \to Z \to ll$ - $\sigma = \mathcal{H} \otimes f_1 \otimes f_1 +$ p.c.




Access the average transverse momenta of quarks and hadrons:

$\langle k_\perp^2 \rangle$ and $\langle P_\perp^2 \rangle$

mapping quarks and hadrons in 3D momentum space

# Outline



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A non-exhaustive personal list of open questions:

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Thank you!

# Some useful references

Some references to further explore these topics:

- SpinQuest collaboration: https://spinquest.fnal.gov/
- A Fixed-Target Experiment at the LHC (AFTER@LHC): [1807.00603]
- EIC websites: https://www.bnl.gov/eic and https://www.jlab.org/eic

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- overview of TMD factorization and evolution: [1509.04766]