# Corrections to nucleon spin structure asymmetries measured on nuclear polarized targets

O. A. Rondon
*Institute of Nuclear and Particle Physics, and Physics Department, University of Virginia, Charlottesville, Virginia 22901*
(Received 15 December 1998; published 27 July 1999)

The nucleon spin structure functions have been extracted from measurements of asymmetries in deep inelastic scattering of polarized leptons on polarized nuclei. The polarized nuclei present in practical targets: H, $^2$H, $^3$He, $^{14}$N, $^{15}$N, $^6$Li, and $^7$Li, are, with the exception of hydrogen, systems of bound nucleons, some of which can attain significant degrees of alignment. All the aligned nucleons contribute to the asymmetries. The contributions of each nuclear species to the asymmetry have to be carefully determined, before a reliable value for the net nucleon asymmetry is obtained. For this purpose, the spin component of the nuclear angular momentum for every nuclear state and the probability of each state have to be known with sufficient accuracy. In this paper I discuss the basic corrections used to estimate the contributions of the different nuclei, with emphasis on the $A=6$ and 7 Li isotopes present in the Li$^2$H polarized target used during SLAC Experiment 155 to study the deuteron spin structure. [S0556-2813(99)04308-3]

PACS number(s): 14.20.Dh, 13.88.+e, 27.10.+h, 27.20.+n

The use of polarized nuclei as targets in spin structure experiments [1–7] requires that proper allowance be made for their nuclear properties. This involves an understanding of the magnitude of the contributions of the polarized nucleons in each nuclear species to the spin asymmetries, as well as of the possible kinematic ($x$) dependence of the nucleon polarization. The polarizable nuclei can be present in the targets in elemental form ($^3$He) or as compounds (deuterated ammonia–N$^2$H$_3$, lithium deuteride–Li$^2$H, butanol). More than one of the nuclear species can attain significant degrees of polarization, as in the case of Li$^2$H, in which both $^6$Li and $^2$H polarize almost equally.

Two approaches can be followed to determine the effective nucleon polarization. Things are considerably simplified by the fortunate availability of practically polarizable light nuclei because the Coulomb interaction can be largely neglected, and isospin symmetry applied. Hence, in the case of isospin singlets, like deuterium, $^6$Li and $^{14}$N, model-independent analyses using only isospin conservation are possible, based on the decomposition of the total nuclear angular momentum $I$ and magnetic moment $\mu$ in terms of spin and orbital components [8,9]. In the case of mirror nuclei, like $^3$H and $^3$He, or $^{15}$N and $^{15}$O, this type of analysis can be supplemented by information from the $\beta$ decay of one of the pair members [10,11]. The cluster model description of light nuclei, such as Li, allows the extension of this method to the cluster components of $^7$Li [12].

The second approach involves models of the nuclear structure of the target nuclei. This approach has been used to describe the nucleon polarization in $^2$H [13], $^3$He [14], $^6$Li [13,15,16], $^7$Li [16], $^{14}$N [1], and $^{15}$N [3]. At the simplest level, the relevant information to be extracted from the models consists of the probabilities of the different nuclear ground states, and their corresponding angular momentum decompositions [17]. This information results in a proportionality factor between the polarization of the nucleon and that of the nucleus. More sophisticated analyses, such as that of Ref. [16], give directly the integrated spin densities of the protons and neutrons.

In a recent article, Melnitchouk, Piller, and Thomas [18] have shown that for the deuteron the use of a constant factor for values of the Bjorken scaling variable $x<0.75$ to correct for the presence of $D$-state nucleons in the deuteron results in a less than 1.5% deviation from the more accurate convolution or covariant descriptions. Since the uncertainty in the $D$-state probability itself is on the order of $\simeq 1\%$ (20% relative), the use of a constant correction factor, at least in the measured ranges of $0.015<x<0.75$ is valid in the case of the deuteron. For $x<0.015$ the extraction of the neutron spin structure may require more refined descriptions of the nucleon polarization, given the small size of the deuteron asymmetry at low $x$ [18]. The level of required refinement becomes more important with increasing nuclear complexity, as in the cases of $^3$He and Li.

## Model-independent analyses

The model-independent analyses are based on some simple properties of mirror nuclei, including the self-conjugate nuclei (i.e., those odd-odd nuclei that are their own mirror). Mirror nuclei are either isospin doublets or singlets (self-conjugate). We first examine isosinglets.

From the decomposition of the angular momentum of a nucleus and its magnetic moment, it can be determined that an orbital contribution to these quantities may be present in isosinglets. Since only the spin component contributes to the nucleon spin asymmetry, the orbital component has to be subtracted. The decomposition of the magnetic moment and angular momentum for the state with $M=I$ is given by [10]

$$\mu=\left\langle \sum_i \left[\frac{1}{2}(1+\tau_3^{(i)})(l_z^{(i)}+\sigma_z^{(i)}\mu_p)+\frac{1}{2}(1-\tau_3^{(i)})\sigma_z^{(i)}\mu_n\right]\right\rangle, \tag{1}$$

$$I=\left\langle \sum_i l_z^{(i)}\right\rangle+\frac{1}{2}\left\langle \sum_i \sigma_z^{(i)}\right\rangle, \tag{2}$$

 

TABLE I. Nucleon polarizations in spin-1 nuclei.

| Nuclide | Magnetic moment ($\mu_N$) | Spin component | Orbital component ($\times 1/2$) |
|---|---|---|---|
| $^2$H | 0.857 | 0.94 | 0.03 |
| $^6$Li | 0.822 | 0.847 | 0.077 |
| $^{14}$N | 0.404 | −0.26 | 0.63 |

where the index $i$ refers to the different nucleons and as usual $\langle \sigma_z \mu_i \rangle = \langle g_s^i S_z^{(i)} \rangle$, etc. In the case of the isosinglets $^2$H, $^6$Li, and $^{14}$N, these equations simplify to [9]

$$\mu = g_s^p \langle S_z^{(p)} \rangle + g_s^n \langle S_z^{(n)} \rangle + g_l^p \langle L_z^{(p)} \rangle, \tag{3}$$

$$I = \langle S_z^{(p)} \rangle + \langle S_z^{(n)} \rangle + \langle L_z^{(p)} \rangle + \langle L_z^{(n)} \rangle, \tag{4}$$

where $g_s^{p(n)} = 2\mu_{p(n)}$ are the $g$ factors for free protons (neutrons). Neutrons do not contribute to the orbital component of $\mu$. Isospin conservation for isosinglets lets us write

$$\langle S_z^{(p)} \rangle = \langle S_z^{(n)} \rangle = \frac{(\mu - I/2)}{0.76}. \tag{5}$$

Substitution of the experimentally measured magnetic moments in Eq. (5) yields the spin and orbital components of the magnetic moment displayed in Table I, which can be interpreted as indicating that the polarization of the unpaired nucleons ($p$ or $n$) in polarized spin-1 nuclei is the fraction of the nuclear polarization shown in column 3 of Table I. For example, the nucleon polarization in $^{14}$N is 26% of the ammonia polarization and the nucleon spin points in the opposite direction as the nuclear spin. As I show below, these values are in very good agreement with the results of the model-dependent method.

In the case of mirror nuclei, like $^3$H and $^3$He, or $^{15}$N and $^{15}$O, this analysis can be supplemented with information from the $\beta$ decay of one of the pair members [10,11]. For the pair $^7$Li and $^7$Be, the magnetic moment of the radioactive $^7$Be (half-life: 53.3 days) disappointingly has not been measured. However, the cluster model description of light nuclei such as $^7$Li [12], allows the extension of this method to the cluster components of $^7$Li ($\alpha + triton$).

For mirror nuclei, assuming isospin conservation and neglecting the Coulomb interaction, the sum of the magnetic moments of the pair is twice the expectation value of the isoscalar component of the magnetic moment, while the difference is twice the isovector component:

$$\mu\left(t_3 = \frac{1}{2}\right) + \mu\left(t_3 = -\frac{1}{2}\right) = \left\langle \sum_i l_z^{(i)} \right\rangle + (\mu_p + \mu_n) \times \left\langle \sum_i \sigma_z^{(i)} \right\rangle,$$

$$\mu\left(t_3 = \frac{1}{2}\right) - \mu\left(t_3 = -\frac{1}{2}\right) = \left\langle \sum_i \tau_3^{(i)} l_z^{(i)} \right\rangle + (\mu_p - \mu_n) \times \left\langle \sum_i \tau_3^{(i)} \sigma_z^{(i)} \right\rangle. \tag{6}$$

The terms in the sum can be obtained from knowledge of the magnetic moments and spin of the nuclei, while the last term in the difference is related to the Gamow-Teller $\beta$ decay matrix element [10,19]

$$\left| \left\langle \sum_i \tau_3^{(i)} \sigma_z^{(i)} \right\rangle \right|^2 = \frac{I}{I+1} \left| \int \sigma \right|^2. \tag{7}$$

Knowledge of the $ft$ value for the decay allows the extraction of the term from

$$ft = \frac{B}{\left| \int 1 \right|^2 + \left( \frac{g_A}{g_V} \right)^2 \left| \int \sigma \right|^2}, \tag{8}$$

where $B = 5967$ s depends on the Fermi coupling constant $G_F$ and $g_V$, $g_A$ are the vector and axial-vector coupling constants for the neutron $\beta$-decay. For the mirror pair $^3$H ($\mu = 2.9789\mu_N$) and $^3$He ($\mu = 2.1276\mu_N$), using the latest $\beta$-decay values [20], we obtain the proton spin contribution to the magnetic moment

$$\left\langle \sum_i (1 + \tau_3^{(i)}) \sigma_z^{(i)} \right\rangle = 0.932, \tag{9}$$

implying that the proton polarization is 93% of the tritium polarization. This analysis also predicts that the contribution of the paired neutrons to the magnetic moment is −0.007. The corresponding values of Ref. [10] are 0.96 and −0.04, based on older values of the $\beta$ decay constants. The analysis on tritium can be applied to its mirror $^3$He, with the substitution of neutrons for protons, so the neutron in $^3$He would be 93% polarized.

As I mentioned before, there is no magnetic moment data for $^7$Be, so the method cannot be used for $^7$Li directly. For $^{15}$N, $\mu_{^{15}\mathrm{N}} = -0.283\mu_N$ (its mirror is $^{15}$O, $\mu_{^{15}\mathrm{O}} = 0.719\mu_N$) Ref. [10] finds the proton polarization to be −24%, similar in magnitude and direction to that of the proton in $^{14}$N. With updated values for the input data, I get −22% for the same quantity, still in good agreement.

### Model-dependent analyses

Models ranging from the single-particle shell model to cluster models to Fadeev's calculations to the very latest Green's-function Monte Carlo [16] have been used to describe nuclear properties. The goal of this work is to extract from the different models the information that is relevant to obtain the net nucleon polarization, which is largely just the polarization of the unpaired nucleons in the shell model. The quantities needed are the spin component of the total angular momentum for every populated level in the ground state, and the corresponding occupation probabilities. I will discuss deuterium and the $A = 3$ nuclei to illustrate the method and apply it to the lithium isotopes and the nitrogen isotopes which are components of solid polarized targets.

### *Deuterium*

The deuteron has $I^{\pi}=1^{+}$, and the nucleons are in the spin triplet state (parallel spins). This implies that only even components of orbital angular momentum are allowed. The deuteron has no shell structure, so its total angular momentum is formed by $L$-$S$ coupling. That is, $L=l_p+l_n$ and $S=s_p+s_n$.

The deuteron has a virtual $D$ state that is occupied by the nucleons some fraction of the time. Since the orbital angular momentum of this state is $L=2\hbar$, it affects the polarization of the nucleons relative to that of the nucleus. In the $S$ state $L=0$, so the nucleons add up their 1/2 $\hbar$ spins to $I=1$, parallel to the deuteron spin. In the $D$ state, $L=2$, so $I=1$ is formed from $L+S=2\otimes 1$.

The vector polarization is the net number of deuterons with spins oriented parallel to an external magnetic field and is given by

$$P_V=N_{(M=1)}-N_{(M=-1)},$$

where $N_{M=\pm 1}$ are normalized numbers of deuterons in states $M=\pm 1$, respectively. I will use the notation $\mathrm{CG}|m_L,m_S;M\rangle$, except when noted. For the $D$ state the $M=\pm 1$ substates with the appropriate Clebsch-Gordan (CG) coefficients are represented by

$$\sqrt{\frac{3}{5}}|2,-1;1\rangle+\sqrt{\frac{3}{10}}|1,0;1\rangle+\sqrt{\frac{1}{10}}|0,1;1\rangle,$$

$$\sqrt{\frac{3}{5}}|-2,1;-1\rangle+\sqrt{\frac{3}{10}}|-1,0;-1\rangle+\sqrt{\frac{1}{10}}|0,-1;-1\rangle. \tag{10}$$

From the Clebsch-Gordan coefficients we see that a net 50% of the nucleons in the $D$ state have polarizations antiparallel to that of the deuteron ($M=1$ but $m_S=-1$)[1] implying that the total number of nucleons in the deuteron with spins parallel to the deuteron is

$$N_{\parallel}=N_S-\frac{1}{2}N_D=N-1.5N_D,$$

where $N=N_S+N_D$ are the total, $S$-state, and $D$-state numbers of nucleons, and $N_D=P_DN$. $P_D$ is the $D$-state probability and it has been variously estimated in the different models of the deuteron, as summarized in Table II.

Thus, on average

$$N_{\parallel}=N(1-1.5P_D)=(0.926\pm 0.016)N,$$

so the net nucleon polarization in the deuteron is reduced to $\gamma\cong 93\%$, a factor that needs to be taken into account in extracting the neutron spin structure function.

[1]The same result is obtained in a somewhat less transparent way, by calculating the matrix elements $P^{(\pm)}=\langle II|\hat{P}^{(\pm)}|II\rangle$, where $\hat{P}^{(\pm)}$ is an aligned nucleon counting operator [14,15].

TABLE II. $^2$H $D$ state probabilities.

| Model | $P_D$(in %) |
|---|---|
| Reid [21] | 6.47 |
| Paris [21] | 5.46 |
| Bonn 76 [21] | 4.32 |
| Bonn 87 [22] | 4.25 |
| Hulthén-Yamaguchi [23] | 4.00 |
| $\langle P_D\rangle$ | 4.90 |
| Standard deviation of the values | 1.04 |

We also note that there is no net contribution from the $M=0$ state nucleons: the nucleons with this quantum number are aligned parallel or antiparallel to the deuteron in equal numbers. The probability of the deuteron to be in this substate is related to the tensor polarization, implying that all the contributions to the nucleon polarization come from the vector polarization. This can be also be seen by computing [24]

$$P_{x_i}=\mathrm{Tr}(\rho^d\sigma_{x_i}),\quad x_i=x,y,z,\quad (\mathbf{B}=B_o\hat{\mathbf{z}}),$$

where $\rho^d$ is the deuteron density matrix, and $\sigma_{x_i}$ are the Pauli matrices. The results are $P_{x,y}=0$ and $P_z=(N_{M=1}-N_{M=-1})*(P_S-P_D/2)$, as before.

### *Tritium and $^3$He*

The work of Ref. [14] (‘‘Neutron polarization in polarized $^3$He targets’’) contains a very detailed analysis of the nucleon polarization in $^3$He. The extensive discussion on the extraction of the neutron asymmetries from polarized deep inelastic scattering (DIS) on $^3$He targets that is given in Ref. [25] combined with Ref. [14] constitute essentially a definitive treatment of the subject, at least as far as DIS is concerned. All the experiments using polarized $^3$He targets [2,5,6] apply the results contained in those works to the neutron spin structure. Although no polarized tritium targets have been built, there is interest in this nucleus [26], which as a mirror pair of $^3$He can give us important insights into the properties of the bound nucleon [27]. Very little new information can be added beyond the work of the authors mentioned, so in this section I will use the $A=3$ nuclei as an illustration of how the method applied to the deuteron can be extended to other nuclei.

Both $^3$H and $^3$He have $I^{\pi}=\frac{1}{2}^{+}$. The ground-state description of tritium is very similar to $^3$He, involving in both cases the usual spin singlet state $S$ plus $S'$ (spin triplet) and $D$ states [28] that have their origin in the tensor forces. Following the approach applied in Refs. [17,8] to the magnetic moments of nuclei, we can in general express the ground-state wave function in terms of $\psi_{LS}$ states

$$\psi=\sum_{L,S}c_{LS}\psi_{LS}, \tag{11}$$

where the coefficients $|c_{LS}|^2=P_L$ represent the probabilities of occurrence of the $L,S$ states. Then one can carry out the same type of angular momentum decomposition that was done for $^2$H to obtain the net nucleon polarization in each state and combine the results with the model-dependent estimates of the corresponding probabilities. For the three-nucleon system, in the $S$ state we have the two identical nucleons (neutrons in tritium) with their spins antiparallel, so the spin of the third nucleon (proton) is always aligned with the nuclear spin. In the $S'$ state, two nucleons can pair to form spin $s_1=1$, which is coupled to the third nucleon's spin $s_2=1/2$ to give the allowed combinations $I,M=\frac{1}{2},\pm\frac{1}{2}$. The nucleons are in the $s_1$ state two-thirds of the time, and the other one-third are in the $s_2$ state. Using the slightly changed notation CG$|m_{s1},m_{s2};M\rangle$ the $M=\pm 1/2$ substates are

$$\sqrt{\frac{2}{3}}|1,-1/2;\;\; 1/2\rangle-\sqrt{\frac{1}{3}}|0,\;\; 1/2;\quad 1/2\rangle$$

$$-\sqrt{\frac{2}{3}}|-1,1/2;-1/2\rangle+\sqrt{\frac{1}{3}}|0,-1/2;-1/2\rangle. \qquad (12)$$

When a nucleon is in the $m_{s1}=\pm 1$ substates its spin is parallel to the tritium spin. When $m_{s1}=0$, the nucleon is equally likely to be aligned or antialigned. From the CG coefficients we see that a nucleon in the $s_1$ state is aligned a net 2/3 of the time, for a $\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$ nucleon polarization. Similarly, the spin of the nucleon in the $s_2$ state is aligned opposite the nuclear spin ($M=1/2$ but $m_{s2}=-1/2$) a net 1/3 of the time, for a net $-\frac{1}{3}\times\frac{1}{3}=-\frac{1}{9}$ polarization. Combining the two spin states the nucleon polarization in the $S'$ state is 1/3. The $D$ state involves the combination of $L=2$ with $S=3/2$ to get $I=1/2$. The corresponding angular momentum components are (original notation from here on)

$$\sqrt{\frac{2}{5}}|2,-3/2;1/2\rangle-\sqrt{\frac{3}{10}}|1,-1/2;1/2\rangle+\sqrt{\frac{1}{5}}|0,1/2;1/2\rangle$$

$$-\sqrt{\frac{1}{10}}|-1,3/2;1/2\rangle$$

$$\sqrt{\frac{1}{10}}|1,-3/2;-1/2\rangle-\sqrt{\frac{1}{5}}|0,-1/2;-1/2\rangle$$

$$+\sqrt{\frac{3}{10}}|-1,1/2;-1/2\rangle-\sqrt{\frac{2}{5}}|-2,3/2;-1/2\rangle. \qquad (13)$$

For $m_S=\pm\frac{3}{2}$ all the nucleon spins are aligned with the total spin $S$. For $m_S=\pm\frac{1}{2}$ the nucleon spin is parallel to $S$ 2/3 of the time and 1/3 is antialigned, for a net 1/3 alignment of the nucleon spin with $S$. Combining these weights (1 for $m_S=\pm\frac{3}{2}$, 1/3 for $m_S=\pm\frac{1}{2}$) with the CG probabilities for $S$ to be parallel or antiparallel to the nuclear spin in each $m_S$ substate, the net nucleon polarization in the $D$ state is $-1/3$.

The number of nucleons in a state with orbital angular momentum $L$ is $N_L=P_LN$, where as before $N=\Sigma N_L$ is the total number of nucleons. $C_L$ denotes the corresponding nucleon polarization, obtained from the angular momentum decomposition. Then the proton polarization in $^3$H (or the neutron in $^3$He) is the sum of the proton polarizations in each state weighted by the probabilities of occupying the given states

$$N_\parallel^p=\sum C_LN_L=N_S+\frac{1}{3}N_{S'}-\frac{1}{3}N_D=N\left(1-\frac{2}{3}P_{S'}-\frac{4}{3}P_D\right). \qquad (14)$$

The corresponding neutron polarization in tritium (or proton in $^3$He) is

$$N_\parallel^n=N\left(\frac{1}{3}P_{S'}-\frac{1}{3}P_D\right).$$

Substituting the values for $P_{S'}=0.0167$ and $P_D=0.0934$ from Refs. [28,29],[2] we obtain the proton polarization in tritium = 86%, and the small neutron contribution (per neutron) = -2.6%. As expected, these values are almost numerically identical to the $^3$He results of Ref. [14]. We also notice that the model-independent calculation for $^3$H overestimates somewhat the proton polarization, and underestimates that of the neutrons.

### *Lithium*

The same approach outlined above can be applied to other nuclei. Next I compute the angular momentum decomposition for $^6$Li.

$^6$Li has also $I^\pi=1^+$, and it can be described very well as an $\alpha$ cluster core plus a neutron and proton pair [15] (which in some cases is combined into a deuteron cluster [30]). Since the ground state of $^6$Li is an isosinglet, the proton and neutron would be in a spin triplet. In a three-body configuration, the antisymmetrization of the wave function allows the orbital angular momenta to also take odd values, so $S$, $P$, and $D$ states are permitted as long as the total orbital angular momentum adds up to an even number to obtain positive parity [15]. The $P$ state can be associated with either the usual spin triplet or with a spin singlet. In the $S$ and $D$ states, the configuration is identical to case of the deuteron, so as before we have $N_S$ and $C_DN_D=-\frac{1}{2}P_DN$. For the $P$ state, when $S=1$, $M=1$ is formed from $m_L=1,0$ and $m_S=0,1$. In detail:

$$\sqrt{\frac{1}{2}}|1,0;1\rangle-\sqrt{\frac{1}{2}}|0,1;1\rangle$$

$$-\sqrt{\frac{1}{2}}|-1,0;-1\rangle+\sqrt{\frac{1}{2}}|0,-1;-1\rangle. \qquad (15)$$

We see that only one half of the nucleons in this state are aligned with the Li spin, so $C_{P(S=1)}=\frac{1}{2}$. As in the $D$ state, the nucleons in the $M=0$ substate have their spins equally aligned in opposite directions. Finally for the $P$ state, when

[2]The work of Ref. [29] mentions a $8.3\times 10^{-4}$ probability of a $P$ state, that we neglect.

$S=0$ the nucleons are in the spin singlet state, so $C_{P(S=0)}=0$. The total number of nucleons in $^6$Li with spins parallel to the nuclear spin is

$$N_{\parallel}=N_S+0N_{P(S=0)}+\frac{1}{2}N_{P(S=1)}-\frac{1}{2}N_D$$
$$=N\left(1-P_{P(S=0)}-\frac{1}{2}P_{P(S=1)}-\frac{3}{2}P_D\right). \qquad (16)$$

The $P_L$ probabilities have been calculated in Ref. [15] solving three-body Fadeev equations for a variety of $NN$ and $\alpha N$ potentials. The probabilities for protons and neutrons are found to be equal to a precision better than a few parts in a thousand. They also find that the Coulomb interaction and isospin breaking have little effect on the calculated magnetic moment and nucleon spin alignment. Taking the average of their results for the eight models that predict $\mu_{^6\mathrm{Li}}$ to better than 2%, I obtain $P_{P(S=0)}=0.0238$, $P_{P(S=1)}=2.9\times 10^{-3}$, and $P_D=0.0725$, which result in a nucleon polarization in $^6$Li of $(0.866\pm 0.012)/3$, or if one prefers, the unpaired nucleons are aligned with the nuclear spin 86.6% of the time. The uncertainty quoted is just the standard deviation of the results of the individual models.

The results of Ref. [15] for the nucleon polarization would agree with the above value, if the symbol $P_n^+$ that these authors call indistinctly the ''polarization of the neutron'' or ''the probability of the neutron to have its spin up,'' actually denoted the latter. Using the same averaged eight models the neutron polarization in $^6$Li would then be $N_{\parallel}=P_n^+-(1-P_n^+)=0.933-0.067=0.866$, as expected.

These results are in very good agreement with both the naive result of the first section, and with the very sophisticated Green's functions Monte Carlo prediction of Ref. [16], which obtained the integrated nucleon spin densities of 1.93 for spin up and 1.07 for spin down in $^6$Li with its nuclear spin up, or a net nucleon polarization of 0.86/3. The simplified estimate of Ref. [13] for the same quantity is 0.82/3.

$^7$Li was present at the 4.6% per weight level in the E155 Li$^2$H target. $^7$Li has $I^{\pi}=(\frac{3}{2})^-$, $\mu_{^7\mathrm{Li}}=3.26\mu_N$. It can be described in the single-particle shell model as a combination of 1 unpaired proton and 2 paired neutrons in the $p_{3/2}$ state, and a closed $s_{1/2}$ shell. $j-j$ (or $i-i$) coupling of the nucleons' spins and orbital angular momenta is dominant. The extreme shell model predicts $\mu_{^7\mathrm{Li}}=3.79\ \mu_N$, and the improved shell model for odd-$A$ nuclei predicts $\mu_{^7\mathrm{Li}}=3.44\mu_N$, with some admixture of other states [19]. In either case the agreement is reasonable (discrepancy is 16% or less). If we take the extreme shell-model configuration, then there are four $M=\pm 3/2,\pm 1/2$ substates, of equal probability. In the 3/2 substates the proton and the Li spins are parallel. The 1/2 substates are combinations of $m_l=\pm 1,0$ with $m_s=\pm 1/2$

$$\sqrt{\frac{1}{3}}|1,-1/2;\ \ 1/2\rangle+\sqrt{\frac{2}{3}}|0,\ \ 1/2;\ \ \ \ 1/2\rangle$$
$$\sqrt{\frac{1}{3}}|-1,1/2;-1/2\rangle+\sqrt{\frac{2}{3}}|0,-1/2;-1/2\rangle. \qquad (17)$$

It is readily seen that 2/3 of the time the proton and Li have parallel spins, and 1/3 antiparallel, for a net 1/3 of the time having parallel spins in the $M=\pm 1/2$ substates. Combining this result with the two $M=\pm 3/2$ substates, the proton and Li spins are aligned a total 2/3 of the time.

An alternative description of $^7$Li is in terms of an $\alpha + triton$ clusters [12]. In this case the $I=3/2$ is formed from $S=1/2$ for the triton orbiting in an $L=1$ state about the $\alpha$ cluster. The angular momentum decomposition discussed above in the shell model is valid in the cluster model as well, with the triton playing the role of the unpaired $p_{3/2}$ proton. Tritium is a very good equivalent of the free proton ($\mu_{^3\mathrm{H}}$ is only 7% off the proton's), because its nucleons have no Coulomb interaction. The results for tritium obtained earlier can be applied here, to correct the 2/3 polarization of tritium in $^7$Li taking into account the reduced (86%) polarization of the proton in $^3$H, to give a net 57% polarization of the proton in $^7$Li, and a $-2$% neutron polarization. More refined figures for both the proton and neutron polarizations in this nucleus have been obtained in Ref. [16], which predicts 88% for the proton and $-4$% for the neutrons. Combining the first figure with the 2/3 spin component of the $^7$Li angular momentum results in a net proton polarization of 59%.

### *Nitrogen*

$^{14}$N has $I^{\pi}=1^+$ like deuterium and $^6$Li. It is reasonably well described in terms of the shell model as a configuration of six nucleons in the $s_{1/2}$ and $p_{3/2}$ shells plus a proton or neutron in the $p_{1/2}$ shell. The straightforward application of $i-i$ coupling, predicts the magnetic moment to be

$$\mu_{o-o}=\frac{1}{2}\left[(g^p+g^n)+(g^p-g^n)\frac{j_p(j_p+1)-j_n(j_n+1)}{J+1}\right], \qquad (18)$$

where $\mu_{o-o}$ represents the magnetic moment for odd-odd nuclei [19], and the $g^{k=p,n}$ factors are

$$g^k=\frac{1}{2}\left[(g_l^k+g_s^k)+(g_l^k-g_s^k)\frac{l(l+1)-3/4}{i(i+1)}\right]. \qquad (19)$$

Substituting the free nucleon $g_s^k$ factors, $\mu_{o-o}=0.373\mu_N$, in good agreement with the measured value for $\mu_{^{14}N}$. The spin and orbital components of $\mu$ for the $p_{1/2}$ nucleons ($i=l-1/2$) can be obtained by examining the substates with $m_l=\pm 1,0$, $m_s=\pm 1/2$ [31]

$$\sqrt{\frac{2}{3}}|1,-1/2;\ \ 1/2\rangle+\sqrt{\frac{1}{3}}|0,\ \ 1/2;\ \ \ \ 1/2\rangle$$
$$\sqrt{\frac{2}{3}}|-1,1/2;-1/2\rangle+\sqrt{\frac{1}{3}}|0,-1/2;-1/2\rangle \qquad (20)$$

from which we see that the nucleon spin is aligned antiparallel to the nuclear spin a net 1/3 of the time (as in the $M=\pm 1/2$ substates in $^7$Li but with opposite nucleon orientation in this case). This result agrees in sign and is not too different in magnitude from the model independent estimate.

TABLE III. Nucleon polarizations.

| | Model independent | | Model dependent | | | | |
|---|---|---|---|---|---|---|---|
| Nuclide | This work | Ref. [10] | This work | Ref. [14] | Ref. [15] | Ref. [16] | Ref. [13] |
| $^2$H | 0.94 | | 0.926 | | | | |
| $^3$H-$^3$He | 0.93 | 0.96 | 0.86 | 0.865 | | | |
| $^6$Li | 0.85 | | 0.866 | | 0.866 [a] | 0.86 | 0.82 |
| $^7$Li | | | 0.57 | | | 0.59 | |
| $^{14}$N | −0.26 | | −0.33 | | | | |
| $^{15}$N | −0.22 | −0.24 | −0.33 | | | | |

[a] $P_n^+=0.933$

Nitrogen contributes with only one third of the polarizable nucleons in ammonia, and it polarizes only up to one sixth of the corresponding hydrogen or deuterium polarization. The net contribution of the polarized nucleons in $^{14}$N to the hydrogen or deuterium spin asymmetries is then $-1/3*1/3*1/6=-0.02$. Given this small size, the accuracy of about 10% of these simple estimates is quite adequate for the corrections needed in DIS spin structure experiments [3,1].

$^{15}$N has $I^\pi=\frac{1}{2}^-$. The shell model gives an even better description of $^{15}$N than of $^{14}$N. Its magnetic moment is less than 7% away from the Schmidt line value $-0.264\mu_N$. Thus we would expect the proton in $^{15}$N to be aligned antiparallel to the nuclear spin 1/3 of the time. The model-independent estimates for this isotope agree in sign and are of similar magnitude as the shell-model prediction. The correction to the spin asymmetries due to the presence of $^{15}$N are even smaller than those due to $^{14}$N, because in the former the neutron contributions are entirely negligible, and its nuclear polarization can be measured with better precision than that of the latter.

Table III summarizes the results of this and other works. The row for $A=3$ lists only the proton polarization for $^3$H (neutron polarization for $^3$He).

I would like to thank Keith Griffioen for calling my attention to Ref. [16], and B. Alex Brown, S. Coon, R. Wiringa, and D. G. Crabb for helpful discussions. This work was supported by Department of Energy Contract No. DEFG05-86ER40261 and by the Institute of Nuclear and Particle Physics of the University of Virginia.

[1] EMC, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).
[2] SLAC E142, P. L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993); Phys. Rev. D **54**, 6620 (1996).
[3] SLAC E143, K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995); **75**, 25 (1995); Phys. Rev. D **58**, 112003 (1998).
[4] SMC, D. Adams *et al.*, Phys. Lett. B **329**, 399 (1994); SMC, B. Adeva *et al.*, *ibid.* **302**, 533 (1993); **357**, 248 (1995).
[5] SLAC E154, K. Abe *et al.*, Phys. Rev. Lett. **79**, 26 (1996).
[6] HERMES collaboration, K. Ackerstaff *et al.*, Phys. Lett. B **404**, 383 (1997).
[7] SLAC E155, P. L. Anthony *et al.*, Phys. Lett. B (submitted).
[8] R. G. Sachs, Phys. Rev. **69**, 611 (1946).
[9] B. Alex Brown (private communication).
[10] Kenzo Sugimoto, Phys. Rev. **182**, 1051 (1969).
[11] B. A. Brown and B. H. Wildenthal, Phys. Rev. C **28**, 2397 (1983).
[12] H. Walliser and T. Fliessbach, Phys. Rev. C **31**, 2242 (1985).
[13] L. Frankfurt and M. I. Strikman, Nucl. Phys. **A405**, 557 (1983).
[14] J. L. Friar *et al.*, Phys. Rev. C **42**, 2310 (1990).
[15] N. W. Schellingerhout, L. P. Kok, S. A. Coon, and R. M. Adam, Phys. Rev. C **48**, 2714 (1993).
[16] B. S. Pudliner *et al.*, Phys. Rev. C **56**, 1720 (1997).
[17] Robert G. Sachs, *Nuclear Theory* (Addison-Wesley, Cambridge, MA, 1953), p. 247.
[18] W. Melnitchouk, G. Piller, and A. W. Thomas, Phys. Lett. B **346**, 165 (1995).
[19] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, MA, 1962), p. 445.
[20] Edgardo Browne and Richard S. Firestone, *Table of Radioactive Isotopes* (Wiley, New York, 1986).
[21] Salvatore Frullani and Jean Mougey, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. W. Vogt (Plenum, New York, 1984), Vol. 14, p. 101.
[22] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **49**, 1 (1987).
[23] Yoshio Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[24] H. Arenhövel, W. Leidemann, and E. L. Tomusiak, Z. Phys. A **331**, 123 (1988).
[25] C. Cioffi degli Atti *et al.*, Phys. Rev. C **48**, 968 (1993).
[26] TJNAF proposal 99-017, spokesperson C. E. Jones.
[27] L. P. Kaptari and A. Yu. Umnikov, Phys. Lett. B **240**, 203 (1990).
[28] G. L. Payne, J. L. Friar, B. F. Gibson, and I. R. Afnan, Phys. Rev. C **22**, 823 (1980).
[29] J. L. Friar, E. L. Tomusiak, B. F. Gibson, and G. L. Payne, Phys. Rev. C **24**, 677 (1981).
[30] J. V. Noble, Phys. Rev. C **9**, 1209 (1974).
[31] Ricardo Piegaia, Ph.D. thesis, Yale University, 1988.